PROBLEM SET 10 (DUE ON THURSDAY, NOV 14)

(All Exercises are references to the July 27, 2024 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 8.3.C(a) (quasicompactness is affine-local on the target)

Problem 2. Let $\pi : X \to Y$ be an affine morphism. Say that an affine open $U \subseteq Y$ has property P if the ring homomorphism $\mathcal{O}_Y(U) \to \mathcal{O}_X(\pi^{-1}(U))$ is surjective. Prove that if there exists a cover of Y consisting of affine opens with property P, then every affine open in Y has property P. (In other words, being a closed embedding can be checked on an affine open cover. As I explained in class, the Affine Communication Lemma converts this into an algebra exercise, which you should do.)

Problem 3. Exercise 8.3.E (an application of the affine-locality of affine morphisms)

Problem 4. Exercise 8.3.P(a) (conditions under which open embeddings are finite type - you will want to read the definition of locally Noetherian schemes in 5.3.4)

Problem 5. A *quadric* $V(f)$ in $\mathbb{P}^n_k$ is a closed subscheme cut out by a single homogeneous polynomial f of degree two (see 9.3.2). Give an example of two quadrics in $\mathbb{P}^2_{\mathbb{R}}$ intersecting in a single point, and compute the scheme-theoretic intersection. Then give a second example of this, with scheme-theoretic intersection not isomorphic (as $\mathbb{R}$-schemes) to that in your first example. Then give a third example with intersection not isomorphic to either of the first two! (Changes from last week's problem: $\mathbb{P}^2$ instead of $\mathbb{A}^2$; $\mathbb{R}$ instead of $\mathbb{C}$. It may be helpful to note that Bezout's theorem now directly applies and says that the intersection must be of the form $\operatorname{Spec} A$, where A is an $\mathbb{R}$-algebra that is 4-dimensional as an $\mathbb{R}$-vector space and has exactly one prime ideal.)