PROBLEM SET 0 (POSTED ON THURSDAY, AUG 28)

(All Exercises are references to the September 8, 2024 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Give a single example of a presheaf $\mathcal{F}$ that simultaneously fails both sheaf axioms (identity and gluability, as in Section 2.2.6).

Problem 2. Exercise 2.2.J (stalks of $\mathcal{O}_X$-modules are $\mathcal{O}_{X,p}$-modules - you will want to read Section 2.2.13 before doing this to learn what $\mathcal{O}_X$-modules are, as well as think through why stalks of sheaves of abelian groups are abelian groups)

Problem 3. Let $X = \mathbb{R}$. Let $\mathcal{F}$ be the sheaf on X of locally rational functions $f : U \to \mathbb{R}$, i.e. of functions $f : U \to \mathbb{R}$ such that for any point $p \in U$ there is an open neighborhood $p \in V \subseteq U$ and a rational function $g(t) \in \mathbb{R}(t)$ such that $f = g$ on V. Describe the stalk of $\mathcal{F}$ at the origin.