PROBLEM SET 2 (POSTED ON THURSDAY, SEP 11)

(All Exercises are references to the September 8, 2024 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Let $\pi : \mathbb{Q}[x] \to \mathbb{C}[x]$ be the ring homomorphism sending x to x. Let $\pi^* : \operatorname{Spec} \mathbb{C}[x] \to \operatorname{Spec} \mathbb{Q}[x]$ be the induced map of spectra. For each point $p \in \operatorname{Spec} \mathbb{Q}[x]$, describe the fiber $(\pi^*)^{-1}(p)$ (as a set).

Problem 2. Exercises 3.2.J and 3.2.K (essential algebra practice with prime ideals if you haven't done it before - see also 3.4.I(b) to take into account the Zariski topology as well)

Problem 3. Let $n > 0$ and let $\pi : \mathbb{Z} \to \mathbb{Z}[x_1, \dots, x_n]$ be the unique ring homomorphism. Let $\pi^* : \operatorname{Spec} \mathbb{Z}[x_1, \dots, x_n] \to \operatorname{Spec} \mathbb{Z}$ be the induced map of spectra. For each point $p \in \operatorname{Spec} \mathbb{Z}$, describe a bijection between the fiber $(\pi^*)^{-1}(p)$ and $\operatorname{Spec} k_p[x_1, \dots, x_n]$ for an appropriate field k_p depending on p. (Exercise 3.2.Q has some discussion and a picture that might be helpful.)

Problem 4. Exercise 3.5.B (covering $\operatorname{Spec} A$ with distinguished open sets)

Problem 5. Exercise 3.5.E (equivalent conditions to $D(f) \subseteq D(g)$)

Problem 6. Exercise 3.6.J(a) (sufficient conditions for the closed points in $\operatorname{Spec} A$ to be dense (As suggested in the hint, you will want to read the statement of Zariski's Lemma in 3.2.6.))