PROBLEM SET 3 (POSTED ON THURSDAY, SEP 18)

(All Exercises are references to the September 8, 2024 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 3.6.K (sometimes functions are determined by their values on closed points)

Problem 2. Exercise 3.7.G (irreducible components of Spec A correspond to minimal prime ideals in A)

Problem 3. Use the structure sheaf $\mathcal{O}_{\operatorname{Spec} A}$ to show that if Spec A is disconnected, then A is isomorphic to the product of two nonzero rings. (Be careful to show that the two rings are nonzero!)

Problem 4. Let $X = \operatorname{Spec} k[x, y, z]/(xz, yz)$ (draw a picture of X!) and let $U \subset X$ be the complement of the closed point $[(x, y, z)]$. Compute the ring $\mathcal{O}_X(U)$ along with the restriction map $\operatorname{res}_{X,U} : \mathcal{O}_X(X) \to \mathcal{O}_X(U)$. Is $\operatorname{res}_{X,U}$ isomorphic to some localization map $A \to S^{-1}A$?

Problem 5. Exercise 4.3.A (classifying isomorphisms of affine schemes)