PROBLEM SET 4 (POSTED ON THURSDAY, SEP 25)

(All Exercises are references to the September 8, 2024 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 4.3.F (functions on locally ringed spaces)

Problem 2. Let $X_1 = \operatorname{Spec} k[x, y]$ and $X_2 = \operatorname{Spec} k[w, z]$ be two copies of the affine plane over a field k. Let X be the scheme formed by gluing X_1 and X_2 along the isomorphism of open subschemes $\operatorname{Spec} k[x, x^{-1}, y] \cong \operatorname{Spec} k[w, w^{-1}, z]$ induced by the ring isomorphism $k[x, x^{-1}, y] \cong k[w, w^{-1}, z]$ given by $x \mapsto w, y \mapsto w^{-1}z$. Compute the ring of global sections of the structure sheaf of X. Is X affine?

Problem 3. Exercise 7.3.M (morphisms from Spec of a local ring)

Problem 4. Let k be a field. Classify all morphisms $\pi : \mathbb{P}^1_k \to \mathbb{P}^1_k$ that commute with the natural morphism $\mathbb{P}^1_k \to \operatorname{Spec} k$. (Note: this is saying that π is a "morphism of k-schemes". Hint: break into cases based on whether π maps the generic point to the generic point. If it does, look at the induced map of stalks there and use that to constrain what the morphism can look like on affine opens. If it doesn't, you can show that the map on points must be constant and work from there.)