

PROBLEM SET 1 (POSTED ON THURSDAY, SEP 10)

(All Exercises are references to the October 21, 2025 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** As in last week's problem set, give a single example of a presheaf $\mathcal{F}$ that simultaneously fails both sheaf axioms (identity and gluability, as in Section 2.2.6). Then compute its sheafification.
- Problem 2.** Exercise 2.2.J (stalks of $\mathcal{O}_X$ -modules are $\mathcal{O}_{X,p}$ -modules - you will want to read Section 2.2.13 before doing this to learn what $\mathcal{O}_X$ -modules are)
- Problem 3.** Exercise 2.7.G (support of a section is closed - you will want to read Section 2.7.6 before doing this)
- Problem 4.** Exercise 2.3.C (sheaf Hom)
- Problem 5.** Exercise 2.4.D (isomorphisms are determined by stalks)
- Problem 6.** Exercise 2.5.E (gluing sheaves together (very important) - the hint given by Vakil will require reading the preceding part of Section 2.5, but you can also do this without using a base if you prefer)