

PROBLEM SET 4 (POSTED ON THURSDAY, OCT 1)

(All Exercises are references to the October 21, 2025 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 4.3.F (functions on locally ringed spaces)

Problem 2. Let $X_1 = \operatorname{Spec} k[x, y]$ and $X_2 = \operatorname{Spec} k[w, z]$ be two copies of the affine plane over a field k . Let X be the scheme formed by gluing X_1 and X_2 along the isomorphism of open subschemes $\operatorname{Spec} k[x, x^{-1}, y] \cong \operatorname{Spec} k[w, w^{-1}, z]$ induced by the ring isomorphism $k[x, x^{-1}, y] \cong k[w, w^{-1}, z]$ given by $x \mapsto w, y \mapsto w^{-1}z$. Compute the ring of global sections of the structure sheaf of X . Is X affine?

Problem 3. Take your solution to Exercise 4.3.A (classifying isomorphisms of affine schemes) from last problem set and see if you can refine it to classify all morphisms between affine schemes. In other words, construct a bijection between morphisms $\operatorname{Spec} A \rightarrow \operatorname{Spec} B$ and ring homomorphisms $B \rightarrow A$. (This is Proposition 7.3.2, so you can read the proof there if you get stuck.)

Problem 4. Exercise 7.3.M (morphisms from Spec of a local ring)

Problem 5. Let k be a field. Classify all morphisms $\pi : \mathbb{P}_k^1 \rightarrow \mathbb{P}_k^1$ that commute with the natural morphism $\mathbb{P}_k^1 \rightarrow \operatorname{Spec} k$. (Note: this is saying that π is a “morphism of k -schemes”. Hint: break into cases based on whether π maps the generic point to the generic point. If it does, look at the induced map of stalks there and use that to constrain what the morphism can look like on affine opens. If it doesn’t, you can show that the map on points must be constant and work from there. In both cases, you will need to actually construct the morphisms in your classification.)