INFO ON FINAL PROJECT FOR 632

First, the basics: you should spend time in the next few weeks reading about and thinking about an additional topic of your choice in algebraic geometry, and then either write a short (3-10 pages) final paper or schedule an informal oral presentation/exam with me (30-45 minutes). In either case, this should be completed by the last day of classes, April 20.

Algebraic geometry is a very wide subject - there are lots of different directions to go, and many things that we won't have time to do this semester. I see this as an opportunity for you to pick one of these directions that interests you and learn a little more about it. You should pick a topic, learn about it, and then pick a key result or interesting example and write an exposition (if choosing the paper option) of the parts of the topic needed to get to the result/example.

The option of writing a paper is intended as practice with mathematical writing, and you may also find that writing things down helps you learn the topic more deeply. I'll also provide feedback on the writing (including any drafts that you ask me to read before the completed product). But if you feel like you are already getting plenty of writing practice, don't want another writing project, or would otherwise prefer to do an oral exam instead, the procedure there is similar in that you should have some key results/interesting examples in mind when you come to our meeting. I'll probably ask you what they are, and then we'll have an informal discussion about what is involved with proving/computing them - if you like you can structure this as a short presentation on the subject, and I'll interject with questions.

A couple final words about the writing: it certainly isn't required to write down every single necessary detail along the way - you will have to use your judgment for that, just as any writer of math does. The idea is just to write a treatment of the topic that feels clear and natural to you. In cases where the topic corresponds to sections in Vakil, this will probably involve figuring out how to prove some of the exercises (or figuring out alternative ways of getting to the target results).

Here are some sample options for topics (though you are very welcome to choose your own topic - almost anything inside or related to algebraic geometry is fine - and if you have vague directions in mind, I can try to help you narrow things down):

- basic intersection theory (Chapter 20 of Vakil, possibly with some of the results in Section 20.4 as a goal)
- blowups/singularities (Chapter 22 of Vakil and also possibly some of Chapter 29; one possibility is to work out some of the ADE-surface singularity examples in 22.4.F and 22.4.5)
- derived functors (Chapter 23 of Vakil - one possible goal is Theorem 23.5.1)
- cubic surfaces (Chapter 27 of Vakil - the idea would be to understand and sketch the argument behind either Theorem 27.1.1 or Theorem 27.1.2)
- the classification of genus 5 curves as hyperelliptic, trigonal, or complete intersections of three quadrics (one possible source for this is Geometry of Algebraic Curves by

Arbarello, Cornalba, Griffiths, and Harris, which outlines how to prove a more general theorem (the Enriques-Babbage Theorem) in Section III.3)

- Hilbert schemes (there is a little bit about these in Section 28.3 of Vakil, but you will need to look elsewhere for an actual construction - one possible reference is `https://homepages.warwick.ac.uk/staff/D.Maclagan/papers/HilbertSchemesNotes.pdf`)
- something from arithmetic geometry (this is a huge field with lots of directions, but one possible source would be to read some of Arithmetic of Elliptic Curves by Silverman)
- connections with topology, e.g. genus of an algebraic curve equals topological genus of the corresponding Riemann surface (Exercise 21.7.I outlines how you might prove this single fact, but you might want to do more than just this - for example, can you give a (probably non-rigorous) explanation of why compact Riemann surfaces should all come from algebraic curves? I can give a reference or hints for this if desired.)
- low genus examples of moduli of curves, e.g. understanding genus 4 curves better by describing how to describe hyperelliptic curves as limits of non-hyperelliptic curves (I can provide more details on request)
- even lower genus examples of moduli of stable curves (this is a very combinatorial and beautiful subject - I like Vakil's introduction to this in the first 8 pages or so of `http://math.stanford.edu/~vakil/files/notices/ams4.pdf`, but there are many other references out there. One possible topic is describing how $\overline{M}_{0,n+3}$ can be viewed as an iterated blowup of $\mathbb{P}^n$, while another is describing the relationship between $\overline{M}_2$ and $\overline{M}_{0,6}$ - I can provide more details on request.)

I'm also happy to talk about any of these topics that might interest you even if you aren't actually choosing them for your final project!