

### PROBLEM SET 3 (DUE ON THURSDAY, FEBRUARY 25)

(All Exercises are references to the November 18, 2017 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** Exercise 16.3.H (the projection formula)
- Problem 2.** Exercise 16.4.B (automorphisms of  $\mathbb{P}^n$ )
- Problem 3.** Exercise 16.4.J (dimensions of images of maps from projective space)
- Problem 4.** Exercise 16.6.C (very ample  $\otimes$  base-point-free is very ample)
- Problem 5.** Let  $X = \text{Bl}_{(0,0)} \mathbb{A}_k^2$  be the blow-up of the affine plane at the origin. Let  $p : X \rightarrow \mathbb{A}_k^2$  be the blow-up map. Let  $E$  be the exceptional divisor of  $X$  (the fiber of  $p$  above the origin), so  $\text{Pic}(X)$  is generated by  $\mathcal{O}_X(E)$ . Determine all integers  $d$  such that there exists a morphism  $\pi : X \rightarrow \mathbb{P}_k^1$  with  $\pi^* \mathcal{O}_{\mathbb{P}_k^1}(1) \cong \mathcal{O}_X(dE)$ . Which of these morphisms factor through  $p$ ?