PROBLEM SET 2 (DUE ON THURSDAY, JANUARY 31)

(All Exercises are references to the December 31, 2022 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Let $D \subseteq X$ be an effective Cartier divisor. Let $\mathcal{L} = \mathcal{I}_D^\vee$. Let s be the global section of $\mathcal{L}$ given by the inclusion of the ideal sheaf $\mathcal{I}_D$ into $\mathcal{O}_X$. Show that the vanishing scheme of s is equal to D.

Problem 2. Exercise 15.2.F (div and $D \mapsto O_X(D)$ are inverses)

Problem 3. Exercise 15.2.Q (computing $\mathrm{Pic}(\mathbb{P}^1 \times \mathbb{P}^1)$ - in the hint given by Vakil, "restricts to" can be taken to mean "pulls back by the inclusion morphism")

Problem 4. Let $X = \mathrm{Bl}_{(0,0)} \mathbb{A}_k^2$ be the blow-up of the affine plane at the origin (as described in Exercise 10.3.F). Compute $\mathrm{Pic}(X)$.