PROBLEM SET 4 (DUE ON THURSDAY, FEBRUARY 16)

(All Exercises are references to the December 31, 2022 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 17.3.H (the projection formula)

Problem 2. Exercise 17.6.C (very ample $\otimes$ base-point-free is very ample)

Problem 3. Let $X = \mathrm{Bl}_{(0,0)} \mathbb{A}^2_k$ be the blow-up of the affine plane at the origin. Let $p : X \to \mathbb{A}^2_k$ be the blow-up map. Let E be the exceptional divisor of X (the fiber of p above the origin), so $\mathrm{Pic}(X)$ is generated by $\mathcal{O}_X(E)$. Determine all integers d such that there exists a morphism $\pi : X \to \mathbb{P}^1_k$ with $\pi^* \mathcal{O}_{\mathbb{P}^1_k}(1) \cong \mathcal{O}_X(dE)$. Which of these morphisms factor through p?