PROBLEM SET 3 (POSTED ON THURSDAY, JANUARY 29)

(All Exercises are references to the October 21, 2025 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 15.5.K (computing $\mathrm{Pic}(\mathbb{P}^1 \times \mathbb{P}^1)$ - in the hint given by Vakil, "restricts to" can be taken to mean "pulls back by the inclusion morphism")

Problem 2. Let $X = \mathrm{Bl}_{(0,0)} \mathbb{A}^2_k$ be the blow-up of the affine plane at the origin (as described in Exercise 10.3.F). Compute $\mathrm{Pic}(X)$.

Problem 3. Classify all morphisms (of quasicoherent sheaves on $\mathbb{P}^1_k$)

$$\mathcal{O}_{\mathbb{P}^1_k}(m) \to \mathcal{O}_{\mathbb{P}^1_k}(n)$$

for $m, n \in \mathbb{Z}$.

Problem 4. Exercise 15.7.H (every quasicoherent sheaf on a projective scheme comes from a graded module)