

PROBLEM SET 4 (POSTED ON THURSDAY, FEBRUARY 5)

(All Exercises are references to the October 21, 2025 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 14.5.L (the projection formula)

Problem 2. Exercise 15.5.A (automorphisms of \mathbb{P}^n)

Problem 3. Exercise 15.5.E (dimensions of images of maps from projective space)

Problem 4. Let $X = \text{Bl}_{(0,0)} \mathbb{A}_k^2$ be the blow-up of the affine plane at the origin. Let $p : X \rightarrow \mathbb{A}_k^2$ be the blow-up map. Let E be the exceptional divisor of X (the fiber of p above the origin), so $\text{Pic}(X)$ is generated by $\mathcal{O}_X(E)$. Determine all integers d such that there exists a morphism $\pi : X \rightarrow \mathbb{P}_k^1$ with $\pi^* \mathcal{O}_{\mathbb{P}_k^1}(1) \cong \mathcal{O}_X(dE)$. Which of these morphisms factor through p ?