PROBLEM SET 7 (POSTED ON THURSDAY, FEBRUARY 26)

(All Exercises are references to the October 21, 2025 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Does there exist a quasicoherent sheaf $\mathcal{F}$ on $\mathbb{P}^2_k$ with $H^1(\mathbb{P}^2_k, \mathcal{F}) \neq 0$?

Problem 2. Let $X = \mathrm{Bl}_{(0,0)} \mathbb{A}^2_k$. Let E be the exceptional divisor. Compute $H^1(X, \mathcal{O}_X(dE))$ for every integer d.

Problem 3. Let $C = \operatorname{Proj} \mathbb{C}[x,y,z]/(x^3+y^3+z^3)$. Compute $V := H^1(C, \mathcal{O}_C)$. Let G be the automorphism group of C (viewing C as a $\mathbb{C}$-scheme). Define a natural action of G on V. (Warning: this is a bit tricky because you will have to compare Cech cohomology computed using two different affine open covers!) Does there exist an element of G acting nontrivially on V? What about a nontrivial element of G acting trivially on V?