

PROBLEM SET 9 (POSTED ON THURSDAY, MARCH 19)

(All Exercises are references to the October 21, 2025 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** Exercise 19.2.C (deleting a point makes a curve affine)
- Problem 2.** Suppose C is an integral regular projective curve of genus 1 over an algebraically closed field k . Let L be a line bundle of degree 4 on C . Show that L identifies C with the intersection of two quadric surfaces in \mathbb{P}_k^3 .
- Problem 3.** Suppose C is an integral regular projective nonhyperelliptic curve over an algebraically closed field k . Let p_1, p_2, p_3 be distinct closed points in C . Show that p_1, p_2, p_3 are collinear in the canonical embedding of C if and only if there exists a degree 3 morphism $\pi : C \rightarrow \mathbb{P}_k^1$ with $\pi(p_1) = \pi(p_2) = \pi(p_3)$.
- Problem 4.** Suppose C is an integral regular projective curve of genus $g \geq 2$ over an algebraically closed field k . Prove that there exists a degree 1 line bundle L on C with $h^0(C, L) = 0$.
- Problem 5.** Suppose C is an integral regular projective curve of genus 2 over an algebraically closed field k . Prove that C is trigonal (i.e. there exists a degree 3 morphism $\pi : C \rightarrow \mathbb{P}_k^1$).