PROBLEM SET 2 (DUE TUESDAY, NOVEMBER 29)

If you have any questions about any of the problems (either clarifications about what I mean, or concerns about not having the right technical knowledge to think about the problem properly), please let me know. In general, these problems are designed to give practice working with moduli spaces of stable curves rather than to challenge your technical knowledge, and you should worry more about understanding what is going on than about having precise justifications for everything.

Problem 1. It is a theorem of Hurwitz that a Riemann surface of genus $g > 1$ has at most $84(g-1)$ automorphisms. We know that stable curves of genus g have finitely many automorphisms, but do they satisfy the same $84(g-1)$ bound? (Can you construct a sequence of stable curves of genus $g = 2, 3, 4, \dots$ with automorphism group size growing faster than linearly in g?)

Problem 2. Describe the (complex) codimension 2 boundary strata in $\overline{M}_{2,1}$. (There are five of them, corresponding to topological types of stable curves with exactly two nodes.) Find two of them whose closures are disjoint. (One trick that might help with thinking about this - if the closures intersect, then the intersection has to contain one of the zero-dimensional boundary strata, which are just three points in $\overline{M}_{2,1}$.)

Problem 3. Recall that M_2 and $M_{0,6}/S_6$ are homeomorphic (though they have different orbifold structures). It turns out that this homeomorphism extends to the compactifications $\overline{M}_2$ and $\overline{M}_{0,6}/S_6$, and it respects the boundary stratifications of these spaces, giving a bijection between the seven boundary strata of $\overline{M}_2$ and the seven S_6-orbits of the boundary strata of $\overline{M}_{0,6}$. Try to describe this homeomorphism and/or bijection, either in a unified way (e.g. by explaining a procedure for constructing a stable genus 2 curve in terms of a double curve of a stable genus 0 curve with six unordered marked points) or by giving it explicitly for each topological type.

Problem 4. Recall that smooth genus 3 curves come in two types - the smooth plane quartic curves (which are dense) and the hyperelliptic curves (which are complex codimension 1). Let $\overline{H}_3 \subset \overline{M}_3$ be the closure of the hyperelliptic locus in $\overline{M}_3$. Since the hyperelliptic locus was codimension 1, $\overline{H}_3$ cannot contain any of the boundary divisors (boundary strata with exactly one node) of $\overline{M}_3$. Does it contain any of the codimension 2 boundary strata?