PROBLEM SET 3 (DUE THURSDAY, DECEMBER 8)

Problem 1. Although the tropical plane curves I drew in class all had every vertex of valence 3, this isn't true in general - for example, the tropical plane conic $V_{\text{trop}}(x^2 + x + y^2 + y + t)$ has one vertex of valence 4 and one vertex of valence 3. What is the maximum possible valence of a vertex in a tropical plane cubic? Give an example that attains this maximum, either by drawing a picture or by writing a cubic equation. (Hint: think about the corresponding regular subdivision of the isosceles right triangle with vertices at $(0,0),(3,0),(0,3)$.)

Problem 2. Describe the maximal cones in $M_{2,1}^{\text{trop}}$ - there are 3 of them. One definition of what it means for a tropical curve to be hyperelliptic is as follows: there exists an involution (a self-isometry of the metric graph) such that the quotient space is a tree. Describe the hyperelliptic locus in $M_{2,1}^{\text{trop}}$. (To make things simpler, feel free to only describe what is happening in the interiors of the 3 maximal cones.) What is its dimension?