PROBLEM SET 5 (DUE ON OCT 19)

(All Exercises are references to *Introduction to Commutative Algebra* by M. Atiyah and I. Macdonald.)

Problem 1. Chapter 7, Exercise 1 (a ring is Noetherian if and only if every prime ideal is finitely generated)

Problem 2. Chapter 7, Exercise 2 (description of nilpotent power series over a Noetherian ring)

Problem 3. Let A be a Noetherian ring. Prove that A has only finitely many minimal prime ideals (i.e. prime ideals that do not contain other prime ideals). (Hint: Let S be the set of all ideals $I \subseteq A$ such that A/I has infinitely many minimal prime ideals. If S is nonempty, you can take a maximal element I of S and use the fact that I cannot be prime to construct a larger element of S, giving a contradiction.)