PROBLEM SET 11 (DUE ON DEC 13)

(All Exercises are references to the December 29, 2015 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 13.7.F (local freeness can be checked at stalks)

Problem 2. Exercise 13.7.K (finite type quasicoherent sheaves of constant rank are vector bundles)

Problem 3. Exercise 14.1.D (classifying invertible sheaves on $\mathbb{P}^1_k$)

Problem 4. Classify all morphisms (of quasicoherent sheaves on $\mathbb{P}^1_k$)

$$\mathcal{O}_{\mathbb{P}^1_k}(m) \to \mathcal{O}_{\mathbb{P}^1_k}(n)$$

for $m, n \in \mathbb{Z}$.

Problem 5. Let $S_\bullet = \mathbb{C}[x,y,z,w]/(xw - yz)$ be a graded ring, where we take $\deg x = \deg y = 0$ and $\deg z = \deg w = 1$. Let $M_\bullet$ be the graded $S_\bullet$-module given by the ideal (xw) of $S_\bullet$. Let $\widetilde{M_\bullet}$ be the corresponding quasicoherent sheaf on $\operatorname{Proj} S_\bullet$ (as defined in Section 15.1). Show that $\widetilde{M_\bullet}$ is a line bundle and compute its base locus. (Hint: Use the affine charts $\operatorname{Proj} S_\bullet = D(z) \cup D(w)$.) (Addendum: If you prefer, you can view $M_\bullet$ as the degree shift $S(-1)_\bullet$, so $\widetilde{M_\bullet}$ also goes by the name $\mathcal{O}_{\operatorname{Proj} S_\bullet}(-1)$.)