PROBLEM SET 3 (DUE ON OCT 6)

(All Exercises are references to the December 29, 2015 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 3.7.E (irreducible closed subsets correspond to prime ideals)

Problem 2. Prove that if $\operatorname{Spec} A$ is disconnected then A is isomorphic to the product of nonzero rings A_1 and A_2. (Hint: use the sheaf axioms for the structure sheaf of $\operatorname{Spec} A$. You might also find Remark 3.6.3 helpful.)

Problem 3. Let $X = \operatorname{Spec} k[x, y, z]/(xz, yz)$ and let $U \subset X$ be the complement of the closed point $[(x, y, z)]$. Compute the ring $\mathcal{O}_X(U)$. Is U a distinguished open set?

Problem 4. Exercise 4.3.A (classifying isomorphisms of affine schemes)

Problem 5. Exercise 4.3.G (functions on locally ringed spaces)

Problem 6. Let $X_1 = \operatorname{Spec} k[x, y]$ and $X_2 = \operatorname{Spec} k[w, z]$ be two copies of the affine plane over a field k. Let X be the scheme formed by gluing X_1 and X_2 along the isomorphism of open subschemes $\operatorname{Spec} k[x, x^{-1}, y] \cong \operatorname{Spec} k[w, w^{-1}, z]$ induced by the ring isomorphism $k[x, x^{-1}, y] \cong k[w, w^{-1}, z]$ given by $x \mapsto w, y \mapsto w^{-1}z$. Compute the ring of global sections of the structure sheaf of X. Is X affine?