PROBLEM SET 2 (DUE ON THURSDAY, OCT 4)

(All Exercises are references to the November 18, 2017 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Let $\pi : \mathbb{Q}[x] \to \mathbb{C}[x]$ be the ring homomorphism sending x to x. Let $\pi^* : \operatorname{Spec} \mathbb{C}[x] \to \operatorname{Spec} \mathbb{Q}[x]$ be the induced map of spectra. For each point $p \in \operatorname{Spec} \mathbb{Q}[x]$, describe the fiber $(\pi^*)^{-1}(p)$ (as a set).

Problem 2. Let $n > 0$ and let $\pi : \mathbb{Z} \to \mathbb{Z}[x_1, \ldots, x_n]$ be the unique ring homomorphism. Let $\pi^* : \operatorname{Spec} \mathbb{Z}[x_1, \ldots, x_n] \to \operatorname{Spec} \mathbb{Z}$ be the induced map of spectra. For each point $p \in \operatorname{Spec} \mathbb{Z}$, describe a bijection between the fiber $(\pi^*)^{-1}(p)$ and $\operatorname{Spec} k_p[x_1, \ldots, x_n]$ for some field k_p. (Exercise 3.2.Q has some discussion and a picture that might be helpful.)

Problem 3. Exercise 3.6.J (when are the closed points in $\operatorname{Spec} A$ dense?)

Problem 4. Exercise 3.6.K (sometimes functions are determined by their values on closed points)

Problem 5. Exercise 3.7.E (irreducible closed subsets correspond to prime ideals)

Problem 6. Let $X = \operatorname{Spec} k[x, y, z]/(xz, yz)$ and let $U \subset X$ be the complement of the closed point $[(x, y, z)]$. Compute the ring $\mathcal{O}_X(U)$ along with the restriction map $\operatorname{res}_{X,U} : \mathcal{O}_X(X) \to \mathcal{O}_X(U)$. Is $\operatorname{res}_{X,U}$ isomorphic to some localization map $A \to S^{-1}A$?