Quantized orbits in weakly coupled Belousov-Zhabotinsky reaction - Supplementary material

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I. STABILITY ANALYSIS FOR TWO SPIRALS ROTATING IN THE SAME DIRECTION

Here we want to examine the stability of the solutions of eq. 9 for which $|\Delta z_{k_0}|$ is constant, *i.e.*, eq. 11. For this, we assume a small variation of the steady solution, of the form

$$\begin{aligned} |\Delta z_k| &= |\Delta z_{k_0}| + \varepsilon_k \\ &= \frac{\lambda}{2\pi}\left(\frac{\pi}{2} + m\pi - \varphi\right) + \varepsilon_k, \text{ with } \varepsilon_k \ll 1. \end{aligned} \tag{14}$$

Inserting 14 into eq. 9 (in the paper) and considering only the square of the absolute values, we write:

$$\begin{aligned} (|\Delta z_{k_0}| + \varepsilon_{k+1})^2 =& (|\Delta z_{k_0}| + \varepsilon_k)^2 + (2h\cos(\Delta\psi))^2 \\ &+ 4h\cos(\Delta\psi)(|\Delta z_{k_0}| + \varepsilon_k)\cdot\cos(\varphi + \frac{2\pi}{\lambda}|\Delta z_{k_0}| + \varepsilon_k\frac{2\pi}{\lambda})]. \end{aligned} \tag{15}$$

Since ε_k and also h are assumed to be small ($\ll 1$), we drop all higher order terms of these quantities. Reformulations of (15) gives then:

$$\varepsilon_{k+1} = \varepsilon_k - (-1)^m 2h\cos(\Delta\psi)\left(1 + \frac{\varepsilon_k}{|z_{k_0}|}\right)\cdot(\varepsilon_k 2\pi/\lambda). \tag{16}$$

ε_k and $|z_{k_0}|$ are positive. Thus, if $\cos(\Delta\psi) > 0$ [$\cos(\Delta\psi) < 0$], for even [odd] m it is $\varepsilon_{k+1} < \varepsilon_k$ and thus $|\Delta z_{k_0}|$ is stable. For other m, $|\Delta z_{k_0}|$ is unstable.

Having this said, we show in fig. 1 the vector field of $\delta\Delta z_k$ as a function of Δz_k (eq. 9) for positive $\cos(\Delta\psi)$. There, the stable and unstable limit cycles are clearly visible.

II. VECTOR FIELD OF $\delta\Delta z$ FOR COUNTER-ROTATING SPIRALS

Figure 2 shows the field of $\delta\Delta z_k$ as a function of Δz_k (eq. 12). It is clearyly visible that there are fixpoints at $|\Delta z_{k_0}|$ as described by eq. 13. Depending on the angle $arg(\Delta z_{k_0})$, these points can be both stable and unstable.

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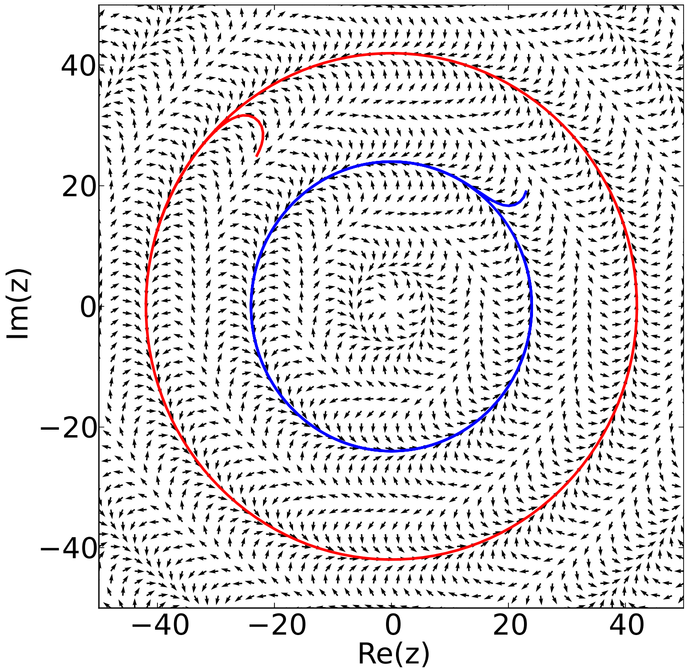

FIG. 1. Orientation of $\delta\Delta z$ as a function of Δz, regarding eq. (9), for positive $h\cos(\Delta\psi) = 1$. The red and blue curves show the evolution of Δz with time, as it settles on the circular limit cycle starting from different initial conditions.

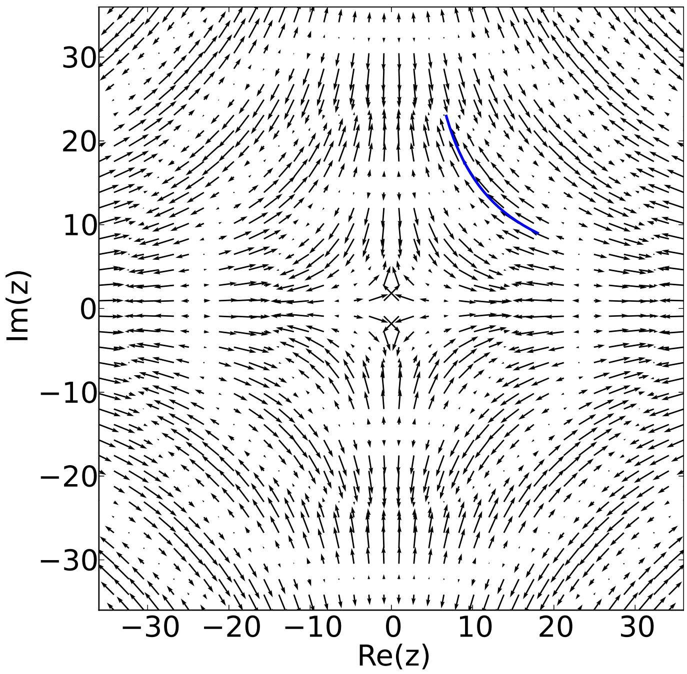

FIG. 2. Orientation of $\delta\Delta z$ as a function of Δz, regarding eq. (12), for $h = 0.1$ and $\cos(\Delta\psi) = 0$. The blue curve shows the evolution of Δz with time, starting from its initial value of (x=18, y= 9).