



The Conditional Probability of Mortgage Default

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This research examines the implications of contingent-claims models for empirical research on default. We focus on the probability of default over a short horizon given the current state of the world, *i.e.*, the conditional probability of default, which more closely resembles the estimates of empirical models. We highlight the differences between the conditional and unconditional approaches and provide guidance for empirical research by illuminating situations where the expected sign reverses over the shorter horizon or where the functional form is highly nonlinear.

While the importance of the embedded options in a mortgage contract has been recognized for two decades (Findlay and Capozza 1977; Asay 1978), the analysis of default probabilities using contingent claims models is more recent (Kau, Keenan and Kim 1993, 1994). Nevertheless, the link between the theoretical models of default and the empirical tests remains weak. Typically, theoretical models have focused on the unconditional probability of default.¹ Unconditional probabilities are most suitably tested using data on the cumulative defaults on mortgage loans over the entire 30-year life of the loans. However, empirical work commonly focuses on conditional probabilities over short horizons by using annual default data for seasoned loans.

Conditional probabilities are important for three reasons. First, most empirical work analyzes conditional probabilities. Second, conditional

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¹ We use the term "unconditional" when the default probabilities are conditioned only on the information available at the time of origination. We use "conditional" to describe the default probability conditioned not just on whether the loan is alive but on all available information that has arrived since origination, such as the current loan-to-value ratio and recent economic conditions. Unconditional probabilities are sufficient for unseasoned loans, while conditional probabilities are essential for seasoned loans.

probabilities are essential for pricing mortgage loans. Third, unconditional probabilities can be obtained from the conditional by integration of the state variables.

In this research we explore the conditional probabilities in the context of a contingent claims model. Our primary goal is to provide guidance on functional form and specification of the independent variables for empirical research on default. Our results highlight situations where the importance of some independent variables is greatly diminished or where care must be taken in specifying functional form. The effect of transactions costs and trigger events is also examined in a precise way.

The results show that not only is the relationship between default and several of its determinants highly nonlinear, but also the magnitude and even the sign depend on the horizon. We show that some variables, like the volatility of house prices, which are important unconditionally, are not very important conditionally. For other variables, like the current interest rate, interaction terms are essential.

This article is organized as follows. The next section presents the contingent claims model that is solved by backward induction to determine the optimal stopping boundary. The third section presents and discusses the simulation results. The final section is the conclusion.

The Model

To capture the competing hazards of prepayment and default, we employ a two-factor model. House prices are assumed to follow geometric Brownian motion, and interest rates to follow the commonly used Ornstein–Uhlenbeck mean reverting process.² The simulations use discrete monthly time steps so that once each month, just prior to a mortgage payment, the mortgagor decides whether to prepay, default or make the scheduled mortgage payment.³ We abstract from issues surrounding delinquency or delay by

² The Ornstein–Uhlenbeck process is one of the two most commonly used interest-rate models. It is a special case of the general class of models described in Chan *et al.* (1992). Both mean reversion and volatility are captured by the model. Chen and Yang (1995) point out that “mortgage pricing models are less sensitive to the underlying interest rate process than a simple coupon bond because of the non-financial prepayment feature.”

³ In actuality, the decision to prepay can occur at any time. It is optimal to default only at a payment date.

assuming that default results in immediate loss of the house in exchange for forgiveness of the debt.⁴

House Price Process

House prices (H) are assumed to follow the process

$$dH = (g - \gamma)H dt + \sigma_H H dW \quad (1)$$

g = Required return on housing given its risk.

γ = Rental rate or "rent-to-price" ratio for the house (analogous to the dividend rate on common stock).

σ_H = Volatility of house prices.

W = Standard Brownian motion.

Hedging arguments (*e.g.*, Hull 1993) yield the risk-neutral pricing process given by

$$dH = (r - \gamma)H dt + \sigma_H H dV \quad (2)$$

r = Risk-free interest rate.

V = An alternate Brownian motion.

The risk-neutralized drift of the house price process depends not on the gross return to housing, but rather on the risk-free interest rate. For consistency, the risk-neutralized house price process is used when modeling the default decision; however, the original process is used when modeling the probability of default.⁵

Interest-Rate Process

Interest rates are also stochastic in the model. Interest-rate changes have two influences on default decisions. First, if interest rates rise, default is less likely because the mortgagor values the low-cost mortgage and will hesitate

⁴ Ambrose, Buttimer and Capone (1996b) point out that when a borrower decides to technically default by missing scheduled payments, the motivation can be either an attempt to exercise the implicit put option or a decision to use mortgage delinquency to finance other expenditures. Default in the context of our model is a foreclosure and sale by the lender, so that delinquency and delay to finance other expenditures is not modeled.

⁵ See Kau, Keenan and Kim (1994) for an elaboration.

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making payments. In this case the transactions costs remain relevant, since default will necessitate a move that need not occur otherwise.⁸

Correlation between Interest Rates and House Prices

Our model for predicting mortgage defaults is a two-stochastic-state-variable model that incorporates transactions costs and exogenous termination. We assume zero correlation between house prices and interest rates. This is consistent with the empirical evidence. Studies of house prices (*e.g.*, Haurin, Hendershott and Kim 1991) show that there is considerable variation in house price movements among metropolitan areas, even though all markets face the same interest-rate environment. Kau, Keenan and Kim (KKK) (1994) present reasons why house prices could be positively or negatively correlated with interest rates. To explore this issue further, we analyzed the large panel data set of 64 metropolitan areas from 1979 to 1994 described in Capozza, Kazarian and Thomson (CKT) (1997). We were unable to find any R^2 greater than .02 when house prices were regressed on interest rates in various specifications, including levels, first differences, real and nominal interest rates and house prices. The lack of correlation at the local level is not surprising, since local markets are known to have housing cycles widely displaced in time (*e.g.*, Texas and New England) while interest rates vary very little by metropolitan area.

Simulating Empirical Default

Empirical analysis of mortgage default data is usually based on annual data for seasoned loans rather than on newly originated loans. The default rate is computed by dividing the number of loans that default by the number of loans at risk that year. To approximate the empirical default rate in the simulations, we compute the probability that a loan is still alive (*i.e.*, neither prepaid nor defaulted at the beginning of each 12-month period). We then sum the default probabilities for the next 12 months to determine the probability that default will occur over the next full year. The annual default rate is the probability of default over the next 12 months divided by the probability that the loan was alive, *i.e.*, not prepaid or defaulted, at the beginning of the period.

We initiate the analysis by requiring that the mortgage be fairly priced. The coupon rate on a loan at origination must set the present value of payments

⁸ Ambrose, Buttimer and Capone (1996a) state that transaction costs can be negative if foreclosure takes time and the borrower can live rent-free until the process is completed.

(including default and prepayment options, and the effect of exogenous terminations) equal to the principal amount. The model parameters are then varied individually from their base-case values to determine the effect of changing a parameter's value when all other parameters are held constant. The results are presented graphically to facilitate assessments of the direction, the strength and the linearity of the relationships.

The Simulations

The model evaluates conditional default probabilities for seasoned mortgages. The ranges of parameter values were chosen to be realistic (*e.g.*, 360-month mortgages) and to encompass those used in other studies. Table 1 presents the base case and range of parameters. Relevant parameters include the levels and volatilities of both house prices and interest rates, the rental rate, the interest-rate reversion parameter, transactions costs and trigger events.

The method of solution is described in the Appendix. The results are presented for 5-year-old mortgages having a term of 10 years. As indicated earlier, since mortgage age conditional on the other parameters does not have a large effect on default probabilities over the next year, the short horizon results for these mortgages are representative of the results for other mortgage ages.⁹

Because few defaults occur for houses with low current loan-to-value ratios (CLTVs), we assess default rates for high-CLTV loans only. We present results for the stochastic house price process (house price volatility and dividend rate), the interest-rate process (level of interest rate, interest-rate volatility, interest-rate reversion), transaction costs of default and refinance, and the exogenous termination rate. In each figure, three panels depict the default rates measured over a 1-year and 10-year horizon for three CLTVs (90%, 100%, 110%). The 10-year horizon is representative of the long-horizon or cumulative probability of default. The 1-year horizon is intended to reflect the conditional default rates found in annual default data. Note that the vertical scale changes from panel to panel in each figure.

⁹ This is not to say that age does not affect default probabilities. Age has a large effect on default probability unconditionally (*i.e.*, given only the information at origination). However, once current information is known, *e.g.*, the current LTV, the expected defaults over the next year are similar for all loan ages.

Table 1 ■ Base-case parameters for numerical modeling.

Variable	Base Case	Range
Initial house price, H_0 (\$)	100,000	
Initial loan amount, MB_0 (\$)	90,000	
Contract mortgage rate (%)	0.85 monthly (10.2 annual rate)	
Monthly mortgage payment, Pmt (\$)	803.15	
Initial spot interest rate, r_0 (%)	0.80 monthly (10.0 effective annual rate)	5–15
Gross return to housing, g	0.11	
House rental rate, γ	0.05	0.03–0.08
House price volatility, σ_H	0.1	0.05–0.15
Reversion parameter, β	0.1	0–0.5
Interest rate equilibrium, $\alpha = r_0$	0.10	
Interest rate volatility, σ_r	0.01	0.01–0.02
Deadweight refinance costs, F (\$)	500	
X	0.5% of loan balance	0–1.5
Transaction cost of default, TC (\$)	5,000	0–10,000
Exogenous prepayment rate, λ_r (%)	3 (50% of PSA)	0–6

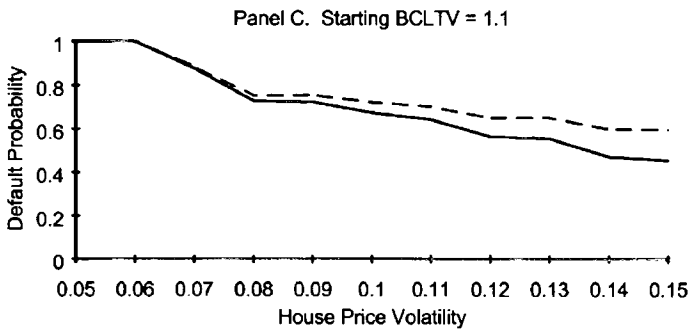
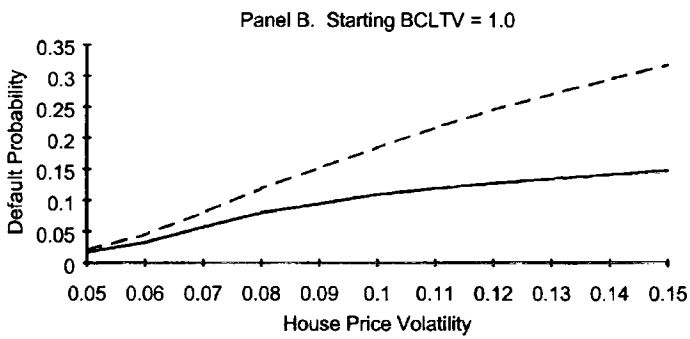
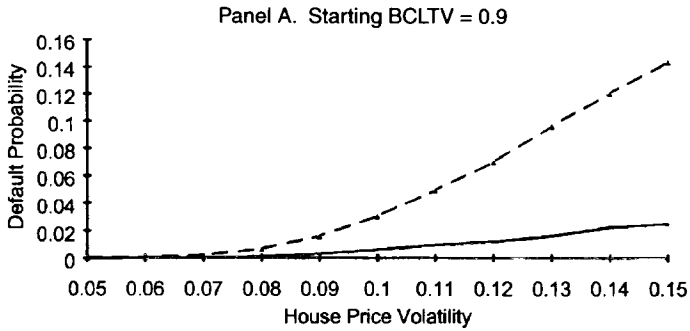
The parameters and ranges were used in the simulations. The results are independent of house price. The implied initial loan-to-value ratio is 90%. The gross return to housing (11%) is similar to discount rates on commercial property (11%–12%). The rental rates bracket the range of values found by CKT (1997). The house price volatility is also based on values in CKT. The ranges of interest-rate reversion and volatility parameters are similar to ranges in Chen *et al.* (1992). Refinance and default costs are similar to those in KKK (1994). The exogenous prepayment rate is set below PSA because optimal defaults are already included in the modeling.

House Price Volatility

Figure 1 summarizes the effect of house price volatility on defaults. There is a sign reversal at high CLTVs that is important and that requires explanation. At low CLTVs, the effect of volatility on defaults is positive, but at high CLTVs the effect is negative and large. This seems counter-intuitive in that option prices increase in volatility. The total number of defaults does increase as house price volatility increases, but it does not follow that the next period's default rate increases conditional on a given CLTV. An increase in volatility has two offsetting effects: (1) the stopping boundary for house prices falls, and (2) the probability of eventually

Figure 1 ■ The effect of house price volatility.

The three panels depict the effect of house price volatility on default for a 5-year-old mortgage and three BCLTVs. The base-case parameters are given in Table 1. The solid line is the default probability for the next year, and the dashed line for the next 10 years.



reaching this boundary increases. It is not obvious which effect will dominate *a priori* over the next year.

Figure 1 shows that the effect of house price volatility depends on the CLTV. When house price volatility is low and the CLTV is high, optimal default occurs immediately, since there is little benefit to waiting to see if house prices fall lower.¹⁰ As house price volatility increases, however, the probability of default in the next period declines, since expected house price changes make waiting more valuable (see Kau and Kim 1994).

Because most empirical data are heavily weighted with low-CLTV loans, it is not surprising that empirical studies find a positive relationship between house price volatility and mortgage default (*e.g.*, Schwartz and Torous 1993). Our results, however, imply that the effect of house price volatility interacts with CLTV and is negative and large at high CLTVs.

Rental Rate

Figure 2 plots the effect of the rental rate on default probabilities. As with house price volatility, there is a sign reversal at high CLTVs. The rental rate is analogous to the dividend rate on a stock. The effect of the rental rate should be similar to the effect of dividend yield on the early exercise of an American put. Again there are two offsetting effects. On one hand, a higher rental rate implies a lower rate of appreciation of the property, which increases the likelihood of hitting the default boundary sometime in the future. On the other hand, a high rental rate makes the existing mortgage payment more attractive relative to renting. When default is imminent, the borrower has an incentive to continue paying longer. The shorter the horizon and the higher the CLTV, the more likely the negative effect on defaults will dominate. Because of the offsetting effects and the small effect in all states as indicated in Figure 2, the anticipated effect of the rental rate in empirical studies is small.

Interest Rates

Figure 3 illustrates that when spot interest rates increase from the level at origination, the computed default rate falls. At CLTVs close to one, the default rate actually declines slightly as interest rates fall. Figure 3

¹⁰ Kau and Kim (1994) show that the reason one delays a current default, even if it is "in the money," is that house prices may fall in the future, so that the present value of defaulting in the future may exceed the value of immediate default.

Figure 2 ■ The effect of the rent-to-price ratio.

The three panels show the effect of the rent-to-price ratio on default for a 5-year-old mortgage and three BCLTVs. The base-case parameters are given in Table 1. The solid line is the default probability for the next year, and the dashed line for the next 10 years.

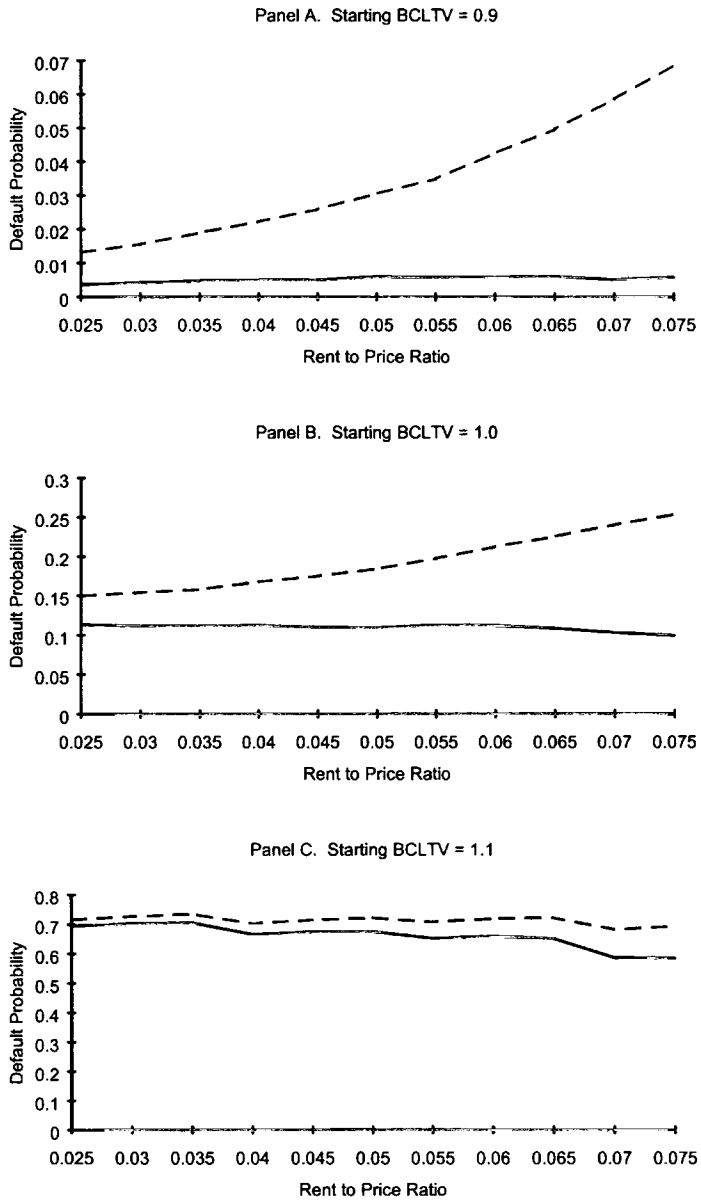
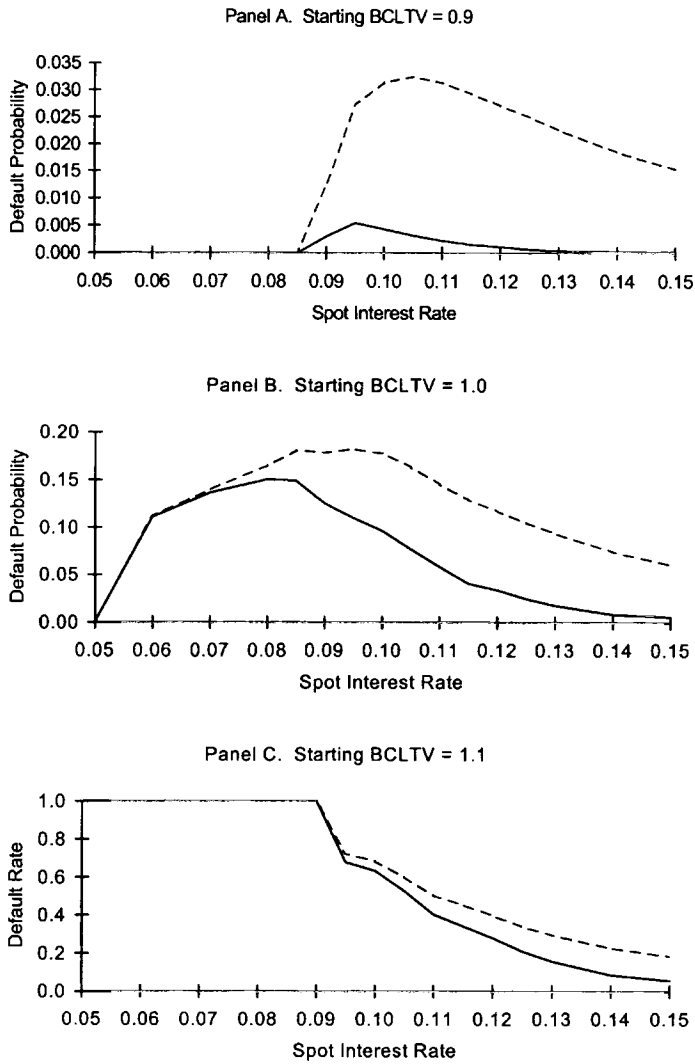


Figure 3 ■ The effect of interest-rate changes.

The three panels illustrate the effect of interest-rate changes since origination on default for a 5-year-old mortgage and three BCLTVs. The base-case parameters are given in Table 1. The solid line is the default probability for the next year, and the dashed line for the next 10 years.



demonstrates that a positive interest-rate spread variable is essential in empirical models that use book CLTV (BCLTV)¹¹ as the measure of CLTV. Notice also that the size of the interest-rate effect is much greater at high CLTVs. An interaction term of interest rates with CLTV will be needed to capture the effect empirically.

Interest-Rate Volatility

Figure 4 shows the effect of interest rate volatility on the probability of default. The effect of volatility is ambiguous *a priori* because refinancings, which foreclose the default option, occur when rates fall. If rates rise, the value of the mortgage is reduced, also leading to fewer defaults. When there is little or no equity in the house, however, default is a likely response to falling rates because the value of the high-coupon loan increases and further reduces effective equity.

In panel A of Figure 4, at low CLTVs, there is minimal effect on default probability. For high CLTVs (panel C), increasing interest-rate volatility slightly decreases the likelihood of default over the next year. As a result, empirical studies may have difficulty distinguishing any statistically significant effect.

Reversion Parameter

Figure 5 indicates that the affect of the interest-rate reversion parameter on next year's default probability is quite small and mostly positive. The stronger the reversion, the less likely a favorable interest-rate environment will present itself, reducing the value of the prepayment option and increasing the probability of default. The effect is more pronounced for high-CLTV loans but still quite small.

Transaction Costs of Default

Transaction costs are particularly interesting because they can vary in three ways. First, each individual faces different transaction costs from family and job characteristics. It is well known that single individuals are more likely to move than other household types. Second, transaction costs vary by location, since the legal remedies available to lenders differ. In one-remedy states, borrowers can default with minimal consequences to their personal

¹¹ BCLTV uses the contractual balance for the value of the mortgage but uses current house prices for the denominator.

Figure 4 ■ The effect of interest-rate volatility.

The three panels present the effect of interest-rate volatility on default for a 5-year-old mortgage and three BCLTVs. The base-case parameters are given in Table 1. The solid line is the default probability for the next year, and the dashed line for the next 10 years.

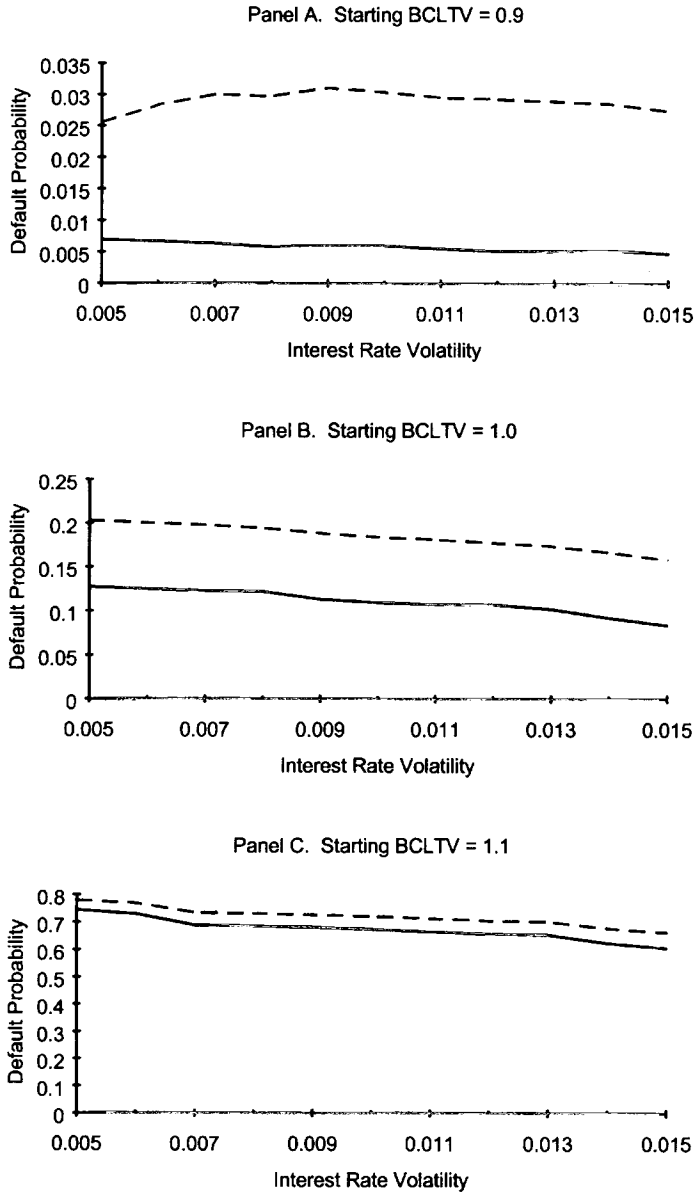
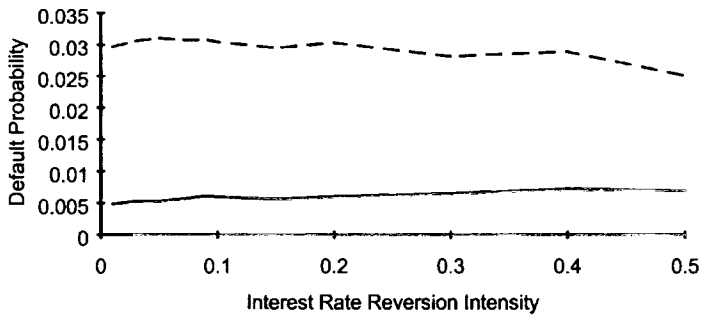


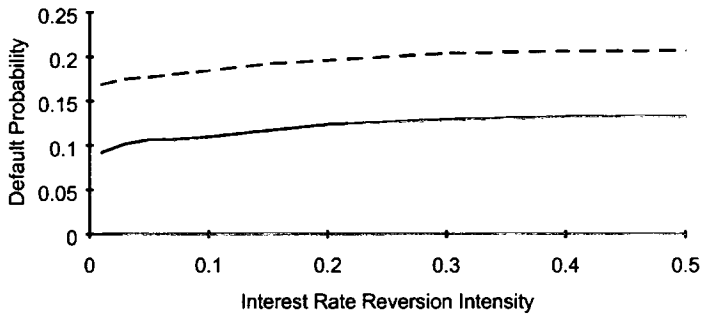
Figure 5 ■ The effect of interest-rate reversion.

The three panels show the effect of interest-rate reversion on default for a 5-year-old mortgage and three BCLTVs. The base-case parameters are given in Table 1. The solid line is the default probability for the next year, and the dashed line for the next 10 years.

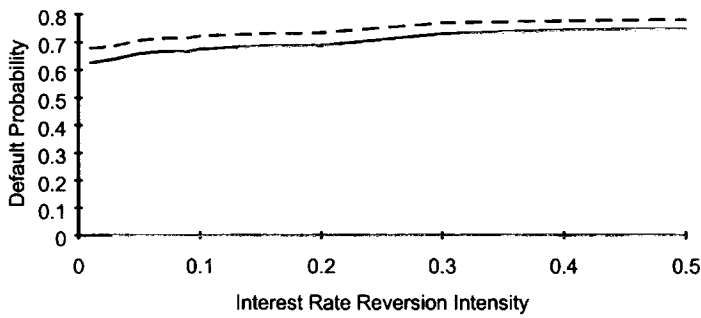
Panel A. Starting BCLTV = 0.9



Panel B. Starting BCLTV = 1.0



Panel C. Starting BCLTV = 1.1



finances (Jones 1993). Third, this cost can vary over time for the same individual when personal circumstances change. A divorce or job change can greatly reduce the cost of default, since the borrower will need to move independently of the default decision. Ambrose, Buttimer and Capone (1996a) point out that transaction costs can even be negative for borrowers who may enjoy a period of free rent before foreclosure is completed.

Figure 6 shows the effect of these transactions costs on default probabilities. The effect on unconditional default is negative and modest in size at the 90% CLTV. Transaction costs reduce conditional default probabilities at all CLTVs, but have their most dramatic effect at a high CLTV (panel C). This difference between the unconditional and conditional may account for some of the disagreement on the importance of transaction costs (Kau, Keenan and Kim 1993; Lekkas, Quigley and Van Order 1993). The results suggest that for empirical models, transaction costs should interact with CLTV, rather than be included as a linear covariate.

Transaction Costs of Refinancing

The transaction costs of refinancing include origination fees, points and legal fees. Absent these costs, refinancing would be optimal whenever interest rates fall below the contract interest rate and borrower equity is positive. The model includes both fixed and variable transaction costs of refinancing. Figure 7 indicates that the short-horizon default rate increases slightly as the transaction cost of refinance increases. Overall, this variable has little effect on the default rate.

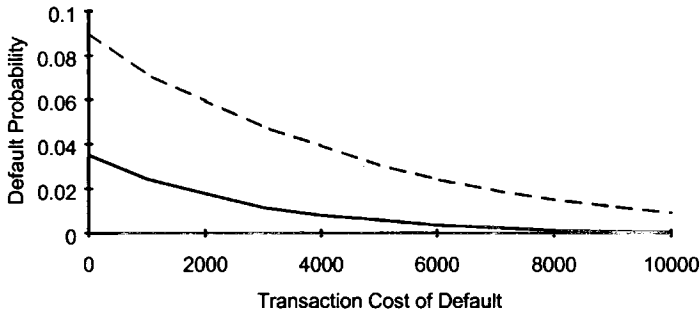
Trigger Events

Historically, industry analysts have assumed that exogenous events (*e.g.*, divorce or unemployment) play a major role in mortgage default. In our model, exogenous events are random events with a given probability of occurrence. The borrower realizes that exogenous events may occur in the future, and adjusts the decision to default or prepay appropriately. We separate the defaults into those that are due to the optimal decision at the time, and those that are a response to an exogenous event. This allows computation of the probability that an exogenous event will cause default. In the base case, it is assumed that transaction costs are present for all default decisions. In many cases, however, an exogenous event may result in several changes for the decision maker—some of which may reduce the transaction costs of defaulting to zero. For example, if a move results from this exogenous event (perhaps a change of employment location), then the moving costs are no longer relevant to the default decision. For this reason,

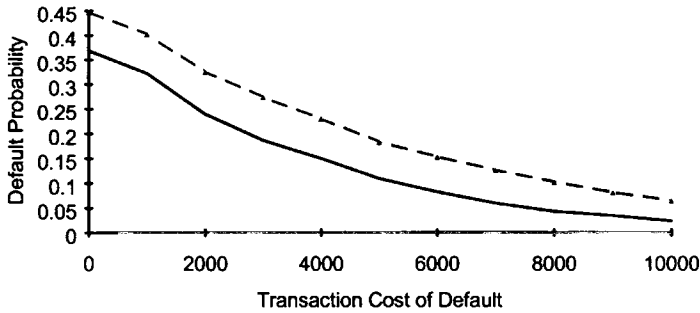
Figure 6 ■ The effect of transaction costs of default.

The three panels illustrate the effect of default-related transactions costs on default for a 5-year-old mortgage and three BCLTVs. The base-case parameters are given in Table 1. The solid line is the default probability for the next year, and the dashed line for the next 10 years.

Panel A. Starting BCLTV = 0.9



Panel B: Starting BCLTV = 1.0



Panel C: Starting BCLTV = 1.1

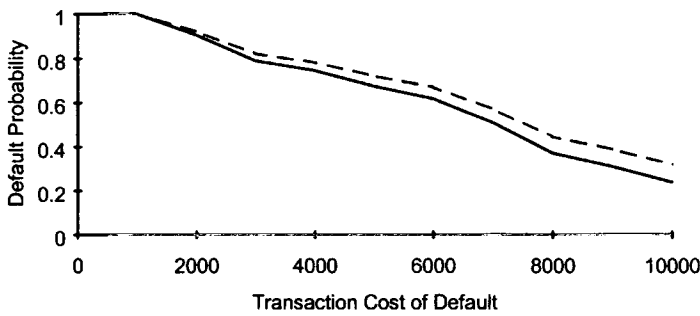


Figure 7 ■ The effect of deadweight refinancing costs.

The three panels depict the effect of deadweight refinancing costs on default for a 5-year-old mortgage and three BCLTVs. The base-case parameters are given in Table 1. The solid line is the default probability for the next year, and the dashed line for the next 10 years.

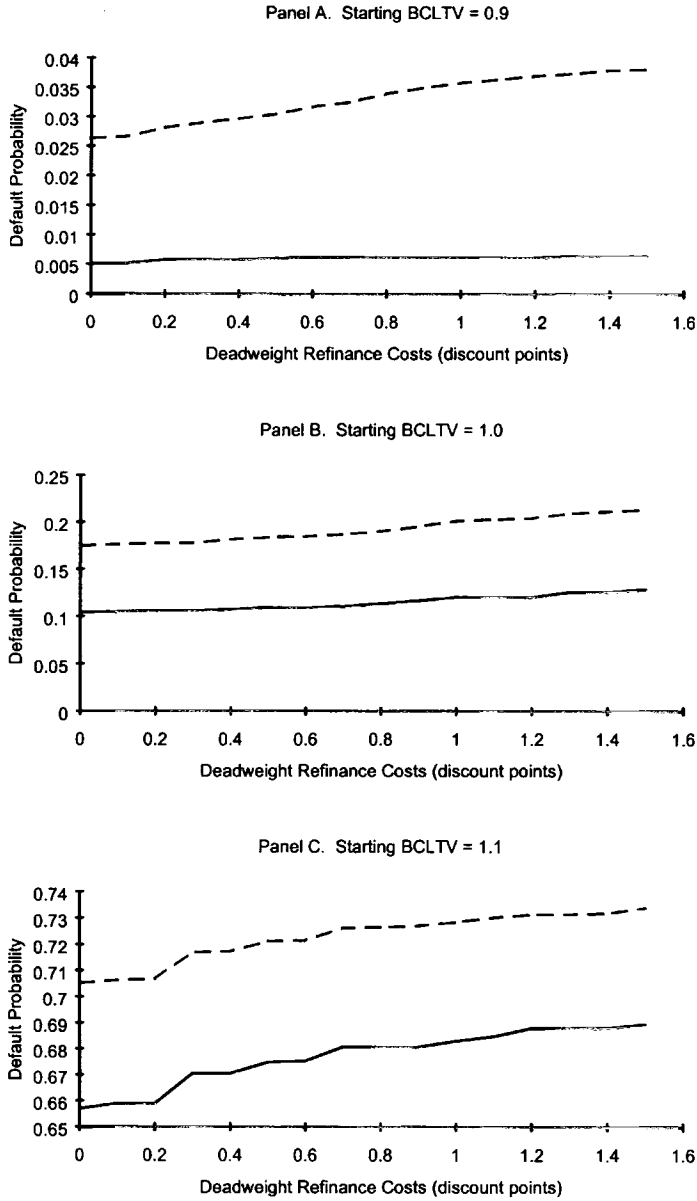


Table 2 ■ The effect of exogenous termination on defaults.

Quantity	Value			
	Transaction costs with a trigger event are same as normal transaction costs		Transaction costs fall to zero with a trigger event	
	BCLTV = 90.0%	110.0%	90.0%	110.0%
Total defaults over the next year (%)	0.6	69.0	0.8	66.0
Proportion of total defaults due to trigger events (%)	7.0	2.0	35.0	4.0
Default rate conditional on a trigger event (%)	0.5	35.0	4.0	75.0

This table illustrates the effect of trigger events on defaults under two assumptions about transaction costs and two loan-to-value ratios.

a modified model is also analyzed when transaction costs are zero if the exogenous event occurs.

The results are summarized in Table 2. The exogenous termination rate is set to a relatively high 9% per year (150% of PSA) for illustrative purposes, and the default rates are those occurring over the next year. The first two columns provide the results for the normal transaction-cost cases; the second two assume that transaction costs fall to zero when the trigger event occurs. For each transaction-cost assumption, results for 90% and 110% LTVs are included. The most useful comparison is between total defaults (row 2) and defaults conditional on a trigger event having occurred. In the first two columns, the default rate given a trigger event is the same as or lower than the overall rate. That is, borrowers are no more likely to default with a trigger event than without.

In the second two columns, when transaction costs are assumed to fall to zero if a trigger event occurs, borrowers default at a higher rate.¹² For the 90% LTV, the default rate rises from 0.75% to 4% with the trigger events,

¹² Notice that by assuming that when a trigger event occurs the borrower's horizon shortens to one year and the transaction costs fall to zero, we have constructed a "worst" case in the sense that less extreme assumptions will result in fewer defaults caused by trigger events.

and for 110% LTV the defaults rise from 66% to 75%. That is, if the trigger event reduces transaction costs to zero, borrowers are more likely to default. The increase is relatively large in the low LTV range where a significant proportion of the total defaults (35%) can be attributed to the trigger events. However, since most defaults occur at high LTVs and since only 4% of the defaults can be attributed to trigger events in this range, trigger events cannot be the primary cause of defaults.

Optimal default (and prepayment) is the first choice available to the borrower. If the borrower does not default or prepay, then an exogenous event may present itself. When an outsider observes both a default and an exogenous event, it may be assumed that the trigger event caused the default, rather than the default occurring as an optimal decision in the same period a trigger event occurs. The default should not be credited to the trigger event unless the event occurs only because of the trigger event. Our overall conclusion is that trigger events play a minor role.¹³ If house prices are low (causing loan-to-value ratios to be high), defaults will be high, regardless of whether trigger events occur or not. Figure 8 illustrates the minor effect on default as the exogenous termination rate is varied when transaction costs do not fall to zero when a trigger event occurs.

Measuring Current Loan-to-Value Ratios in Empirical Studies

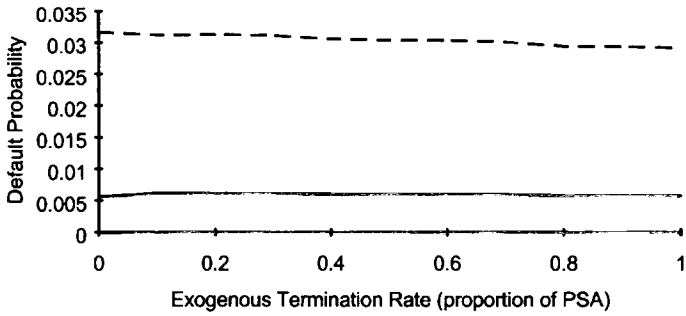
The CLTV is the current value of the loan divided by the current value of the house. The BCLTV is the contractual balance on the note divided by the current house value. The market CLTV (MCLTV) is defined as the market value of the loan divided by the current house value. The BCLTV is easily computed as long as the initial loan parameters and current house prices are available. The BCLTV explicitly allows for the amortization of the mortgage and changing house prices. However, it does not allow for the effect of interest rates on current loan values. For example, when interest rates rise, the market value of a mortgage falls. The market CLTV will fall while the book CLTV is unaffected. The change in market value will enter the borrower's decision. This suggests that the MCLTV may be a better measure for default analysis. The three ways to deal with the difference between BCLTV and MCLTV in empirical analysis follow.

¹³ This is consistent with the empirical findings in CKT (1997). They find that trigger events are orders of magnitude less economically important than current loan-to-value ratios.

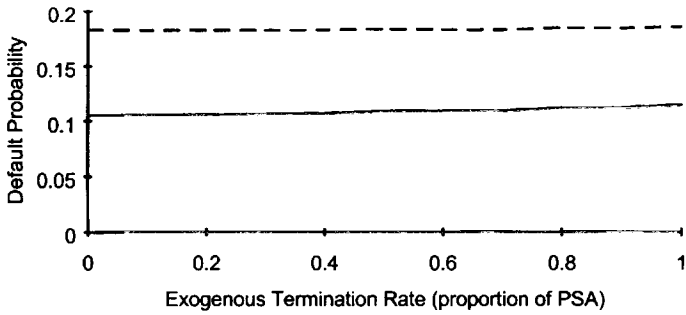
Figure 8 ■ The effect of exogenous terminations.

The three panels show the effect of exogenous terminations on default for a 5-year-old mortgage and three BCLTVs. The base-case parameters are given in Table 1. The solid line is the default probability for the next year, and the dashed line for the next 10 years. The base PSA rate is 6% per year.

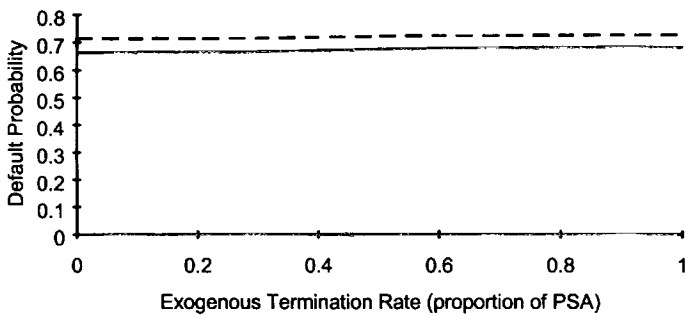
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Panel B. Starting BCLTV = 1.0



Panel C. Starting BCLTV = 1.1



*Simple Ad Hoc Rules for Computing the MCLTV**Simple Ad Hoc Rules for Computing the MCLTV*

Foster and Van Order (FVO) (1984) recognized that BCLTV and MCLTV might diverge when interest rates rise. To adjust for this effect, they compute the value of the current mortgage by discounting the payments using the current mortgage rate if interest rates have risen. A further assumption is that the loan will be repaid after 40% of its remaining life. If interest rates have stayed the same, or fallen, no adjustment is used. This *ad hoc* procedure has been used in several subsequent studies.

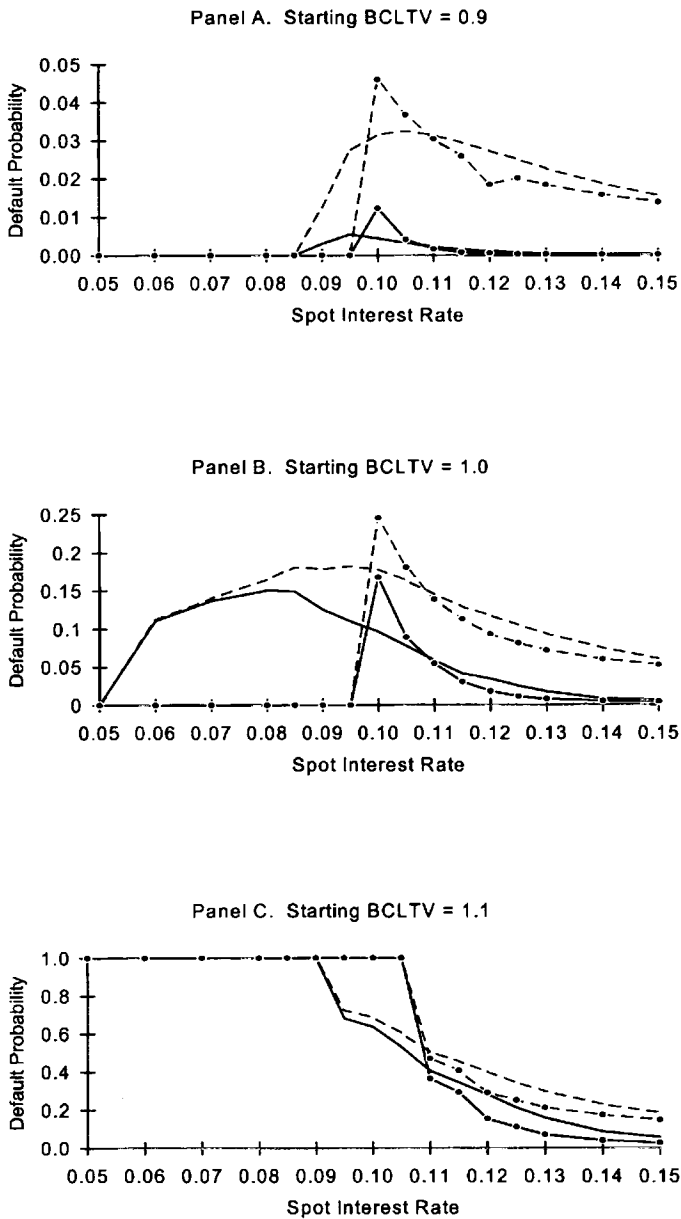
To assess the change in mortgage value in response to an increase in interest rates, the model was rewritten to approximate the FVO procedure in a dynamic model. If interest rates fall or remain the same, no changes are made in the model.¹⁴ When interest rates rise, the model is adjusted by using the FVO mortgage value calculation to compute the value of continuation, $P^*(H, r_t)$, as seen in Equation (A4) in the Appendix.

Figure 9 presents the effects of interest rate changes using both the results presented earlier and those computed using the FVO adjustment. For low CLTVs, panel A shows that the results are quite similar between models. Panel B, with results for CLTV = 1, also illustrates a similar trend, though the FVO-predicted default rate is lower when the spot rate is above 11%. In the 8½%–9½% range, however, the contingent claims model waits to see if refinancing is the best choice, while the FVO approach chooses immediate refinancing. For CLTV = 1.1, the FVO approach chooses immediate default if spot rates are 10.5% or lower, while the contingent-claims model does not choose immediate default unless the spot rate is 9% or lower. For interest rates above 11%, the projected default rate in the contingent-claims model is higher, suggesting that the FVO model over-corrects for interest-rate changes. This over-correction is confirmed empirically by Cooperstein, Redburn and Myers (1991) and CKT (1997). The reason for this result is that in the contingent-claims model, the interest-rate process is mean-reverting. If the interest rate rises, it will naturally tend toward its mean. The FVO adjustment, on the other hand, assumes the higher interest rate will prevail until the mortgage is paid off. When the base model is run with a smaller reversion parameter, the results are closer to the FVO version.

¹⁴ This is not exactly the same as assuming that no adjustment to the perceived mortgage value is needed, since the model evaluates the potential of a refinance if interest rates fall. If the FVO assumption of no adjustment is used, refinancing will never be chosen, regardless of how low interest rates fall, since there are costs to refinance with no computed benefit.

Figure 9 ■ The effect of interest-rate changes modeled as an FVO correction.

The three panels illustrate the effect of interest-rate changes since origination on default for a 5-year-old mortgage and three BCLTVs. The base-case parameters are given in Table 1. The solid line is the default probability for the next year, and the dashed line for the next 10 years. Dots on lines represent the FVO analysis.



Use the MCLTV in Empirical Analysis

An alternative approach to measuring the CLTV is to use a contingent-claims model to compute the MCLTV for each observed set of values of the independent variables. Given that an empirical study may have 50,000 or more combinations of independent variable values, it would be impractical to make the computations that would be required to compute the MCLTV.¹⁵ Computing the BCLTV is quicker, since the only variables that affect BCLTV are mortgage age, coupon rate and house value. The MCLTV also complicates the interpretation of results in that the source of the change in the MCLTV may not be apparent. Any of the independent variables could have caused the change in the MCLTV.

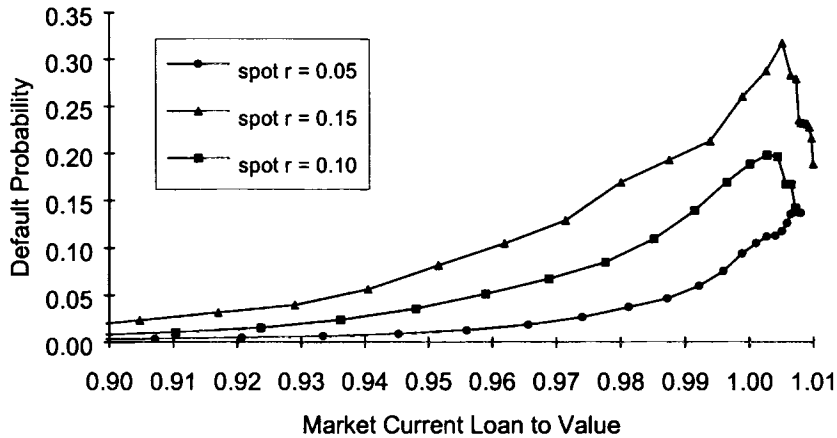
Nevertheless, if the MCLTV fully captures the many dimensions of default, it may be worth the computational effort. To test whether the MCLTV is a sufficient statistic for predicting default rates, we run the model with various combinations of house prices, coupon rates and current interest rates to create a range of MCLTV values. Loans with different coupons but subject to the same spot rate will have different MCLTVs. Figure 10 illustrates the computed default rate over the next year as MCLTV is varied by adjusting the mortgage coupon rate from rates below the spot rate to coupon rates above the spot rate. Three cases are presented for spot rates of 5%, 10% and 15%. If the MCLTV is sufficient for assessing default, then all lines should plot on top of each other (*i.e.*, for a given MCLTV the default rates will be equal).

In general, if the coupon rate is high relative to the spot rate, default becomes more likely. However, near $MCLTV = 1$ the default rate falls because refinancing, which extinguishes the default option, becomes increasingly likely. The MCLTV changes most in response to house price changes or interest rate changes. Near an MCLTV of one, with default imminent, as spot interest rates fall, the probability of default first rises because borrowers are increasingly unhappy with the existing high coupon rate. However, if rates continue to fall and the borrower has equity in the house, the borrower will refinance and extinguish the default option. If the loan is "under water," however, the optimal response will be to default. For the same measured MCLTV, one observes in one case more defaults, and the other sharply fewer. The MCLTV is not a sufficient statistic. Thus, despite the sophistication of the MCLTV, it still is not ideal for empirical analysis.

¹⁵ The computation uses more than five minutes of CPU time on a Sun 670MP. 50,000 computations would require about six months of CPU time.

Figure 10 ■ MCLTV and default.

This figure plots the default probability over the next year for three spot interest rates as MCLTV varies for a 5-year-old loan and for a BCLTV ratio of 1.0. The model parameters are as given in Table 1 with the following exceptions: (1) house value = loan balance = \$100,000; (2) mortgage coupon rate varies from about 2% below the indicated spot rate to 2% above the indicated spot rate to vary the MCLTV. The figure illustrates that MCLTV does not fully capture the effect of interest-rate changes on default.



Use the BCLTV and Include Other Covariates

A third alternative is to include other variables, such as the current coupon rate, in empirical models to mitigate the known shortcomings of the BCLTV. This approach to measuring the CLTV also has its advantages and disadvantages. On one hand, it is simple and efficient to implement; but on the other hand, care must be taken to include the relevant variables and properly specify the functional form. The analysis suggests that using the BCLTV in conjunction with other variables can be an effective alternative to using *ad hoc* rules. In sum, no one approach dominates the others.

Conclusions and Empirical Implications

This article develops a contingent-claims model of mortgage default and applies the model to obtain insights for empirical modeling. There are four types of results. First, some variables suggested by options pricing matter unconditionally but are secondary or tertiary in importance conditionally. These include the rental rate, interest-rate volatility and interest-rate reversion.

Second, some effects reverse signs under certain conditions. These include, most notably, house price volatility, but also the rental-rate and interest-rate volatility. The sign reversal for volatility from positive to negative occurs when the default option is in the money (high CLTV). In this range, volatility causes borrowers to delay default in anticipation of the possibility of more favorable exercise conditions in the future. This result contrasts with the unconditional probability, which is positively affected by volatility.

Third, some parameters have little effect on default and can be excluded from empirical models with less risk of misspecification bias.¹⁶ The interest-rate volatility and reversion, as well as the transaction costs of refinancing, are in this category. Trigger events, traditionally considered important precipitators of default, have relatively little influence. The empirical findings in CKT (1997) confirm that trigger events have only a small effect.

Fourth, the functional form is complicated for some independent variables. Interest-rate declines, for example, have only an indirect effect on defaults through prepayments. However, increases from the origination rate reduce defaults and reduce them further when the option is in the money.

Some parameters such as volatility and rental rate, as noted earlier, reverse sign at high CLTVs. Some, such as transaction costs and trigger events, have more impact at high CLTVs.

Trigger events do have an effect, albeit small, on defaults. The increase in defaults from trigger events is most pronounced when transaction costs are low and the default option is only slightly out of the money. In other cases, the default rate conditional on an exogenous event is similar to the rate without the trigger event.

The key variable in predicting default is the CLTV. The BCLTV and MCLTV, however, are not equivalent. Some empirical studies have adjusted the BCLTV for changes in spot interest rates. Alternatively, a contingent-claims model could be used to compute an MCLTV. The computer resources

¹⁶ The bias from a left-out variable depends on the regression coefficient of the left-out variable in the true relation and the co-movements of the left-out variable with the included variables. Thus, if a variable has only a small effect on the dependent variable (the regression coefficient of the left-out variable is small), the bias of the included variable is small. At the same time, the variance of the estimate of the included variable may be improved by excluding the variable (see Johnston 1972, p. 169).

required, however, prevent extensive use of this approach. In addition, the MCLTV is not sufficient, because it does not fully explain observed defaults. Our results suggest that all the available approaches have shortcomings. When using the BCLTV as an alternative to *ad hoc* rules, it is advisable to add appropriate covariates with a suitable functional form to adjust for the complex effects of BCLTV on defaults.

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Appendix

Numerical Solution

The model is solved using backward recursion of a stochastic dynamic programming model. Dynamic programming requires discretizing the two stochastic processes. The house price process of Equation (2) is discretized using the standard equations of Cox, Ross and Rubinstein (1979):

$$u = \exp(\sigma_H \sqrt{\Delta t})$$

$$d = 1/u$$

$$p_u = \frac{\exp[(r - \gamma) \Delta t] - d}{u - d}$$

u = Next period house price multiplier if an up state occurs.

d = Next period house price multiplier if a down state occurs.

p_u = Probability of an up state.

The interest rate process of Equation (3) is discretized using the equations of Nelson and Ramaswamy (1990):

$$r_u = r + \sigma_r \sqrt{\Delta t}$$

$$r_d = r - \sigma_r \sqrt{\Delta t}$$

$$q_u = \begin{cases} 0.5 + \sqrt{\Delta t} \beta(\alpha - r)/2\sigma_r, & \text{if } 0 \leq 0.5 + \sqrt{\Delta t} \beta(\alpha - r)/2\sigma_r \leq 1 \\ 0 & \text{if } 0.5 + \sqrt{\Delta t} \beta(\alpha - r)/2\sigma_r \leq 0 \\ 1 & \text{otherwise} \end{cases}$$

r_u = Interest rate next period if the up state occurs.

r_d = Interest rate next period if the down state occurs.

q_u = Probability of the up state occurring.

The borrower's problem at each time is to determine which action provides the lowest discounted housing cost. In other words, just prior to a mortgage payment due date, the mortgagor assesses whether it is less costly to default, to refinance or to make the scheduled mortgage payment. This decision can be written as

$$P_t(H_t, r_t) = \min[P_t^d(H_t, r_t), P_t^r(H_t, r_t), P_t^w(H_t, r_t)] \quad (\text{A1})$$

where

$$P_t^d(H_t, r_t) = H_t + TC \quad (\text{A2})$$

$$P_t^r(H_t, r_t) = Pmt + (1 + X)MB_t + F \quad (\text{A3})$$

$$\begin{aligned} P_t^w(H_t, r_t) = & [p_u q_u P_{t+1}(uH_t, r_u) + (1 - p_u) q_u P_{t+1}(dH_t, r_u) \\ & + p_u (1 - q_u) P_{t+1}(uH_t, r_d) + (1 - p_u) (1 - q_u) P_{t+1}(dH_t, r_d)] \delta_t \\ & + Pmt \end{aligned} \quad (\text{A4})$$

$P_t^d(H_t, r_t)$ = Value of the house plus the transaction costs, which is the value of the loan (to the mortgagor) if the mortgagor chooses to default on the loan now.

TC = Transactions costs of default.

$P_t^r(H_t, r_t)$ = Cost to obtain a fair value new mortgage, which is the value if one refinances the mortgage now. The lender receives a fair exchange mortgage for providing the funds to refinance the mortgage balance. The nature of the new loan is not specified. The new loan could be a nonprepayable or prepayable, a fixed or variable rate loan, etc. The only requirement is that it be fairly priced, so that it is equal to the mortgage balance that must be refinanced. The deadweight cost of a new mortgage includes any excess points (X) that are not priced in the mortgage instrument itself, and a fixed fee (F). Dunn and Spatt (1986) emphasize the importance of the mortgage refinancing costs for refinancing decisions, and in determining the value of

the mortgage. In particular they note that prepayable mortgages may trade at values above par when it is not advantageous for a mortgagor to refinance in response to a small interest-rate decrease due to the transaction costs of refinancing. In the absence of refinancing costs, any decrease in interest rates would lead to refinancing.

X = Deadweight refinancing costs that are a function of the loan size (*e.g.* the commonly used one-point origination fee for a mortgage).

F = Deadweight fixed refinancing cost (*e.g.* cost of a property survey or credit check).

MB_t = Mortgage balance at time t .

Pmt = Monthly mortgage payment.

$P_t^w(H_t, r_t)$ = Present value of the loan, assuming the decision maker makes the contractual mortgage payment and thus continues the mortgage at least one more period. This is the expected discounted future value of the loan, given that the house price and interest rate change in the next period according to the processes noted.

δ_t = One-period discount factor for the current spot interest rate.

The model is revised to allow for events which are exogenous to the optimizing decision as described thus far. These events cause premature termination of the mortgage, in the sense that when it is not otherwise optimal to choose to default or prepay, an exogenous event may precipitate one of these decisions. The exogenous event is assumed to follow the pattern established by the PSA¹⁷ function, though it may be scaled by a chosen multiple. When an exogenous event occurs, the decision maker decides whether it is better to prepay or to default by simply assessing what is best at that instant, as there is no future to be considered. The best choice is the minimum-cost choice—either to pay off the mortgage balance, or to default if the house price plus transaction costs of default is lower than the mortgage balance. Equation (A1) is modified to allow for exogenous events as

¹⁷ The PSA model is the prepayment model produced by the Public Securities Association, which projects the prepayment rate as rising linearly from 0% to 6% per annum over the first 30 months of the loan and then remaining at 6% per annum for the remainder of the loan. Because our analysis is for seasoned loans, 100% of PSA is a constant 0.5% per month, or $\lambda_t = 0.005$.

$$P_t(H_t, r_t) = \min\{P_t^d(H_t, r_t), P_t^r(H_t, r_t), (1 - \lambda_t) P_t^w(H_t, r_t) + \lambda_t \min[MB_t, H_t + TC]\} \quad (\text{A5})$$

λ_t = Exogenous event rate at period t .

To complete the model one must specify the appropriate boundary condition at maturity. In the final period, the wait choice vanishes. The value of the mortgage, if default is chosen, is the value of the house plus transactions costs. The alternative is to pay off the mortgage by making the final payment. The mortgagor chooses the lower cost alternative, which in general is to make the final mortgage payment. The mortgagor will make the final mortgage payment if the transaction cost of default exceeds the mortgage payment. The boundary equation at maturity is

$$P_T(H_T, r_T) = \min(H_T + TC, Pmt). \quad (\text{A6})$$

The solution for the optimal decision sequence and values is obtained by backward induction starting with Equation (A6) and working to the present. The model determines the optimal decision at each stage (time period) and computes the value of the loan. The primary purpose of this study, however, is to determine default probabilities, which requires the use of lattice process probabilities to compute projected default rates. Because default at any stage is conditional on what has happened in the past, this computation is made as a forward recursion on the lattice using the optimal default and prepayment boundaries as stopping points for the process. While hedging arguments provide that the default option and optimal stopping boundary are determined using the risk-neutralized house price process of Equation (2), determining the probability of default is done using the actual house price process of Equation (1). To implement the computation of the default probabilities, the probability of an up state in house prices is computed using the same CRR equation, but with the gross return to housing used in place of the risk-free interest rate, that is,

π_u = actual probability of an up state

$$= \frac{\exp[(g - \gamma) \Delta t] - d}{u - d}$$

Operationally, the model is run and the optimal default and prepayment boundaries are stored. Then, starting with a probability of 1 from the initial node, the probability of reaching each interest rate and house price node in the next period is computed. If that node involves a prepayment, the

probability of reaching that node is credited to the probability of a prepayment, and then the probability of that node is set to zero. A similar approach is used for default nodes. For nodes for which neither a prepayment nor default is chosen, their probability is reduced by the exogenous termination rate. The exogenous event probability is credited to either prepayment or default, depending on whether the mortgage balance exceeds the house value plus the transaction cost of default. The process is then continued for another stage. The correct conditional probabilities are computed because nodes where a default or prepayment has occurred have their probability set to zero, so forward movements from these nodes are made with zero probability. The probability of default at each stage is computed by summing the probabilities of all default nodes at that stage.