Name: Date:

Lab # 1

Measuring the Acceleration Due to Gravity

Instructions: *Numerical answers require units and appropriate numbers of significant digits.* ***Remember to show your work!***

PRELAB

1 Review:

In studying moving objects – like planets – we consider the relations between acceleration, velocity, and position. As you recall from calculus, the velocity can be expressed as

$$v(t) = \int a(t)dt \tag{1}$$

where $a(t)$ is the acceleration of an object as a function of time t. Similarly, the object's position as a function of time, t, is

$$x(t) = \int v(t)dt = \int \int a(t')dt'dt \tag{2}$$

Questions:

(a) To see how this works, consider dropping a ball from the top of a building. If the acceleration of gravity is g and the ball is dropped at time $t = 0$ with downward velocity v_0, integrate equation (1) to find its velocity at time $t = T$. Start by drawing a simple sketch. Check your result by verifying that $v(t = 0) = v_0$.

(b) Similarly, integrate equation (2) to find how the ball has fallen at time $t = T$.

2 Introduction

To constrain Earth's average density ρ_0, we need to know its mass M and volume V. The mass is the volume integral of the density, which can be written as an integral over Earth's radius, R, assuming that density varies only with depth:

$$M = 4\pi \int_0^R \rho(r) r^2 dr \tag{3}$$

Question:

Using the approximation that Earth is a perfect sphere of radius R, show that the average density of Earth, ρ_0, is

$$\rho_0 = \frac{3 \int_0^R \rho(r) r^2 dr}{R^3} \tag{4}$$

As shown above, a primary constraint on Earth's density is its mass, which can be found from the acceleration of gravity at the surface using Newton's law of gravitation. This law states that the force F exerted by a body of mass m_1 on another body with mass m_2 depends on the two masses, the distance r between them (i.e. between their respective centers of mass), and the universal gravitational constant G (see Fig. 1):

$$F = \frac{G m_1 m_2}{r^2} \tag{5}$$

$$G = 6.67 \times 10^{-11} \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{kg}^2} \tag{6}$$

where force is given in Newtons (N).

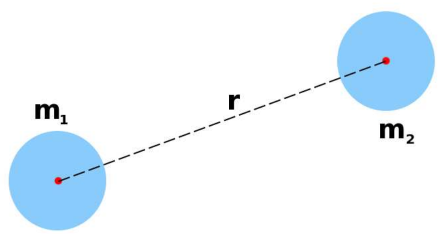


Figure 1: Schematics for Newton's law of gravitation.

G has been measured through several types of experiment and has a constant value in the universe.

Question:

Using Newton's second law which states that force equals mass multiplied by acceleration, $F = ma$, show that N=kg.m/s^2.

Name: Date:

SI vs. CGS

Read the piece on MKS vs CGS units at the University of North Carolina's web page `https://www.unc.edu/~rowlett/units/cgsmks.html` and list the CGS and MKS units for length, mass and time.

The SI[a] unit system extends the MKS system to other physical quantities.

[a] The International System of Units or in French, Système International d'Unités

Questions:

1. Considering the fact that Newton (N) is the unit for force in the SI system (from Newton's second law), calculate the value of G in the cgs units system.

2. Using Newton's law of motion, show that an object of mass m at the surface of the Earth (with mass M and radius R) is subject to an acceleration due to gravity g:

$$g = \frac{GM}{R^2} \tag{7}$$

There are many methods to measure the acceleration due to gravity, among which we will use two in this lab:

(1) dropping a ball
(2) swinging a pendulum.

Once we determine g, we can then compute Earth's mass from Eq. (7) and consequently its density, ρ_0 which will give us valuable insight into Earth's internal structure. All of this is possible if we know Earth's volume or, more precisely, its radius.

2.1 Earth as a Sphere

When calculating the gravitational force between two objects, the distance between the respective centers of mass of objects plays an important role ((5)). This distance is the sum of the objects' radii and the distance between their surfaces. For planets, as for example the Earth and the Moon, the latter (and consequently the radius of the Moon) can be measured rather easily as we will see in the lectures and the first problem set (P-2).

Also, as we saw above, calculating the Earth's volume is crucial in studying its internal structure. If we consider the Earth as a perfect sphere, then calculating its volume corresponds to finding its radius.

Measuring the radius of the Earth has been the subject of many studies throughout history. One of the first efforts to measure the radius of the Earth (of course, considering it as a sphere) was made by Eratosthenes in 200 BC. He knew that at the summer solstice the sun shone directly into a well at Syene (now Aswan) at noon. At the same time, in Alexandria, Egypt, approximately 5,000 stadia ($\approx 787km$) due north of Syene (1 stadion is approximately equal to 157 m), the angle of inclination of the sun's rays was about $7°12'$ (see Fig 2).

Question:

If 1 degree is equal to 60 arc-minutes, what decimal fraction of a degree is each arc-minute?

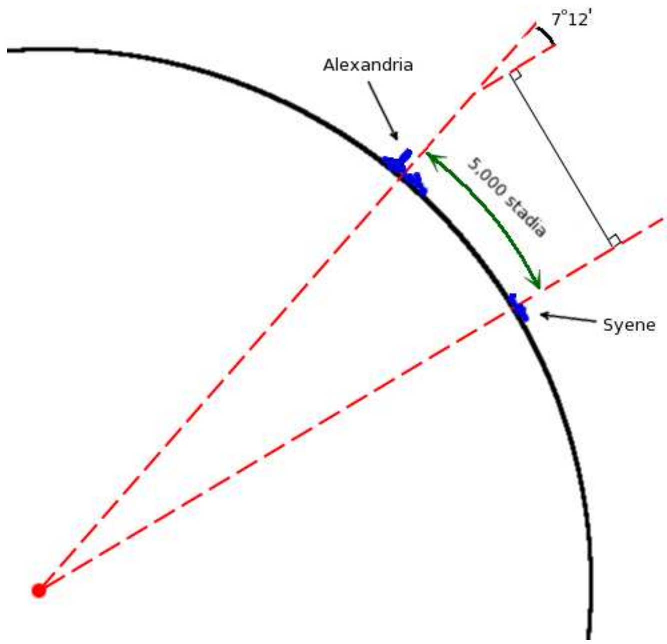


Figure 2: Eratosthenes measurement.

With these measurements, Eratosthenes computed the diameter and circumference of the earth.

Questions:

1. Use Eratosthenes' measurements to calculate the circumference of the Earth in stadia.

2. What is the resulting circumference in km?

3. What is the resulting radius in km?

4. Compare your answer to the modern value of 6371 km (measured through Geodesy which is the science of the shape of the Earth).

2.2 Simple Vertical Pendulum

One day while attending Mass, Galileo noticed a chandelier above him was swaying in a draft. Comparing to his heartbeat, he noticed how the period of motion compared for large and small swings. The period, T, is the amount of time taken for the swinging motion of the lamp to repeat (see Fig. 3).

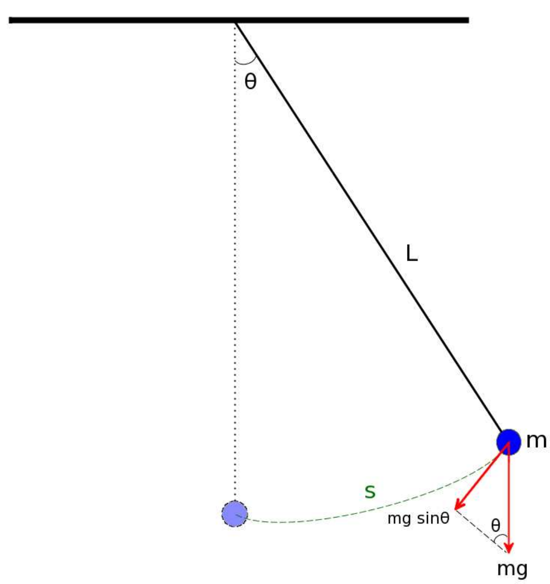


Figure 3: Dynamics of simple vertical pendulum.

Just as we will find out in this lab, Galileo noticed that period of the oscillation for any pendulum depends only on the length of the string used to suspend the mass, and not on the mass.

Let's take a look at the dynamics of the simple vertical pendulum to see how it works. In Fig. 3, the arc-path of the mass, s, is related to the product of the length of the string l and the angle θ of the swing:

$$s = l\theta \tag{8}$$

The force acting on the pendulum tangential to the arc-path of the mass is:

$$F = -mg\sin\theta \tag{9}$$

where mg is the weight of the mass.

Using Newton's law of motion, this force is also equal to the product of the mass and the tangential acceleration, d^2s/dt^2:

$$F = -mg\sin\theta = ma = m\frac{d^2s}{dt^2} \tag{10}$$

which can also be written as:

$$\frac{d^2s}{dt^2} = -g\sin\theta \tag{11}$$

By using the small-angle approximation (when angle θ is small, $\sin\theta \sim \theta$), we now have:

$$\frac{d^2s}{dt^2} = -g\theta = -g\frac{s}{l} \tag{12}$$

This equation shows that the pendulum is a type of *simple harmonic oscillator*, with the square of the angular frequency ω^2 equal to g/l. Using a relationship between the angular frequency ω and the period T:

$$T = 2\pi/\omega = 2\pi\sqrt{\frac{l}{g}} \tag{13}$$

We can rearrange this equation to find the value of g:

$$g = \frac{4\pi^2 l}{T^2} \tag{14}$$

Name: Date:

Questions:

1. Use an EXCEL spreadsheet or a computer program to calculate the sines of angles from 0° to 40° with 0.1° increments. (Hint: Remember that many of the programming languages as well as EXCEL use *radians* instead of *degrees* for angles.)

2. Make a plot with angles in radians on the horizontal and the sine values on the vertical axis, respectively. Add the $y = x$ line to the plot.

3. Upload an electronic copy of your spreadsheet and plot on Canvas and also "submit a hard copy of your plot along with the handout".

4. Based on your diagram, where does the small angle approximation work?

2.3 Moving Coordinate Systems

Motion is relative and therefore motion along with all its attributes such as position, velocity and acceleration are measured with respect to a point of space and time, or more precisely, to a *frame of reference.*

Let's assume a point of space is located by two vectors with respect to two separate origins, O and O^* (Fig. 4). Let's call the coordinates $\vec{r}$ and $\vec{r^*}$, respectively.

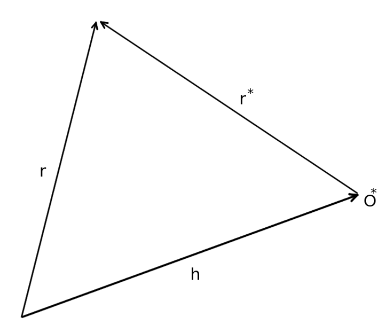


Figure 4: Different coordinate systems for a single point of space.

If $\vec{h}$ is the vector connecting O to O^*, then the relation between the too coordinates $\vec{r}$ and $\vec{r^*}$ will be

$$\vec{r} = \vec{r^*} + \vec{h} \tag{15}$$

or in Cartesian coordinates:

$$x = x^* + h_x \tag{16}$$
$$y = y^* + h_y \tag{17}$$
$$z = z^* + h_z \tag{18}$$

Question:

Show that if the origin O^* is moving with respect to origin O, then by simply taking derivatives from Eq. (15) we get

$$\vec{v} = \vec{v^*} + \vec{v_h} \tag{19}$$
$$\vec{a} = \vec{a^*} + \vec{a_h} \tag{20}$$

where $\vec{v}$ an $\vec{a}$ are velocity and acceleration respectively.

If we multiply both sides of Eq. (20) by mass, m, and rearrange, then from Newton's second law we have

$$\vec{F}^* = \vec{F} - m\vec{a_h} \tag{21}$$

Eq. (21) is the general equation for Newton's second law in a moving frame of reference. According to Eq. (21), the force experienced by a body in a moving coordinate system is different from that measured from outside the system by the amount $-m\vec{a_h}$ which obviously depends on the *relative* acceleration of the moving system with respect to the stationary one.

Hence, the term $-m\vec{a_h}$ is called a *fictitious force*. Great examples where such forces are experienced in daily life are elevators. Consider, for example, when an elevator starts to move from a stationary position downward. In this case, the person inside the elevator will feel an upward stretch due to his/her inertia and he/she will momentarily *feel lighter* until the elevator reaches a constant velocity. We know from Eq. (21) that this is because the elevator is accelerating in the same direction as the gravitational force (downward) and therefore the total "experienced" gravitational force, $\vec{F}^*$, will be less than the real force which is always constant at the same location. Note the minus sign in Eq. (21) (Fig. 5).

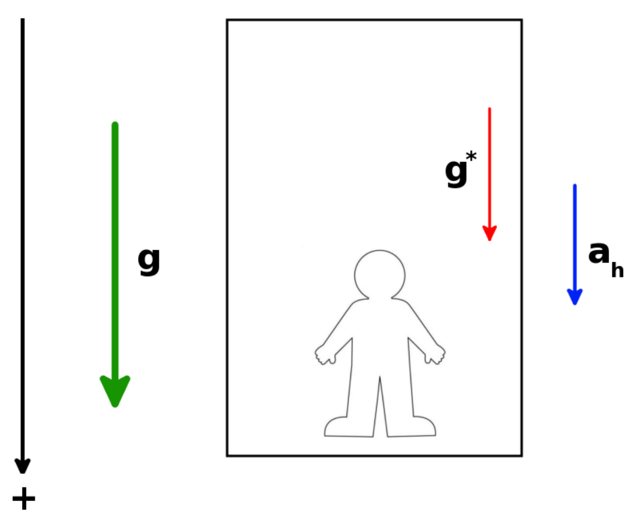


Figure 5: Apparent g in an elevator. The elevator is undergoing an acceleration in the direction of gravity and as a result, the experienced gravitational force by the person inside the elevator, $\vec{F}^*$ is going to be less than the actual value (note the minus sign in Eq. (21)).

Name: Date:

Questions:

1. Draw a simple sketch similar to Fig. 5 for the case when the elevator starts to move upward from a stationary position.

2. If you were to measure gravity in this elevator, would your measured value of g be smaller or bigger than the accepted value (9.8 m/s^2)? (Hint: Use your sketch to decide.)

Name: Date:

IN LAB

3 PART I: The Ball-Drop Experiment

A ball is dropped between two infrared sensors (gates) A and B connected to a timer (Fig. 6). This system yields:

1. the time the ball takes to pass through gate A, t_A
2. the time the ball takes to pass through gate B, t_B
3. the total time the ball takes to go from gate A to gate B, t_{AB}

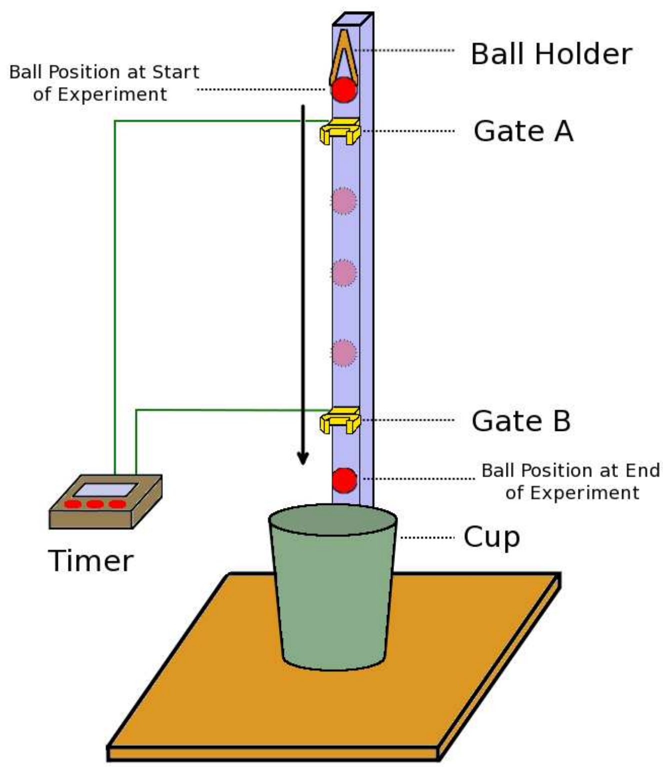


Figure 6: Schematics for the ball-drop experiment.

The ball's velocity at each gate is the diameter of the ball, D, divided by the time it takes to pass through the gate. Thus, the velocity of the ball at each gate is:

$$v_A = D/t_A \qquad\qquad v_B = D/t_B \tag{22}$$

Because acceleration is the time derivative of velocity,

$$a = dv/dt \tag{23}$$

the ball's acceleration due to gravity can be found by approximating the derivative with the differences:

$$g = \frac{v_B - v_A}{t_{AB}} = \frac{\frac{D}{t_B} - \frac{D}{t_A}}{t_{AB}} = \frac{1}{t_{AB}} \frac{D(t_A - t_B)}{t_A t_B} \tag{24}$$

Questions:

Here are a few things to think about before performing the experiment:

1. Using the equation for acceleration due to gravity, Eq. (7), which one of the two balls with different weights would you expect to fall faster?

2. How would positioning the time gates further apart affect the estimate of g?

Name: Date:

3.1 Experiment

1. Measure and record t_A, t_B, and t_{AB} using the steel ball and record your results in Table 1.
2. Find and record the average for these measurements.
3. Upload your "average" results onto the "Ball Drop" tab of an online spreadsheet at `goo.gl/UQuwv9`.
4. Repeat steps 1 to 3 using the plastic ball.

The diameter D of both balls is 1.9 cm (0.75 in).

Table 1. Ball-drop experiment results.

		Steel Ball			Plastic Ball		
Trial	Gate Distance (cm)	$t_A(s)$	$t_B(s)$	$t_{AB}(s)$	$t_A(s)$	$t_B(s)$	$t_{AB}(s)$
1	20						
2	20						
3	20						
4	20						
5	20						
Avg.	20						
1	30						
2	30						
3	30						
4	30						
5	30						
Avg.	30						
1	40						
2	40						
3	40						
4	40						
5	40						
Avg.	40						
1	50						
2	50						
3	50						

4	50						
5	50						
Avg.	50						

Questions:

1. For each gate distance, calculate the *mean*, the *median* and the *standard deviation* of the average data uploaded by all the groups.

   Important:

   - Do this part only when all the groups from both lab sessions have uploaded their results.
   - Do NOT do this part in the online spreadsheet.

2. How do the times for the two balls compare?

3. Calculate *four* values of g from the average times for the steel ball.

4. What is the average of the four?

5. How well do your measurement of g agree with the accepted value of 9.8 m/s^2? List at least *three* causes for a difference?

4 PART II: The g Ball

As we saw in the lectures, you can ideally measure g by simply dropping an object and measuring the time it takes to hit the ground. The challenge, however, is measuring the time accurately. In this part, we are going to use a nifty tool called the "g ball" to calculate the acceleration due to gravity. The g ball is slightly bigger than a baseball with a timer inside which is sensitive to impact (Fig. 7).

Figure 7: The g ball.

To use the g ball:

– Push the reset botton (aptly labeled "PUSH") to make sure the timer is set to zero.

– Hold the balls steadily in your hand.

– Carefully push the button and release the ball *immediately*.

Note: The latter takes some practice to do perfectly. You need to drop the ball right after you push the button in order to reduce time measurement error.

4.1 Experiment

1. Drop the g ball from stationary positions at various heights according to Table 2 and report the times.

   Important: As we saw in the lectures, when working with volume forces, the distance between objects are measured from their respective centers of mass. Therefore, here you should measure both the initial and final heights from the center of the ball, rather than from its side (Fig. 8).

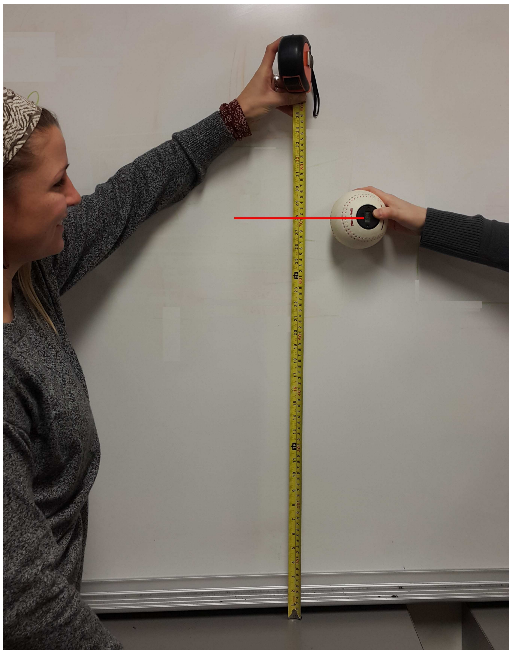


Figure 8: The g ball experiment. Note that measurements are done at the ball's center (red line).

Name: Date:

Table 2 g ball experiment results.

Initial Height (cm)	Drop Time (s)	Drop Time Squared (s^2)
40		
40		
40		
50		
50		
50		
60		
60		
60		
70		
70		
70		
80		
80		
80		
90		
90		
90		
100		
100		
100		
120		
120		
120		
150		
150		
150		

2. Using EXCEL, plot the data in Table 2 with squared times and initial heights on the horizontal and vertical axes, respectively.
3. On the same plot, add a trend line to your points.
4. Use the trend line equation to find the slope of this line.

slope = __________

5. Upload your spreadsheet and plot onto Canvas. Also submit a hard copy of the plot along with your labs.
6. Show that the slope of this line is proportinal to g. (Hint: Check your answer to the question in the review section.)

7. From the slope you obtained above, calculate g (watch for the units!).

$$g = \underline{\hspace{3cm}} \text{ m/s}^2$$

5 PART III: The Pendulum Experiment

As we saw in the pre-lab section, another good way to measure g is the simple pendulum. This is done by measuring the period of oscillation for a pendulum and using Eq. (14).

5.1 Experiment A

1. Using the pendulum set-up, measure the period of oscillation, T, when the string makes a $\theta = 10°$ angle with the vertical direction and report the result in Table 3 (Hint: For better results, measure the oscillation time for 10 periods and then divide the time by 10.)

2. Repeat the measurement for a $\theta = 40°$ angle.

Table 3 Pendulum experiment (A) results.

Initial Angle (deg)	No. of Oscillations	Total Oscillation Time (s)	Period (s)
	10		
	10		

Questions:

1. How do the values for period compare?

2. Justify your answer, using Eq. (14).

3. Why would you calculate the period by measuring the time for a number of oscillations rather than directly measuring a single oscillation time (= period)?

5.2 Experiment B

1. Time various numbers of oscillations of the pendulum with different lengths and fill out Table 3.

2. Average the values for period in Table 3 for every given length.
3. Upload all of your results for periods onto the "Pendulum" tab of the online spreadsheet at `goo.gl/UQuwv9`.

Table 3. Results for the pendulum experiment.

Trial	No. of Oscillations	Length (cm)	Time (s)	Period (s)
1	10	20		
2	10	20		
3	10	20		
4	20	20		
5	20	20		
6	20	20		
7	30	20		
8	30	20		
9	30	20		
10	40	20		
11	40	20		
12	40	20		
Avg.	-	-	-	
1	10	56		
2	10	56		
3	10	56		
4	20	56		
5	20	56		
6	20	56		
7	30	56		
8	30	56		
9	30	56		
10	40	56		
11	40	56		
12	40	56		
Avg.	-	-	-	

4. Calculate the acceleration due to gravity, g, for the averaged values of the period.

Name: Date:

Calculated value of g for the short pendulum :____________

Calculated value of g for the long pendulum :____________

Questions:

1. For each pendulum length, calculate the *mean*, the *median*, and the *standard deviation* of the periods uploaded by all the groups.

   Important:

   – Do this part only when all the groups from both lab sessions have uploaded their results.

   – Do NOT do this part in the online spreadsheet.

2. How do the periods for the two pendulum lengths compare?

3. Calculate g using the values from the average periods. How well do your measurements of g agree with the accepted value of $9.8\ \mathrm{m/s^2}$? List at least *three* causes for a difference?

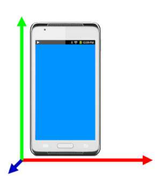
4. How well do your two measurements of g (from the ball drop and the pendulum) agree? List two possible explanations for any discrepancy.

6 PART IV: Measuring g in a Non-Inertial Frame of Reference

Another interesting way to measure gravity is using *accelerometers*. As we will see in the next lab, accelerometers are essentially mass-spring setups that are used to measure the acceleration a body is undergoing. They are also widely used in smartphones and therefore you can measure g with your phone.

Smartphone Apps

In this section, we will use smartphone apps to record g. You can use your favorite app to do so, but make sure that the app you choose can "save" the displayed values in a file that you can later download or share. A good example of such apps for Androids is *G-Sensor Logger*.



Name: Date:

6.1 Experiment

In groups of two, use your smartphones to do the following:

1. Pull up the "*g*" app on your phone and prepare to start it up.
2. Get on the Elevator # 3 in the Tech building (shown below) and go to the 4th floor.

Figure 9: Map view of the 2nd floor of the Tech building. The lecture room and elevator #3 are marked with green and red, respectively.

3. Put your phone (face up) on the elevator's floor (perhaps on a piece of paper – but not a napkin – to keep it clean).
4. Push the button to the first floor and wait for the elevator door to close.
5. Quickly start the app.

6. Wait until the elevator reaches the first floor and then stop recording.

   Note:

   You may need to repeat this several times until you get a "clean" shot, that is when the elevator does not stop in between the 1st and the 4th floor.

7. Make sure to save the record file.

Questions:

1. Use the space below to draw a simple sketch of what you would expect the "record" to look like.

2. Use EXCEL or your favorite plotting program to plot the recording values of g with respect to time. Consult with your lab session coordinator to make sure you have recorded the correct values.
3. How many distinct segments of acceleration can you identify in your record? Mark them on your plot?

4. If you were to measure g in the elevator using a pendulum, how would you expect the period of oscillation change at each segment?

5. What is the base value of g in the record?

6. What are the minimum and maximum values of g in the record?

7. Calculate the minimum and maximum periods of oscillation (in seconds) for a pendulum with the length of 50 cm in the elevator?

8. Upload the CSV time series as well as your plot onto Canvas. Please also turn in a hard copy of your plot along with your lab report.

   Note: Please mention the names of all the people in your group while submitting your files on Canvas.

7 PART V: Interpretation

7.1 Earth's Structure

Using the values you determined for g, the Earth's average radius of 6371 km, and the gravitational constant G of 6.67×10^{-11} Nm^2/kg^2, we can find the Earth's mass and average density. By rearranging equation (7), Earth's mass is found using:

$$M = \frac{gR^2}{G} \tag{25}$$

Name: Date:

Questions:

1. Using the relationship above (with the average g from your three experiments), calculate the mass M and the average density ρ_0 of Earth (see your answer to the question on page 2).

2. How do these results compare with the accepted values of mass and average density of 5.98×10^{24} kg and 5.5×10^3 kg/m^3, respectively?

3. Crustal Earth rocks have an average density of $\rho_c = 3$ g/cm^3. How does this compare with ρ_0? Use a simple sketch (Hint: What does this imply about Earth's density structure?)

Name: Date:

7.2 Gravity Anomalies

We expect from Eq. (7) that the measured values for gravity to be the same everywhere on the surface of the Earth (why?). However, accurate measurements have proved this assumption to be wrong. Below is a map of the so-called gravity anomalies on the surface of our planet (Fig. 10). Gravity anomalies are deviations from the expected acceleration of gravity at the surface. They can be either negative or positive as shown in Fig. 10 which correspond to less or more than expected values of g.

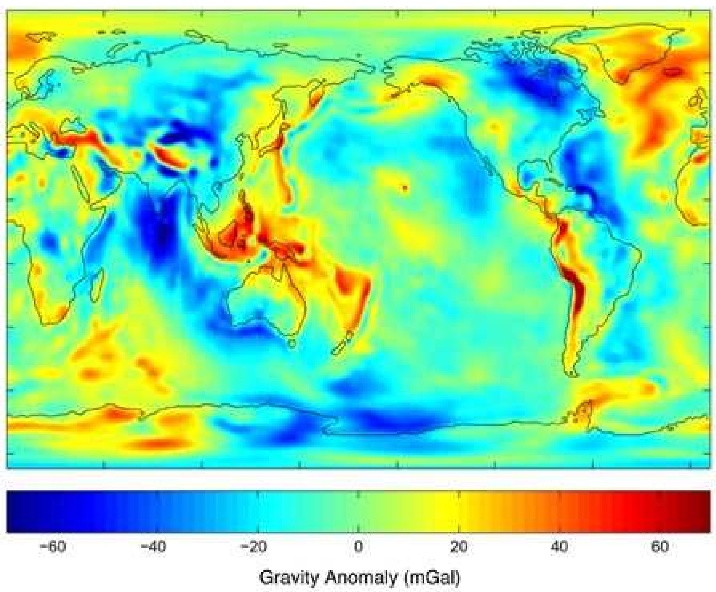


Figure 10: Gravity anomalies on the surface of the Earth measured by the GRACE satellite (image courtesy: NASA).

Questions:

1. Going back to Eq. (7), why do you think we observe these anomalies? (Hint: think of each variable in the equation)

2. We will revisit this concept while studying plate tectonics. Plates are assumed to be huge rigid bodies of rocks on the surface of the Earth, sometimes spanning entire continents. What do you think "positive" or "negative" anomalies tell us about these plates? Use a simple sketch (Hint: remeber Lab #0). Draw a cross section.

7.3 Comparison

Find the average density in g/cm^3 to two significat figures of:

1. Earth: radius 6371 km, mass 6×10^{24} kg
2. Venus: radius 6051 km, mass 5×10^{24} kg
3. Mars: radius 3389 km, mass 0.6×10^{24} kg
4. Mercury: radius 2439 km, mass 0.3×10^{24} kg
5. Moon: radius 1738 km, mass 0.07×10^{24} kg

Name: Date:

- Compare the "mean" density of each body to the density of Earth's surface rocks. What can you conclude?
- Identify each planet on the graphic below.
- Plot the density versus mass for each planet. What can you infer?