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Lab # 6

Heat

Instructions: *Numerical answers require units and appropriate numbers of significant digits.* ***Remember to show your work!***

PRELAB

1 Introduction

As we saw in the lectures, the evolution of planets' temperature distribution (their thermal history) and the different means of heat transfer inside and in/out of them play crucial roles in their dynamic systems. Therefore, we can consider heat as "the geological lifeblood of planets". Again, as we saw in class, it is of utmost importance to recall two key concepts.

- **Heat** is a measure of the internal *energy* of a body.
- **Temperature** is a measure of the heat contained and the ability to transfer heat.

A useful physical analogy is Feynman's:

> *"heat is like the amount of water in a towel, while temperature is the 'wetness.' Given two towels of the same 'wetness' the bigger one contains more water. A small towel can be much 'wetter' but contain less water than a big towel."*

Heat can be transferred through one or more of the following processes:

Radiation: Heat is transfered by electromagnetic waves and therefore the process does not require any material between the source and the receiver

Conduction: Heat is transferred by molecular collisions through a solid object. If you touch something hot, the heat is transferred into your fingers by conduction.

Convection: Heat is transferred by motion of the material - for example, by a hot fluid. As the bottom of the pot heats up the water starts to convect, as hot water rises, transporting heat upward. Solid rocks convect at temperatures found in the mantle.

Among these processes, conduction and convection are controlled by temperatures of all of the bodies involved.

Question

Explain by which of the processes listed above, is heat transferred

- From the Sun to the Earth

- From the Moho to the Earth's crust

- From the CMB to the upper mantle

- From the upper mantle to the CMB

In this lab, we are going to investigate some principles behind the heat transfer methods.

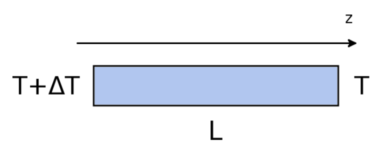
1.1 Conduction

As we saw in class, due to *Fourier's Law of Conduction*, in any given system the direction of spontaneous heat flow is from the hot parts to the colder parts.

$$q = -k\frac{dT}{dz} \tag{1}$$

where q is the heat flux and k is *thermal conductivity*.

For example, if a stripe of length L and temperature T is heated at one end so that end would have a $T + \Delta T$ temperature,



the heat will flow from the hot end to the colder end. Then

$$\begin{aligned}\frac{dT}{dz} &= \frac{T - (T + \Delta T)}{L} \\ &= -\frac{\Delta T}{L}\end{aligned} \tag{2}$$

and by combining Eqs. (1) and (2) we get,

$$q = k\frac{\Delta T}{L} \tag{3}$$

Notice that the minus sign is set up to make heat go the right way – from hot to cold.

1.1.1 Heat Transfer in Layered Media:

Transfer of heat from an arbitrary source through layered media is a complex problem. However, we can simplify this problem by assuming a large-enough plane source from which a uniform flow of heat is passing perpendicularly through a non-dissipative ($Q_{in} = Q_{out}$) layered medium as shown in Fig. 1. In this simple 1-D case, we can use the discussion in section 1.1 and write Eq. (3) for individual layers (see Fig. 1).

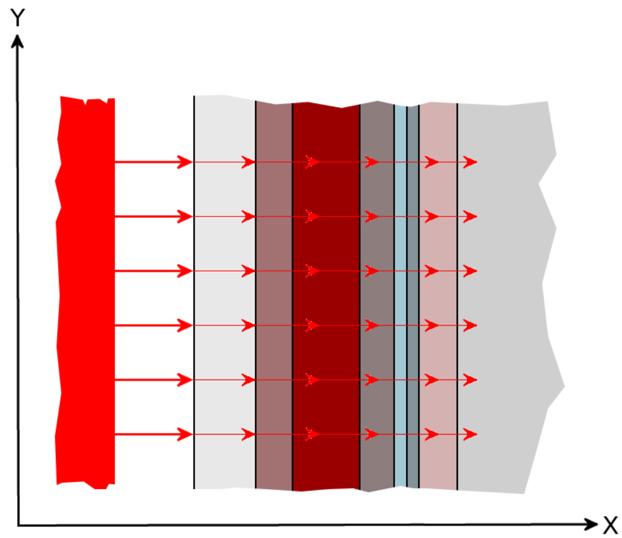


Figure 1: Heat from a *homogeneous* source passes through a non-dissipative layered medium.

Questions

1. Show that for two non-dissipative consecutive layers with identical thicknesses (made from different materials), the temperature gradients of layers are inversely proportional to their thermal conductivities, or

$$\frac{\Delta T_1}{\Delta T_2} = \frac{k_2}{k_1}$$

2. Show that for two non-dissipative consecutive layers made from the same material, the temperature gradients of layers are directly proportional to their thicknesses, or

$$\frac{L_1}{L_2} = \frac{k_1}{k_2}$$

1.2 Radiation

From the black body radiation theory, it is known that a body of absolute temperature T emits a thermal energy of Q_R through radiation as

$$Q_R = \sigma T^4 \tag{4}$$

where $\sigma = 5.670373 \times 10^{-8}$ Wm^{-2}K^{-4}, is called the Stefan-Boltzmann constant.

1.3 Convection

Convection is an important process for heat transfer in the Earth's mantle and outer core. Even though the Earth's mantle is solid and made up of rocks, on geological time-scale, we can consider it as a fluid and thus capable of convection. The basic difference between convection and conduction is that in conduction, the kinetic energy is transferred between individual atoms or molecules which are vibrating around their constant centers of massed in order to get from one part of the system to another, whereas the atoms or molecules of the convecting will move and *carry* the energy to the colder parts.

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IN LAB

IMPORTANT NOTES:

When working with the IR thermometers, please pay attention to the following points:

- DO NOT expose the thermometer to direct sources of heat for extended periods.
- DO NOT point laser-beams at another person.
- DO NOT point laser-beams at another person's eyes.
- NEVER look into the laser beam.
- The optimum and most accurate result can be obtained if the distance between the thermometer and the concerned surface is ~ 30 cm (~ 12 in).

  This is because the IR sensor has a distance-spot ratio of 12:1 which means the surface area measured has a diameter of roughly 1/12 of the distance. Since the sensor will cover different areas of the target object at different distances (similar to the penny & Sun problem in problem set 1), changes in distance will affect the effective target area.

- MEASURE THE TEMPERATURE FROM THE SIDE. Always aim at the heated stuff from the side, to avoid unwanted heat transfer factors (can you identify them?).
- Measuring temperatures:
  - Press and hold the trigger to activate the thermometer and point the IR sensor at desired point on the surface of the object.
  - Release the trigger to get an initial value of the surface temperature.
  - Press the trigger again for at least one second to get additional readings.
  - You can use the laser-pointing system for better aiming accuracy.

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CAUTION: NEVER touch the hot plate of the heater. Be careful when handling the hot coins!

Note:

- You need to divide into groups of 4 or 5 to do this lab.
- *[optional]* You may find it easier to record the readings directly into EXCEL spreadsheets and make the plots from those data.

2 Conduction

2.1 Structure of the Heat Source

Measure the temperatures for "four" points at different distances from the center (see the Fig. 2) and write the results in *Kelvins* Table (1).

NOTE: The Electric heaters used for this lab are already set to "WARM".

Figure 2: Measurement points on a heating plate.

Point	T(K) (trial # 1)	T(K) (trial # 2)	T(K) (trial # 3)	$T_{average}$ (K)
A				
B				
C				
D				

Table 1: Temperature measurement for the heating plate.

Questions

1. How do the temperatures for the four points compare?

2. From your temperature readings for the heating plate, where do you think the heating element is located below the plate? Why?

3. Calculate the temperature gradient for the disk.

2.2 Layered Medium

Build a tower of 20 *pennies* on the heater's plate. Follwing your results in section 2.1, make another tower of 16 *nickels* at an identical thermal geometry on the plate. Follow through these steps to fill out Table (2).

1. Use the IR thermometers to measure the temperature at the base and top of both towers.

2. Repeat the measurements every minute for 10 minutes.

	PENNIES		**NICKELS**	
Time (min)	Tower Base	Tower Top	Tower Base	Tower Top
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Table 2: Measurements over time.

Questions

1. Using your favorite plotting software, plot the base & top temperatures for both towers over time.

2. Measure/Calculate the length of each tower.

   Length of the Penny Tower = ______________ mm

   Length of the Nickel Tower = ______________ mm

3. Why did we use different numbers of coins for each towers?

4. Use the last row from Table (2) to calculate thermal conductivities for the each of the towers.

5. Would the k values you have calculated for penny and nickel towers be different from those of *a single* penny and *a single* nickel? Explain.

6. Using your results from above as well as the table below, estimate what metals are used in nickels and pennies.

Metal	*Density* (*kg/m*^{3})	*Melting point* (*°C*)	*Thermal conductivity* (*W/m°C*)	*Coefficient of linear expansion at 20°C* (*μm/m/°C*)
Aluminium	2700	660	220	23.0
Brass	8450	950	130	16.7
Bronze	8730	1040	67	17.3
Cast iron	7250	1300	54.5	9.0
Copper	8900	1083	393.5	16.7
Lead	11 400	327	33.5	29.1
Monel metal	8600	1350	25.2	14.0
Nickel	8900	1453	63.2	12.8
Silver	10 500	960	420	18.9
Steel	7850	1510	50.2	11.1
Tin	7400	232	67	21.4
Tungsten	19 300	3410	201	4.5
Zinc	7200	419	113	33.0
Cobalt	8850	1490	69.2	12.4
Molybdenum	10 200	2650	13	4.8
Vanadium	6000	1750	—	7.75

7. According to what you have learned in class, explain why you had to wait for 10 minutes?

8. Using the same concept, we can do calculations for the Earth and derive the temperature at the Earth's core. Based on the approximations you used here as well as what you have learned in class, explain why this method (at least as is) cannot be applied to the Earth.

3 Radiation

Fill out Table 3 by following through these steps:

1. Measure the diameter of the cental part of the plate that has an almost uniform temperature by a ruler (you may think of your results from section 2.1.
2. Put the heater's dial on "LOW" and wait for 2 minutes.
3. Measure the surface temperature of that segment of the heating plate using an IR thermometer.
4. Use a piece of paper to clamp a penny on the stand.
5. Place the penny at 1 cm above the heating plate (see Fig. 3).

Figure 3: Use a piece of paper to clamp the penny.

6. Measure the temperatures of both the plate and the penny using the IR thermometer every minute for 10 minutes and continue until both reach a constant value. Write your observations in the table below.

Hints:

- Try to measure the temperatures at the same points on the plate and the penny (why?).
- To measure the penny's temperature, quickly rotate the stand so that the penny will move out and away from the plate (why?).

- After measuring the penny's temperature, move it back to its initial place quickly.

7. Using the Stefan-Boltzmann equation, calculate the emitted heat for each measurement.
8. Calculate the heat flux (= heat per unit area) for the plate at each step.

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Time (min)	Plate T (°C)	Heat (J)	Heat Flux (J/m^2)	Penny's T (°C)	Heat (J)	Heat Flux (J/m^2)
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

Table 3: Measurements over time

Questions

1. If "all" of the thermal energy from the heater was transferred to the penny, would you expect the penny to have a lower, equal or higher temperature compared to the heater? Explain.

2. Assuming that both the penny and the heating plate radiate the heat homogeneously (which you know is not true), since the penny is very close to the radiating disk, show that a good estimate for the portion of the total heat emitted by the disk that is received by the penny is given by (see Fig. 4)

$$\frac{Q_{\text{penny}}}{Q_{\text{disk}}} = \left(\frac{r}{R}\right)^2 = \left(\frac{T_{\text{penny}}}{T_{\text{disk}}}\right)^4 \tag{5}$$

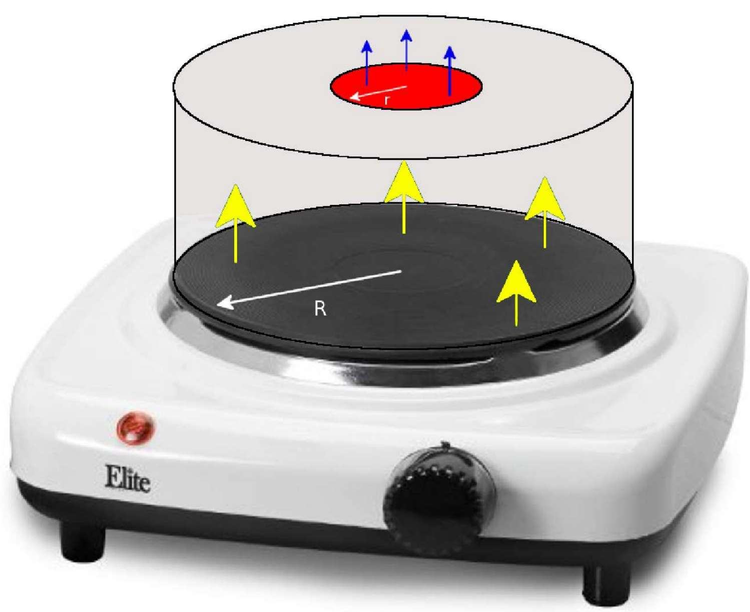

Figure 4: Penny over the heater.

3. Use the data from the last row of Table 3 to find the right-hand side ratio in Eq. (5). Is it equal to the ratio in the middle term? Explain.

4. A similar method can be applied to the Sun and the Earth. Derive a similar equation for the Sun (radius R_S) and the Earth (radius R_E), assuming that *the Sun is very far from the Earth* and the cylindrical estimate (as in Fig. 4) is no longer valid (Hint: You may start by considering a *spherical* system). Draw a simple sketch to visualize the problem. How different is your equation from Eq. (5)?

5. The Sun radiates $\sim 60,000,000$ W/m^2 from its surface. Using your answer to the previous question, and assuming the Sun and the Earth have radii 700,000 km and 6,000 km respectively, and that the Earth is orbiting the Sun at 150,000,000 km, calculate the heat (in W/m^2) received by the Earth.

3.1 Convection

Note: For this part of the lab you will need an app on your phone to either record a video at high speed or convert a recorded video to higher frame rate.

Follow through these steps:

1. Fill $\frac{2}{3}$ of a 300 mL measuring cup with tap water (i.e. to 200 mL).
2. Fill a second 300 mL cup to the 75 mL line with tap water.
3. Add about a fistful of forzen green peas to the 200 mL cup and almost half that amount to the 75ml cup.
4. Measure the height of the water column in the cup.
5. Set the heater's dial to "MEDIUM" and put the 200 mL cup at the plate's center.
6. Wait for 7 minutes and then put the 75 mL cup on the burner, next to the first one.
7. Measure the temperatures at the bottom (T_B) and the surface (T_S) of the water column in both cups every minute for 10 minutes and fill out Table 4.

8. Record the process using your phone. An example snapshot of such a video is shown in Fig. 5.

Figure 5: A snapshot of the convection experiment.

	200 mL Cup		**75 mL Cup**	
Time (min)	T_B(K)	T_S(K)	T_B(K)	T_S(K)
1			-	-
2			-	-
3			-	-
4			-	-
5			-	-
6			-	-
7			-	-
8				
9				
10				

Table 4: Temperature measurement for the top and the bottom of the cups.

Questions

1. What happens to the water and peas in each cup after ~ 5 minutes? Why?

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2. Can you identify the upper boundary layer? Take a picture of the convection cell and circle this layer in the picture. Attach a copy of this figure to your lab report.

3. Use your favorite software to edit your recorded video to speed it up and then upload the result to Canvas (ideally, it should not exceed a few MB in size).

4. Using the values from below, and your measurements, calculate the Rayleigh number (Ra) for both cups (be careful with the units!). How do these values explain your answer to the previous question?

   density of water = 1 g/cm^3, thermal expansion coefficient for water = 4.2×10^{-2}, heat capacity of water = 4.2 J/gK, viscosity of water = 9×10^{-4} Pa.s, thermal conductivity of water = 0.6 W/mK, acceleration due to gravity = 9.8 m/s^2

5. Watch the video at `https://goo.gl/2bPWsH` and

   (a) Explain the reason for the difference in the behavior of the two convection cells.

(b) Describe the main difference between what you observe in the three playback speeds (normal, 15x, 30x).

(c) Use your answer to (b) to describe the time-scale of the convection of Earth's mantle.