Name: Date:

Problem Set # 5

Instructions: *Numerical answers require units and appropriate numbers of significant digits.* ***Remember to show your work!***

Review Problems:

R-1. (12 points) The process of accreting a planet involves converting potential energy to kinetic energy. To see how this works, let's review some basics:

a. The kinetic energy of an object with mass m moving at velocity v is $\frac{1}{2}mv^2$, and the gravitational potential energy of an object with mass m at height h above the Earth's surface is mgh. Show that both potential and kinetic energy have the dimension of joules.

b. If an object of mass m is dropped from a height h, find its kinetic energy when it hits the ground.

c. How does this result depend on the Earth's mass and radius?

d. Compare the potential and kinetic energies when the object is dropped and when it hits the ground. Explain the result from the view of conservation of energy.

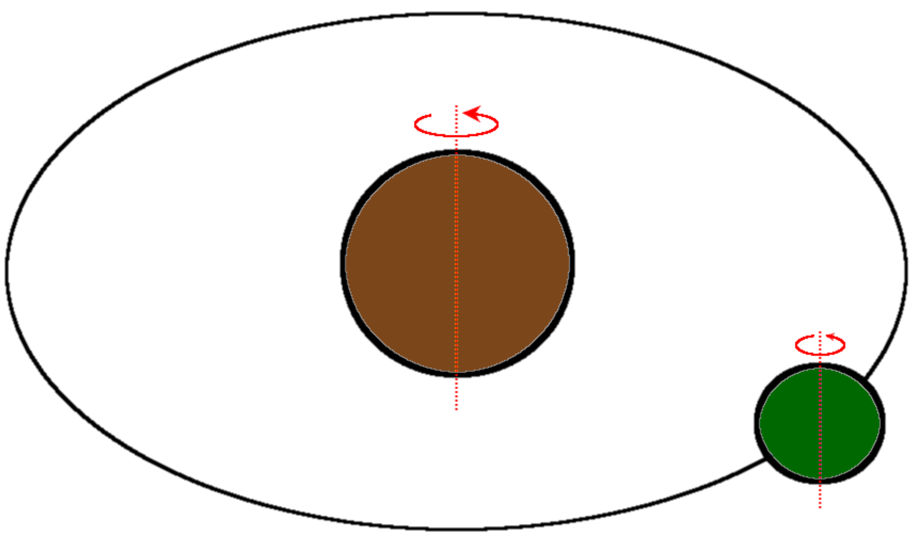
R-2. (6 points) Momentum, the product of an object's mass and velocity, affects its behavior in collisions like those between bodies in the nebula as the solar system forms. Considering momentum explains how in the Bible, David killed the giant Goliath with a stone launched by a sling. Compare the momentum of an 80 grams stone fired from a sling at 30 m/s to that of a 6 grams bullet fired from a gun at 400 m/s. What do you conclude?

1. (8 points) The formula for the gravitational potential energy that binds an object of mass m to a planet with mass M and radius r is $U = \frac{GmM}{r}$. This is also the energy released when bringing an object in to a planet from an infinite distance away, and thus is used when calculating the energy of an accreting planet.

(a) An object from an infinite distance away is brought to the Earths surface. How fast is it traveling when it reaches the planet?

(b) An object from an infinite distance away was brought to the Earth when it was still accreting. If this proto-Earth had a radius of only 200 km, how fast was it traveling when it hit the surface?

2. (26 Points) Much like linear momentum, angular momentum is the momentum associated with rotating bodies. In the absence of external torques, its value is always conserved.



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(a) Rotation about a spin axis: The angular momentum is $L_{rot} = I\omega_{rot}$, where I is the moment of inertia, and ω_{rot} is the angular velocity in radians per second. Treat the sun and the planets as rigid bodies, and remember that the moment of inertia factor for a body of mass M and radius a is $k = I/ma2$. Angular velocity is calculated by dividing the number of radians (2π) by the time it takes to complete one rotational period. Using these facts and the values given below, fill in the blanks below **with the units used in the table**. To see where most of the rotational angular momentum resides, compare the angular momentum ratio of the Earth/Sun and Jupiter/Sun.

No.	Body	Rotational Period (days)	Equatorial Radius (km)	Mass (10^{24} kg)	Moment of Inertia Factor	ω_{rot} (rad/s)	Moment of Inertia kg.m^2	L_{rot} kg.m^2/s
1	Sun	27	6.96×10^5	2×10^6	0.006			
2	Earth	1	6371	5.97	0.331			
3	Mars	1.02	3400	0.64	0.366			
4	Jupiter	0.40	71900	1900	0.25			
5	Saturn	0.43	60200	520	0.22			

(b) Orbital motion of a body about the center of mass: Treat the planets as point masses that resolve around the center of mass for the solar system (a very good approximation for the center of the sun). Also assume that this orbiting is confined to the ecliptic, a plane that is perpendicular to the spin axis of the sun. These two assumptions allow you to calculate the angular momentum associated with revolution as $L_{orb} = mr^2\omega_{orb}$ where m is the mass of the planet, r is the orbital radius, and $\omega_o rb$ is the angular velocity of the planet's orbit (2π radians divided by the planetary orbit time). Using these facts and the values given below, fill in the blanks below **with the units used in the table**. Where does the most orbital angular momentum reside?

No.	Body	Revolution Period (years)	Mean Orbital Radius 10^6 km	ω_{orb} rad/s	L_{orb} kg.m^2/s
1	Earth	1	150		
2	Mars	1.9	228		
3	Jupiter	11.9	778		
4	Saturn	29.5	1427		

(c) Sum up the total angular momentum (both rotational and orbital) for the bodies we considered: the Sun, Earth, Mars, Jupiter, Saturn. Compare this sum to the angular momentum found for each body. Where does the most angular momentum reside?

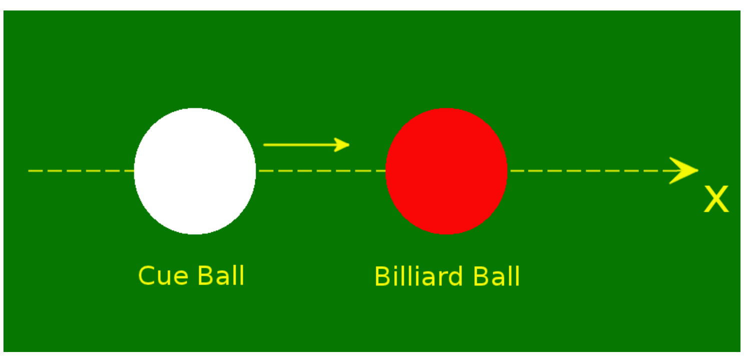
3. (18 points) Consider these two momentum equations for elastic and inelastic collisions:

$$m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4 \qquad \text{(Elastic Collision)}$$
$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_{12} \qquad \text{(Inelastic Collision)}$$

Elastic collisions are what occur when kinetic energy is conserved, inelastic collisions occur when some of the energy is lost (e.g. as heat). m_1 and m_2 are the masses of the objects with initial velocities v_1 and v_2. These change to velocities v_3 and v_4 in an elastic collision, or v_12 when the masses combine in an inelastic collision. We will illustrate elastic collisions with billiard balls, and while planetary bodies behave almost entirely inelastically (but can be modeled as purely inelastic).

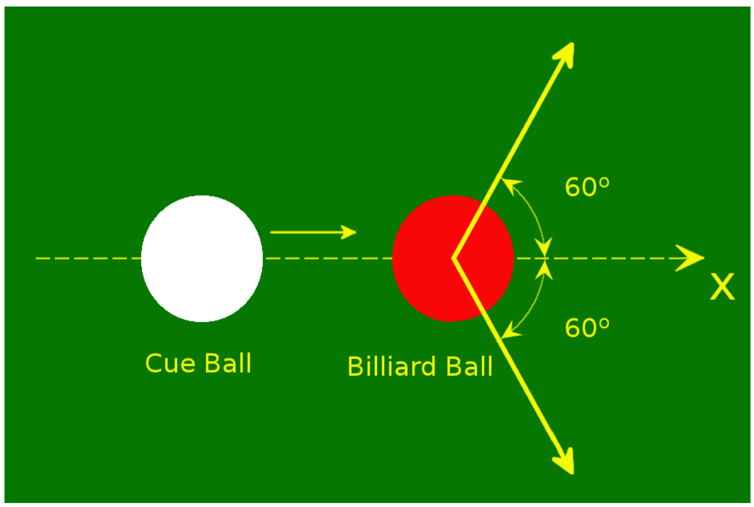
(a) Elastic Collisions

A cue ball traveling with a **speed** (the magnitude of the velocity) of 4 m/s collides with a stationary billiard ball and imparts a speed of 2.4 m/s to the billiard ball. If the billiard ball is sent forward in the same line as the incident cue ball, what will be the speed of the cue ball after the collision? (Hint: Assume both balls have the same mass and conserve the systems momentum).



(b) Velocity and Elastic Collisions

Now assume that instead of a straight shot, you wish to send the billiard ball 60° to the side. This requires that you consider the magnitude and direction of the motion (termed **velocity**). Given the same two balls, and the initial cue ball velocity of 4 m/s along the x-direction (as shown below), and a velocity for the billiard ball of 2.4 m/s 60° off the x-direction (as shown). Determine the velocity (magnitude and direction) of the cue ball after collision (Hint: Conserve momentum independently in both the x- and y-direction).



(c) Inelastic Collisions

Consider the collision of two bodies of masses m_1 and m_2 that stick together upon collision. Let m_2 be at rest initially (as if you were standing on the second body watching the first), and let v_1 be the velocity of m_1 before the collision.

(i) Describe the motion of the system after the collision (give the magnitude and direction of the velocity after the collision).

(ii) What is the ratio of the final kinetic energy to the initial kinetic energy (Hint: Use the results from Part (i), you should end up with a function that is only dependent on the masses involved).

(iii) What is the numerical value of this ratio for a planetesimal accreting into a planet (or for that matter for a meteorite striking the Earth)? Explain why this is.