Rayleigh Wave Displacement Fields
from Finite Sources

> The outputs of this package can be readily
> used in tsunami simulations.

1　Motivation

Saito et al. (2019) have used an elastic wave approach (without actually explaining any of the details[1]) to produce Rayleigh wave amplitudes along the Nankai trough. Then they incorporate these amplitudes in a tsunami simulation model to calculate total tsunami wave heights. However, their approach is cumbersome utilizing a finite difference scheme – which is obviously very time-consuming.

An alternative and much simpler approach – which they avoid without a good explanation – is using normal modes of the Earth. Given an appropriate velocity model and knowing the focal geometry of an earthquake, one can compute Raleigh wave displacements on the Earth's surface at any point with known distance and azimuth from the source. This is possible through calculating spheroidal modes of the Earth and stacking them up to any desired period to obtain appropriate excitation coefficients.

The caveat here, is that calculation of higher harmonic degrees (ls) becomes difficult and unstable due to the numerical issues in handling large numbers (at such large ls using the asymptotic solutions is not possible).

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[1]Very much a black box!

However, due to the long period nature of tsunamis, it is not necessary to continue the computation of excitation coefficients up to very large ls.

Here, a method similar to that proposed by Saito (1967) is used to compute the excitation coefficients up to $_0S_{60}$ – corresponding to periods of $\sim 150 - 200$ s. Then using these coefficients, long period synthetics can be calculated *for a point source* (e.g., Kanamori, 1970; Kanamori & Cipar, 1974; Okal, 1976). Fig. 1 shows a synthetic seismogram calculated *at the epicenter* of the 2011 Japan earthquake.
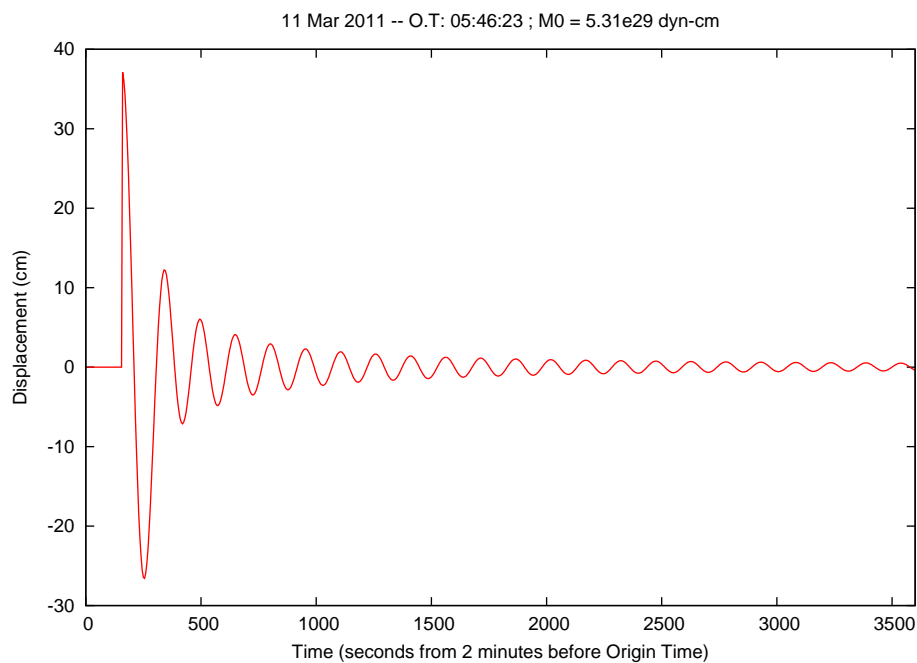

Figure 1: Synthetic displacement of Rayleigh waves from a point source (CMT) solution of the 2011 Japan earthquake calculated by exciting up to $_0S_{60}$ resulting in a dominant period of ~ 150s. The horizontal axis is time in seconds measured from 2 minutes before the origin time.

2   Displacement Field

In order to calculate displacement fields, we prepare a topography/bathymetry grid for the area of study. While this grid does not need to have the same

resolution as that used in further simulations (e.g., tsunami studies), it is a good idea to keep the purpose of preparing this field in mind. We also recall that fewer points will result in faster computations.
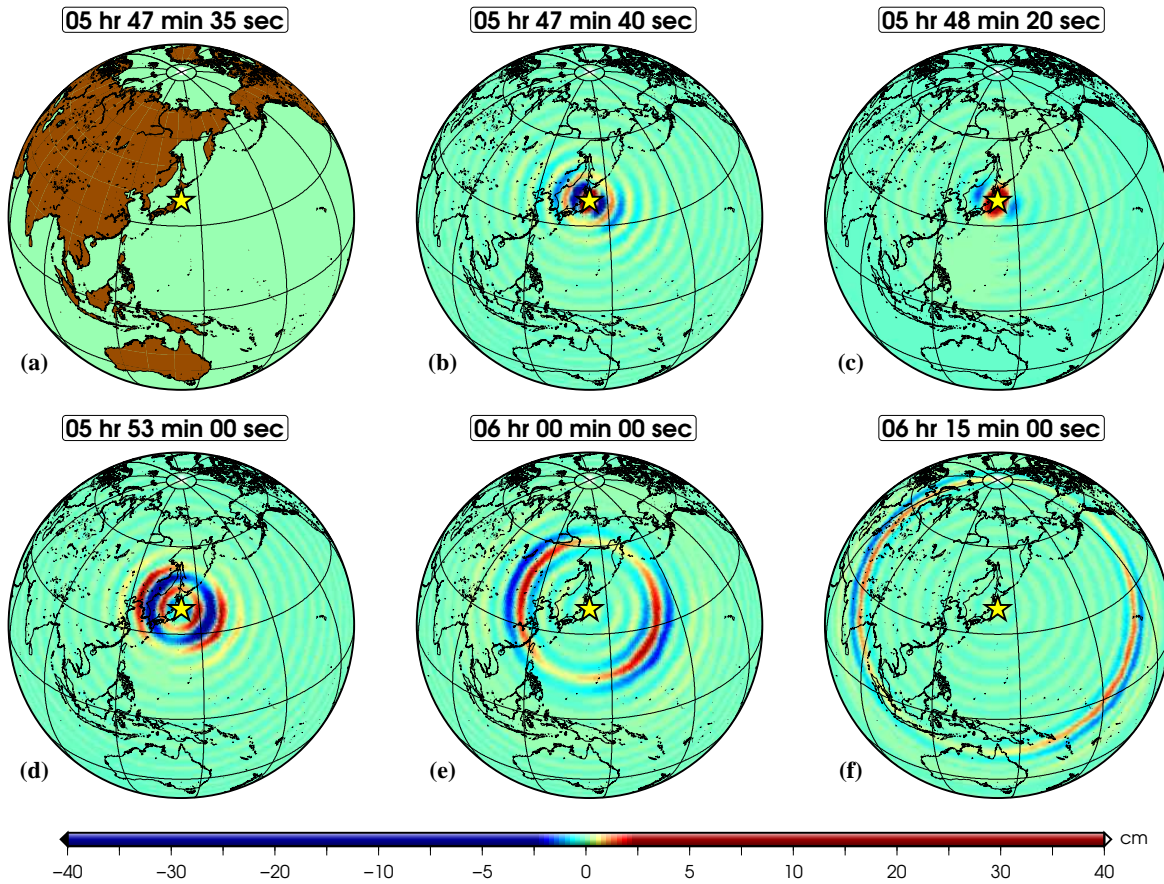
As the next step, we calculate distances and azimuths of each grid point, i.e., each station with respect to the our epicenter, as for example shown in Fig. 2.
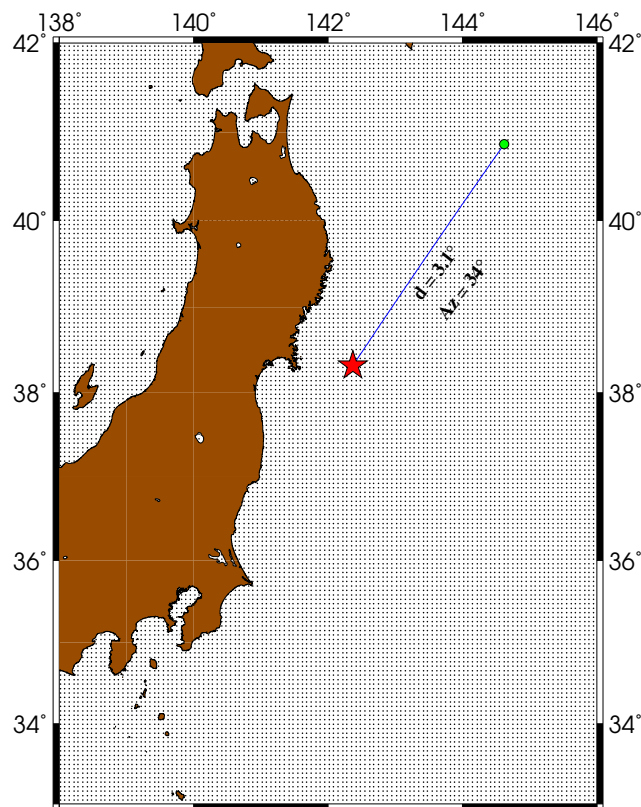

Figure 2: Distance and and azimuth of each grid point (as our "stations") are calculated with respect to the epicenter. Black dots, red star, and green dot represent grid point, epicenter and a sample target *station* for which distance and azimuth are calculated.

We then sync the calculated synthetics at each station and create synchronous grids for each time step. In this way, at each time step and in each

grid, all the grid points belong to the same moment of time. Figs. 3 and 4 show such calculated *snapshots* of the surface at various moments of time around the epicenter and in the world, respectively.

Figure 3: Rayliegh waves from a point source (CMT) solution of the 2011 Japan earthquake propagating on the Earth's surface at six different moments of time shown as snapshots from (a) through (f). Yellow star shows the epicenter.
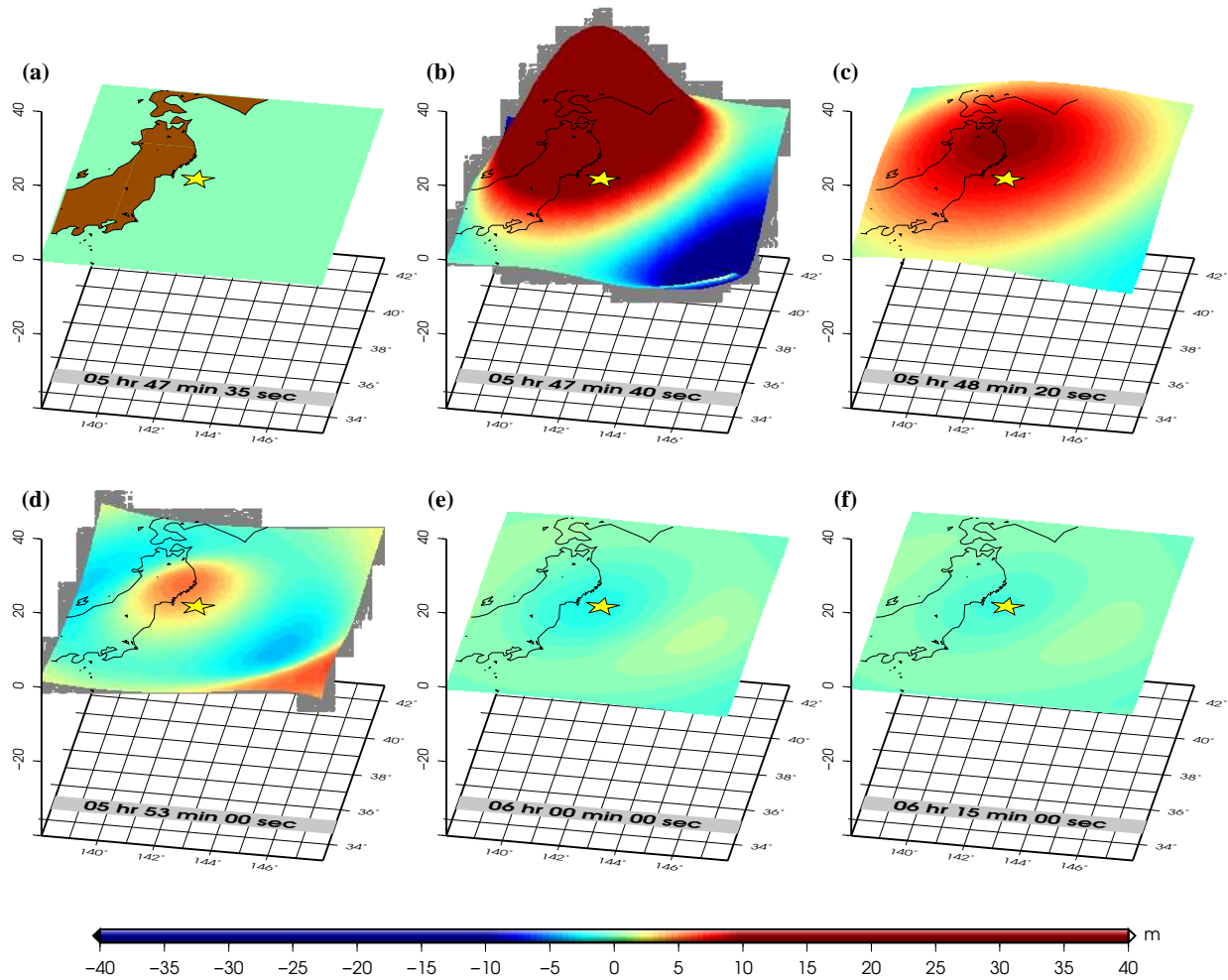
Figure 4: Similar to Fig. 3 for a local scale. The Yellow star shows the epicenter.

As we can see through Figs. 1, 3, and 4, extremely large amplitudes reaching ~ 40 cm are computed for the Rayleigh waves even for a large CMT moment of 5.31×10^{29} dyn-cm (compared to the values of a few centimeters reported, e.g., by Grapenthin & Freymueller, 2011). This is due to the fact that in reality (especially for large earthquakes) the source has a finite length which cannot be ignored, whereas in Fig. 1 we have used a point source. In a finite fault length, destructive superposition of waves will significantly

Therefore, the FRUN script was developed to apply a similar method to

finite sources.

3 FRUN

To use the `FRUN` script, you need to approximate the finite source as a set of smaller *subevents* with identical geometries but variable durations and seismic moments.

The `FRUN` script uses programs that already exist in the `bin/` directory in the `UM` space, and so does not require any further compiling. To use the package you simply need the following few input files:[2]

3.1 Input Files

- *epicenter.xy:* List of epicenter points as a 4-column file:

  [lon] [lat] [delay after the previous segment] [M_0 weight]

  The last column is a float number between 0 and 1 that shows the portion of the total moment to be assigned to that "subevent".

  NOTE: Always check that the sum of these fractions is equal to 1.

  synt.in: Contains the input parameters for the synthetic program.

  NOTE: Manually edit the origin time in this file to that of the first epicenter.

- Altitude grid: XYZ file; the program asks for it in the beginning.

  NOTE: the path MUST be absolute.

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[2] The excitation coeficient file *fort.7.final* is also needed, but it is read from the *bin/* directory. It contains the coefficients up to $_0S_{60}$. You can change it if you need different resolutions.

- Total seismic moment: XXXeYY; the program asks for it in the beginning. Portions of this value will be allocated to each subevents based on their respective listed fractions (see the first item on this list).

Remember that *epicenter.xy* determines the location, duration and size of each epicenter (Fig. 5).
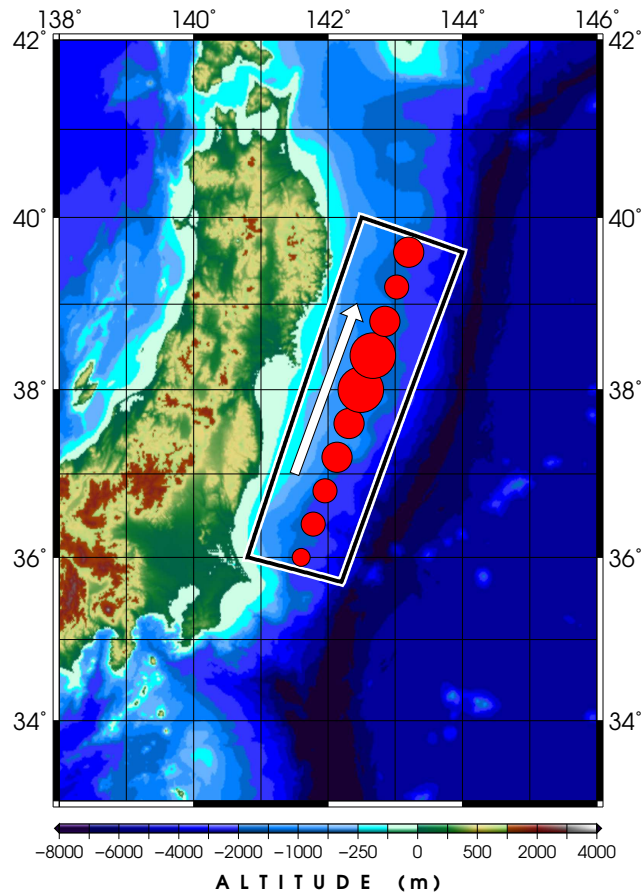
Figure 5: Example loci and relative sizes (e.g. from Koketsu et al. (2011)) of the epicenters representing the unilateral rupture along with a chosen direction (yellow arrow). Delay times are set to reflect the total rupture time of ~ 180 s as reported by Grapenthin & Freymueller (2011). Red circles are epicenters whose sizes are proportional to their seismic moment. The box in this figure is a rendition of a similar box in Fig. 4 of Koketsu et al. (2011).

To run the program simply

```
./frun
```

3.2   Output Files

After successfully running the script you will end up with the following directories:

1. *EPXXX:* Containing synthetic seismograms for all the grid points from each subevent.

2. *raysup:* Containing superimposed seismograms from all the subevents.

3. *raysup/save:* Contains synchronous grids in chunks. This is done to accommodate the allowed maximum number of opened files by the operating syetem. Therefore in each `part.XX` directory, the data for 1000 grid points are stored.

4   Plotting the Fields

To plot the deformations and make a movie, `cd` to the `save` directory and run the `script`. Note that you will need a CPT file named `surf.cpt` according to the vertical resolution of your results. Also, please tweak the parameters at the begnning of this script to suit your need.

After running the script, in the same terminal window you will see a list of target directories. Please remove any unwanted directories to be excluded from the movie.

5   What to Pick?

After running the plotting script, please use the created graphics to pick the right moment(s) of time for your work. You can look for the snapshot number and use the `fetch` script to combine the appropriate grid points to construct a full grid of that snapshot. For example, for the snapshot #32,

```
./fetch 32
```

A grid file named `grid.mix` of that snapshot will be created.

6   Results

Preliminary results from simulating ground displacement using a finite source reveals a significant decrease in amplitude. Fig. 6 shows snapshots of Rayleigh surface displacements for a finite fault with weighted moments as shown in Fig. 5.

Depending on the application, each of the calculated fields in Fig. 6 can be readily used in tsunami simulations.
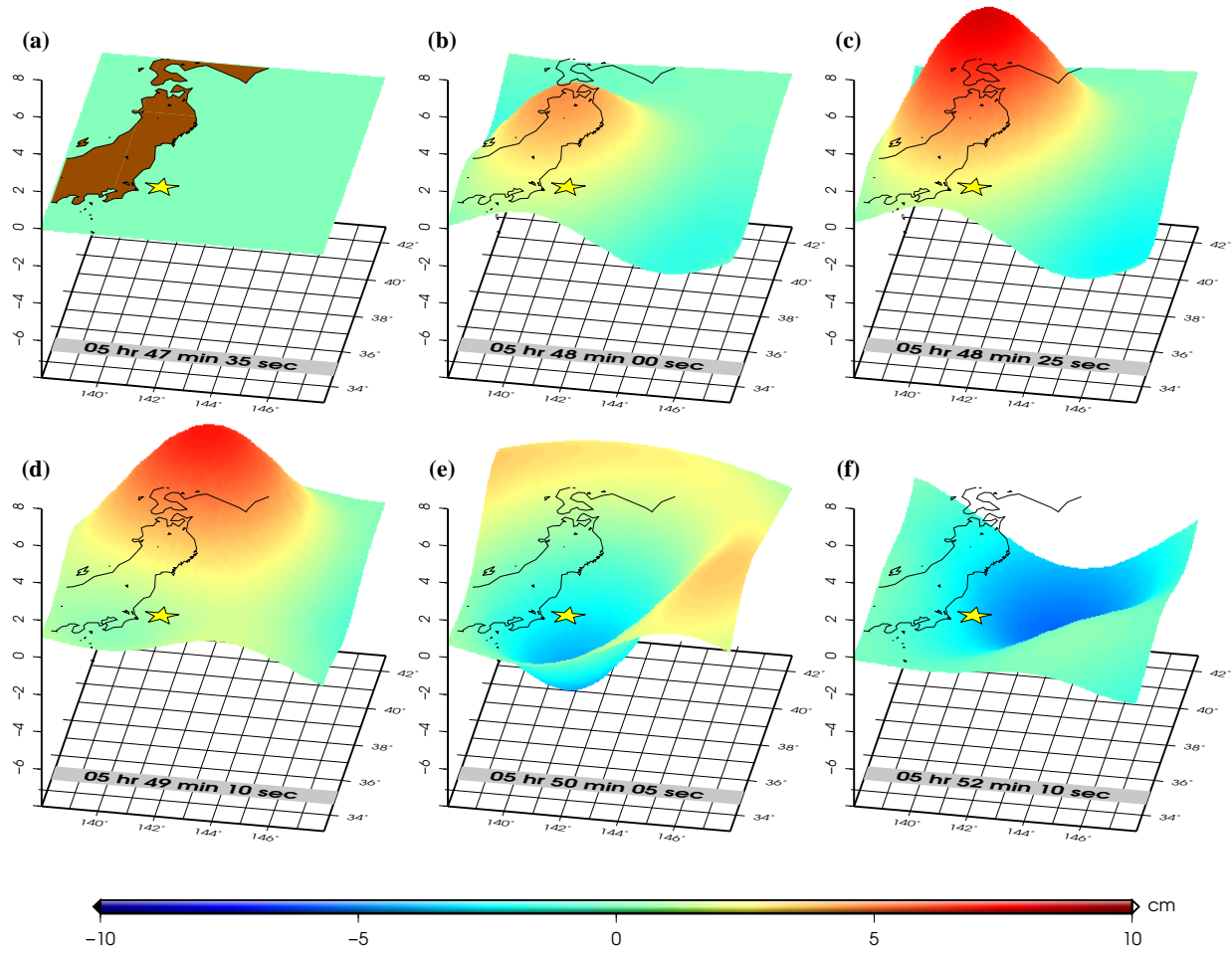
Figure 6: Simulated Rayleigh displacements from a finite source approximated as a series of scaled point sources.

(18).Kanamori, H., 1970. Synthesis of the long-period surface waves and its application to earthquake source studies–Kurile Islands earthquake of October 13, 1963, *Journal of Geophysical Research*, **75**(26), 5011–5027.

Kanamori, H. & Cipar, J. J., 1974. Focal process of the great Chilean earthquake May 22, 1960, *Physics of the Earth and Planetary Interiors*, **9**(2), 128–136.

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Okal, E. A., 1976. A surface-wave investigation of the rupture mechanism of the Gobi-Altai (December 4, 1957) earthquake, *Physics of the Earth and Planetary Interiors*, **12**(4), 319–328.

Saito, M., 1967. Excitation of free oscillations and surface waves by a point source in a vertically heterogeneous Earth, *Journal of Geophysical Research*, **72**(14), 3689–3699.

Saito, T., Baba, T., Inazu, D., Takemura, S., & Fukuyama, E., 2019. Synthesizing sea surface height change including seismic waves and tsunami using a dynamic rupture scenario of anticipated Nankai trough earthquakes, *Tectonophysics*, **769**, 228166.