Note on Gamma Function:

$$\Gamma(z) = \int_0^\infty \underbrace{e^{-t}}_{dv}\, \overbrace{t^{z-1}}^{u}\, dt \longrightarrow \text{if } z \in \mathbb{C} \Rightarrow z = a + ib$$

MUST: $a > 0$

Integration by parts

$$\Gamma(z) = \underbrace{\left(e^{-t} t^{z-1}\right)\Big|_0^\infty}_{0-0=0} - \int_0^\infty e^{-t}(z-1)\,t^{z-2}\,dt$$

$$\rightarrow \Gamma(z) = (z-1)\int_0^\infty e^{-t}\, t^{z-2}\,dt$$

$$\Rightarrow \Gamma(z+1) = z\int_0^\infty e^{-t}\, t^{z-1}\,dt = z\,\Gamma(z) \Rightarrow \boxed{\Gamma(z+1) = z\,\Gamma(z)}$$

If $z = n \in \mathbb{N}$

$$\Gamma(n) = (n-1)\,\Gamma(n-1) = (n-1)(n-2)\,\Gamma(n-2)$$

Definition $\Gamma(1) = 1$

$$= (n-1)(n-2)(n-3)\,\Gamma(n-3)$$

$$= (n-1)(n-2)(n-3)\cdots 3 \times 2 \times 1$$

$$= (n-1)!$$

$$\Rightarrow \boxed{\Gamma(n) = (n-1)!}$$

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}$$

Relation to Beta Function