

> restart;
 Define the matrices for the steady problem as before but with the different values for the elongation problem

```
> Tau:= matrix(3,3,[tauxx,tauxy,tauxz,
   iauxy,iauyy,iauyz,
   iauxz,iauyz,iauzz]);
```

$$T := \begin{bmatrix} \text{tauxx} & \text{tauxy} & \text{tauxz} \\ \text{tauxy} & \text{iauyy} & \text{iauyz} \\ \text{iauxz} & \text{iauyz} & \text{iauzz} \end{bmatrix}$$

The vector v is derived from conservation of mass

```
> V:=vector([gamma*x,-gamma*y,0]);
```

$$V := [\gamma x, -\gamma y, 0]$$

```
> gradV:= evalm(gamma*matrix(3,3,[1,0,0,0,-1,0,0,0,0]));
```

$$\text{grad}V := \begin{bmatrix} \gamma & 0 & 0 \\ 0 & -\gamma & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

```
> Gamma:=evalm(1/2*(gradV+transpose(gradV)));
```

$$\Gamma := \begin{bmatrix} \gamma & 0 & 0 \\ 0 & -\gamma & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$-\frac{\gamma}{2}$ $-\frac{\gamma}{2}$ if axisymmetric

```
> Omega:=evalm(1/2*(gradV-transpose(gradV)));
```

$$\Omega := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

```
> TDeriv:= (T) -> map(diff,T,t)+V[1]*map(diff,T,x)+V[2]*map(diff,T,y)+V[3]*map(diff,T,z);;
```

$$TDeriv := T \rightarrow \text{map}(\text{diff}, T, t) + V_1 \text{map}(\text{diff}, T, x) + V_2 \text{map}(\text{diff}, T, y) + V_3 \text{map}(\text{diff}, T, z)$$

```
> JDeriv:= (T) -> evalm(TDeriv(T)+Omega*T-T*Omega);
```

$$JDeriv := T \rightarrow \text{evalm}(TDeriv(T) + \Omega * T - T * \Omega)$$

substitute a for the upper-convected maxwell model

```
> a:=1;
```

$$a := 1$$

Derive the system of equations

```
> unassign('tauxx','tauxy','tauxz','tauyy','tauyz','tauzz');
```

```
eq:=evalm(Tau+lambda[1]*(JDeriv(Tau)-a*(Tau*Gamma+Gamma*T))-2*eta[0]*(Gamma+lambda[2]*JDeriv(Gamma)));
```

$$eq := \begin{bmatrix} \text{tauxx} - 2\lambda_1 \text{tauxx} \gamma - 2\eta_0 \gamma & \text{tauxy} & \text{tauxz} - \lambda_1 \text{tauxz} \gamma \\ \text{tauxy} & \text{iauyy} + 2\lambda_1 \text{iauyy} \gamma + 2\eta_0 \gamma & \text{iauyz} + \lambda_1 \text{iauyz} \gamma \\ \text{tauxz} - \lambda_1 \text{tauxz} \gamma & \text{iauyz} + \lambda_1 \text{iauyz} \gamma & \text{iauzz} \end{bmatrix}$$

Solve the system of equations

```
> unassign('tauxx','tauxy','tauxz','tauyy','tauyz','tauzz');
```

```
tauxx:=solve(eq[1,1]=0,tauxx);
tauxy:=solve(eq[1,2]=0,tauxy);
tauxz:=solve(eq[1,3]=0,tauxz);
tauyy:=solve(eq[2,2]=0,tauyy);
tauyz:=solve(eq[2,3]=0,tauyz);
tauzz:=solve(eq[3,3]=0,tauzz);
```

$$\text{tauxx} := -\frac{2\eta_0 \gamma}{-1 + 2\lambda_1 \gamma}$$

$$\tau_{xy} := 0$$

$$\tau_{xz} := 0$$

$$\tau_{yy} := -\frac{2 \eta_0 \gamma}{1 + 2 \lambda_1 \gamma}$$

$$\tau_{yz} := 0$$

$$\tau_{zz} := 0$$

Show the value of $\tau_{xx} - \tau_{yy}$

> factor(simplify(expand(tauxx-tauyy)));

$$-\frac{4 \eta_0 \gamma}{(-1 + 2 \lambda_1 \gamma)(1 + 2 \lambda_1 \gamma)}$$

Substitute in $\lambda_1 = 0$ so that it may be compared with a newtonian fluid

> subs(lambda[1]=0,%);

$$4 \eta_0 \gamma$$

>