Stochastic Modeling and Approaches for Managing Energy Footprints in Cloud Computing Service

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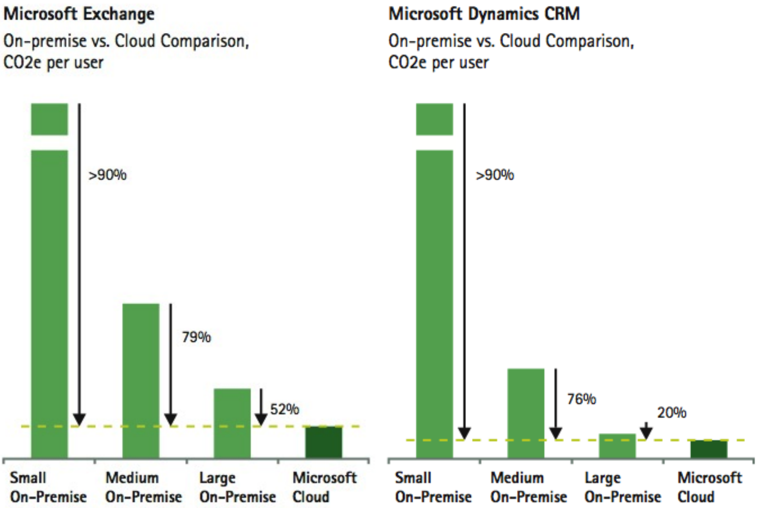
Emerging Trends of Cloud Computing (CC)

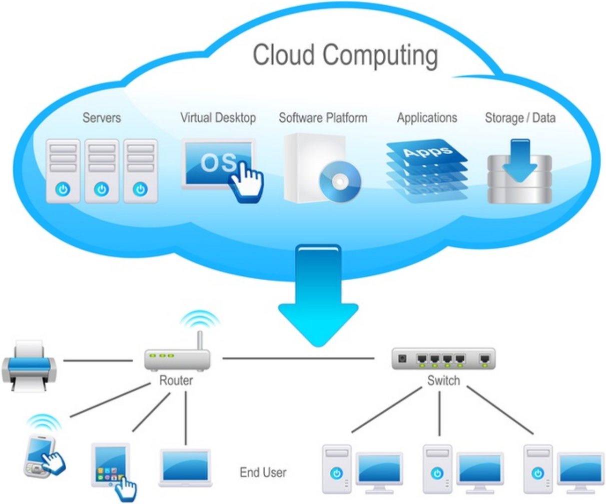
Source: www.cloudtweaks.com by David Fletcher

CC Advantages: Reducing Carbon Emission



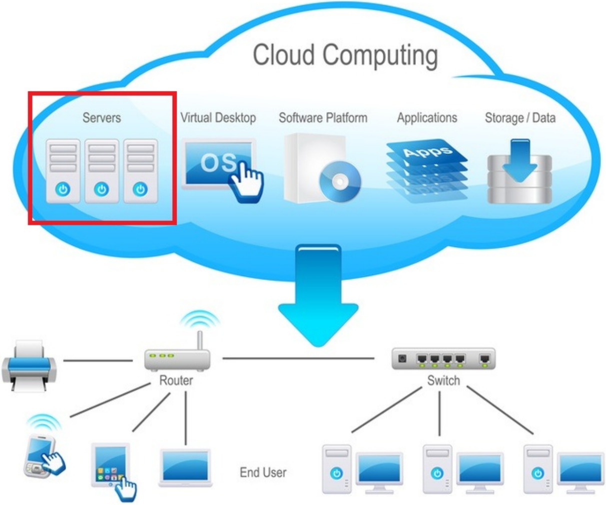
Source: Accenture (2010) “Cloud Computing and Sustainability: Environmental Benefits of Moving to the Cloud”

How CC Works...



Source: www.veterangeek.com

How CC Works...



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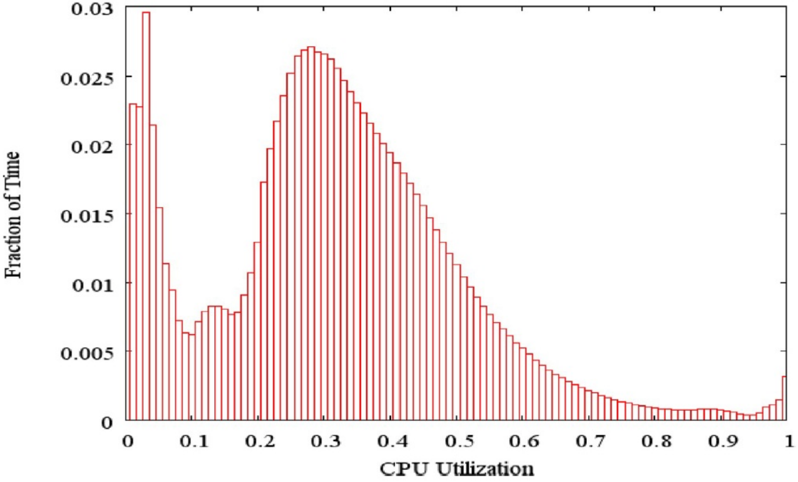

- Moreover, an idle server consumes 60%+ energy at full mode.

Virtual Machine Consolidation

Large-scale servers with low utilization

Consolidate the work to fewer Cloud servers

Source: Google's official blog - Energy efficiency in the cloud.

Our data centers use 50% less energy than typical data centers through server (Virtual Machine) consolidation. — Google.

Other benefits:

- more robust operations schedules
- more idle servers reacting to demand surges

Our Work

- Stochastic mixed-integer programming models to optimize energy footprints while ensure various Quality of Service (QoS) guarantees for managing servers in Cloud Computing service.

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- Estimate demand based on distributions of historical data, and dynamically consolidate or distribute jobs on servers through operational scheduling.
- Vary QoS levels by using joint/multiple chance constraints, to bound chances of job delay and incompleteness.

Outline of Our Research

- Formulations: Stochastic & Chance-Constrained Programs
- Algorithms: the Benders Decomposition and Heuristics
- Computational Design
- Result Analyses
- Conclusions and Future Research

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$$
\begin{aligned}
\text{min:} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) && \text{(1a)} \\
\text{s.t.} \quad & \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \widetilde{d}^t \quad \forall 1 \leq t \leq T && \text{(1b)} \\
& \ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m,\, 1 \leq t \leq T && \text{(1c)} \\
& y_i^1 \geq z_i^1 \quad \forall i \in \mathcal{N}_m && \text{(1d)} \\
& y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in \mathcal{N}_m,\, 2 \leq t \leq T && \text{(1e)} \\
& 0 \leq x_i^t \leq 1 \quad \forall i \in \mathcal{N}_m,\, 1 \leq t \leq T && \text{(1f)} \\
& y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m,\, 1 \leq t \leq T && \text{(1g)}
\end{aligned}
$$

The basic model consolidates demand on severs to minimize the total energy consumed by all servers over $t = 1, \ldots, T$.

$$
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\end{aligned}
$$

$g_i y_i^t$: energy used for booting machine i at period t.

$y_i^t \in \{0,1\}$: $= 1$ if server i is switched to "on" at period t.

$$\text{min:} \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \tag{1a}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \widetilde{d}^t \quad \forall 1 \leq t \leq T \tag{1b}$$

$$\ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T \tag{1c}$$

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$$y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T \tag{1g}$$

$v_i x_i^t$: energy for job processing in machine i at period t.
$x_i^t \geq 0$: percentage of server i's capacity used at period t.

$$
\begin{aligned}
\text{min:} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) && \text{(1a)} \\
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\end{aligned}
$$

$f_i z_i^t$: energy used at "idle" of machine i at period t.
$z_i^t \in \{0,1\}$: $= 1$ if server i is "idle" at period t.

$$
\begin{aligned}
\text{min:} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) && \text{(1a)} \\
\text{s.t.} \quad & \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \widetilde{d}^t \quad \forall 1 \leq t \leq T && \text{(1b)} \\
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\end{aligned}
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Computational time allocated to each period t is no less than the random demand $\widetilde{d}^t$.

$$
\begin{aligned}
\text{min:} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) && \text{(1a)}\\
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$$

If $\widetilde{d}^t$ is discretely distributed,

$$
\begin{aligned}
\text{min:} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) && \text{(1a)} \\
\text{s.t.} \quad & \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \max_{\omega \in \Omega} d^{t\omega} \quad \forall 1 \leq t \leq T && \text{(1b)} \\
& \ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T && \text{(1c)} \\
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If $\widetilde{d}^t$ is discretely distributed, and let $d^{t\omega}$ represent a realization of $\widetilde{d}^t$ in scenario $\omega \in \Omega$,

- reformulate (1b) as a set of deterministic constraints

$$
\begin{aligned}
\text{min:} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) && \text{(1a)} \\
\text{s.t.} \quad & \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \max_{\omega \in \Omega} d^{t\omega} \quad \forall 1 \leq t \leq T && \text{(1b)} \\
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\end{aligned}
$$

The total “on” time of server i at period t is no less than computational time plus the time of booting the server (if there is any).

$$\text{min:} \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) \tag{1a}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \max_{\omega \in \Omega} d^{t\omega} \quad \forall 1 \leq t \leq T \tag{1b}$$

$$\ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T \tag{1c}$$

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$$y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T \tag{1g}$$

Server i is "on" at period 1 if we switch it to "on."

$$
\begin{aligned}
\text{min:} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) && \text{(1a)} \\
\text{s.t.} \quad & \sum_{i \in \mathcal{N}_m} e_i \ell^t x_i^t \geq \max_{\omega \in \Omega} d^{t\omega} \quad \forall 1 \leq t \leq T && \text{(1b)} \\
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& y_i^t, z_i^t \in \{0, 1\} \quad \forall i \in \mathcal{N}_m, \, 1 \leq t \leq T && \text{(1g)}
\end{aligned}
$$

If server i is "off" at $t-1$ but "on" at t, then it means that

- server i is switched to "on" at period t

GOAL:

- Minimize energy consumption of all servers over $1, \ldots, T$ + the expected penalty cost of backlogging.

Allow **backlogging** such that

- Job (j, t) can be partitioned and processed on multiple servers, at any time that is no more than L periods after period t ("time of submission").

Define Sets:

- $B_1(t)$: backlogging periods such that if $t = 1, \dots, T - L$, then $B_1(t) = t, \dots, t + L$; if $t = T - L + 1, \dots, T$, then $B_1(t) = t, \dots, T$.
- $B_2(t)$: possible periods for submitting jobs due at t, such that if $t \le L$, then $B_2(t) = 1, \dots, t$; if $t = L + 1, \dots, T$, then $B_2(t) = t - L, \dots, t$.

Additional Parameter:

- $\mathcal{N}_c$: Set of user groups who submit computational demand.
- $\widetilde{d}_j^t$: random job (j, t) submitted by user j at period t.
- p_j^{tk}: unit penalty of unfinished job (j, t) at period k, $\forall k \in B_1(t)$.

New Variables:

- u_{ji}^{tk}: percentage of ℓ^t for processing job (j, t) on server i in period k, $\forall i \in \mathcal{N}_m$, $j \in \mathcal{N}_c$, $t = 1, \dots, T$, and $k \in B_1(t)$.
- $b_j^{tk\omega}$: unfinished job (j, t) at period k in scenario ω, $\forall k \in B_1(t)$ and $\omega \in \Omega$

$$\min: \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) + \sum_{\omega \in \Omega} \rho^\omega \left(\sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p_j^{tk} b_j^{tk\omega} \right)$$

s.t. (1c)–(1g) $\Rightarrow$ Constraints from Model (1)

$$\sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \tilde{d}_j^t \quad \forall j \in \mathcal{N}_c,\ 1 \leq t \leq T \tag{2a}$$

$$x_i^t \geq \sum_{k \in B_2(t)} \sum_{j \in \mathcal{N}_c} u_{ji}^{kt} \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T \tag{2b}$$

$$b_j^{tk\omega} = \max\left\{0,\ d_j^{t\omega} - \sum_{l=t}^{k} \sum_{i \in \mathcal{N}_m} e_i \ell^l u_{ji}^{tl}\right\} \quad \forall j \in \mathcal{N}_c,\ 1 \leq t \leq T,\ k \in B_1(t),\ \omega \in \Omega \tag{2c}$$

$$0 \leq u_{ji}^{tk} \leq 1,\ b_j^{tk\omega} \geq 0. \tag{2d}$$

ρ^ω: the probability of scenario $\omega \in \Omega \Rightarrow$ penalize unfinished job requests in the objective, and minimize the expected penalty.

$$\text{min:} \quad \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) + \sum_{\omega \in \Omega} \rho^\omega \left(\sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p_j^{tk} b_j^{tk\omega} \right)$$

s.t. (1c)–(1g) $\Rightarrow$ Constraints from Model (1)

$$\sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \max_{\omega \in \Omega} d_j^{t\omega} \quad \forall 1 \leq t \leq T \tag{2a}$$

$$x_i^t \geq \sum_{k \in B_2(t)} \sum_{j \in \mathcal{N}_c} u_{ji}^{kt} \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T \tag{2b}$$

$$b_j^{tk\omega} = \max\left\{0,\ d_j^{t\omega} - \sum_{l=t}^{k} \sum_{i \in \mathcal{N}_m} e_i \ell^l u_{ji}^{tl}\right\} \quad \forall j \in \mathcal{N}_c,\ 1 \leq t \leq T,\ k \in B_1(t),\ \omega \in \Omega \tag{2c}$$

$$0 \leq u_{ji}^{tk} \leq 1,\ b_j^{tk\omega} \geq 0. \tag{2d}$$

$d_j^{t\omega}$: the realization of $\widetilde{d}_j^t$ in scenario $\omega \in \Omega \Rightarrow$ replace stochastic constraints (2a) by equivalent deterministic constraints.

Model 3: Backlogging with a Joint Chance Constraint

Relax Model (2) by allowing job incompleteness after L backlogging periods, however, **bounded by a certain risk tolerance**.

That is, replace Constraint (2a)

$$\sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \widetilde{d}_j^t \quad \forall j \in \mathcal{N}_c,\, 1 \leq t \leq T$$

with

$$\mathbb{P}\left(\sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \widetilde{d}_j^t, \forall j \in \mathcal{N}_c,\, 1 \leq t \leq T \right) \geq \alpha$$

Model 3: Backlogging with a Joint Chance Constraint

$$
\begin{aligned}
\text{min:} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) + \sum_{\omega \in \Omega} \rho^\omega \left(\sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p_j^{tk} b_j^{tk\omega} \right) \\
\text{s.t.} \quad & \text{(1c)–(1g)} \;\Rightarrow\; \text{Constraints from Model (1)} \\
& \text{(2b)–(2d)} \;\Rightarrow\; \text{Constraints from Model (2)} \\
& \mathbb{P} \left(\sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \widetilde{d}_j^t, \forall j \in \mathcal{N}_c,\, 1 \leq t \leq T \right) \geq \alpha
\end{aligned}
$$

Model 3: Backlogging with a Joint Chance Constraint

$$\begin{aligned}
\text{min:} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) + \sum_{\omega \in \Omega} \rho^\omega \left(\sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p_j^{tk} b_j^{tk\omega} \right) \\
\text{s.t.} \quad & \text{(1c)–(1g)} \ \Rightarrow \ \text{Constraints from Model (1)} \\
& \text{(2b)–(2d)} \ \Rightarrow \ \text{Constraints from Model (2)} \\
& \sum_{\omega \in \Omega} \rho^\omega \zeta^\omega \leq 1 - \alpha
\end{aligned}$$

where, for each $\omega \in \Omega$, binary variables $\zeta^\omega = 1$ if $\forall j \in \mathcal{N}_c, \ 1 \leq t \leq T$, there exists at least one

$$\sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} < d_j^{t\omega},$$

and 0 otherwise.

Model 3: Backlogging with a Joint Chance Constraint

$$
\begin{aligned}
\text{min:} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t) + \sum_{\omega \in \Omega} \rho^\omega \left(\sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p_j^{tk} b_j^{tk\omega} \right) \\
\text{s.t.} \quad & \text{(1c)–(1g)} \ \Rightarrow \ \text{Constraints from Model (1)} \\
& \text{(2b)–(2d)} \ \Rightarrow \ \text{Constraints from Model (2)} \\
& \sum_{\omega \in \Omega} \rho^\omega \zeta^\omega \leq 1 - \alpha \\
& \sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} + M_j^t \zeta^\omega \geq d_j^{t\omega} \\
& \qquad\qquad \forall \omega \in \Omega,\ j \in \mathcal{N}_c,\ 1 \leq t \leq T \\
& \zeta^\omega \in \{0, 1\} \quad \forall \omega \in \Omega.
\end{aligned}
$$

where M_j^t is set as the maximal standard time for processing job (j, t), e.g., $M_j^t = \max_{\omega \in \Omega} d_j^{t\omega},\ \forall j \in \mathcal{N}_c,\ 1 \leq t \leq T$.

Model 4: Backlogging with Multiple Chance Constraints

Instead of a joint chance constraint

$$\mathbb{P}\left(\sum_{k\in B_1(t)}\sum_{i\in\mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \tilde{d}_j^t,\ \forall j\in\mathcal{N}_c,\ 1\leq t\leq T\right) \geq \alpha,$$

we formulate a series of job-based constraints, each of which is associated with a risk tolerance α_j^t, for job (j,t), $\forall j\in\mathcal{N}_c$ and $1\leq t\leq T$.

$$\mathbb{P}\left(\sum_{k\in B_1(t)}\sum_{i\in\mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \tilde{d}_j^t\right) \geq \alpha_j^t \quad \forall j\in\mathcal{N}_c,\ 1\leq t\leq T.$$

Solution Algorithms

Computational challenges from:

Large-Scale Time Intervals $(1, \ldots, T)$

Large Number of Users and Servers ($|\mathcal{N}_c|$ and $|\mathcal{N}_m|$)

Large Number of Scenarios ($|\Omega|$) for Describing the Uncertainty ($\tilde{d}$)

Binary Server Operational Decisions (y and z)

$$\text{min:} \quad \boxed{\sum_{t=1}^{T} \sum_{i \in \mathcal{N}_m} (g_i y_i^t + v_i x_i^t + f_i z_i^t)} + \boxed{\sum_{\omega \in \Omega} \text{Prob}^{\omega} \left(\sum_{t=1}^{T} \sum_{j \in \mathcal{N}_c} \sum_{k \in B_1(t)} p_j^{tk} b_j^{tk\omega} \right)}$$

$$\text{s.t.} \quad \ell^t x_i^t + s_i y_i^t \leq \ell^t z_i^t \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T$$

$$y_i^1 \geq z_i^1 \quad \forall i \in \mathcal{N}_m$$

$$y_i^t \geq z_i^t - z_i^{t-1} \quad \forall i \in \mathcal{N}_m,\ 2 \leq t \leq T$$

$$0 \leq x_i^t \leq 1 \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T$$

$$y_i^t, z_i^t \in \{0,1\} \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T$$

Major Operations Decisions (Relaxed Master Problem)

$$\sum_{k \in B_1(t)} \sum_{i \in \mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \tilde{d}_j^t \quad \forall j \in \mathcal{N}_c,\ 1 \leq t \leq T$$

$$\hat{x}_i^t \geq \sum_{k \in B_2(t)} \sum_{j \in \mathcal{N}_c} u_{ji}^{kt} \quad \forall i \in \mathcal{N}_m,\ 1 \leq t \leq T$$

$$b_j^{tk\omega} = \max\left\{0,\ d_j^{t\omega} - \sum_{l=t}^{k} \sum_{i \in \mathcal{N}_m} e_i \ell^l u_{ji}^{tl}\right\}$$

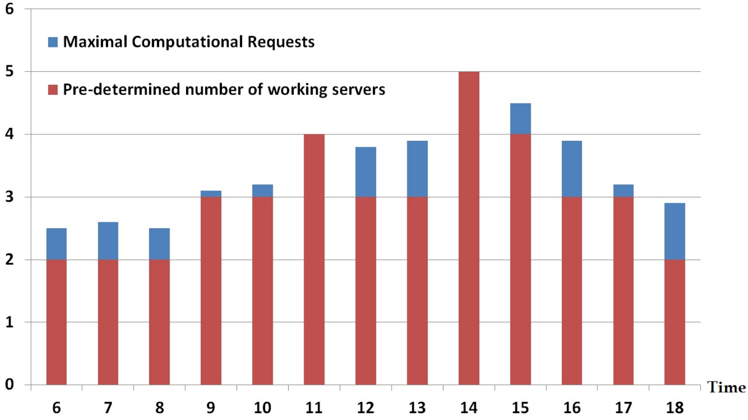
$$\forall j \in \mathcal{N}_c,\ 1 \leq t \leq T,\ k \in B_1(t),\ \omega \in \Omega$$

$0 \leq u_{ji}^{tk} \leq 1,\ b_j^{tk\omega} \geq 0.$ **Continuous!**

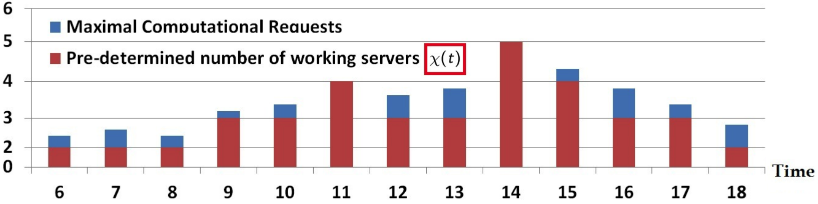
Scenario-based Decisions (Second Stage Subproblem)

A Heuristic Approach

Idea: fix schedules of a subset of servers. Then optimize schedules for the rest of servers using math modeling.



A Heuristic Approach



We pre-determine a subset of servers' schedule by setting

$$x_i^1 = 1 - s_i/\ell^t \quad \forall i = 1, \ldots, \chi(1),$$

$$x_i^t = 1 \quad \forall 2 \le t \le T,\ i = 1, \ldots, \chi^-(t),$$

$$x_i^t = 1 - s_i/\ell^t \quad \forall 2 \le t \le T,\ i = \chi^-(t)+1, \ldots, \chi^-(t)+\chi^+(t) \quad \text{if } \chi^+(t) > 0,$$

where for $t = 1, \ldots, T$,

$$\chi(t) = \left\lfloor \sum_{j \in \mathcal{N}_c} \max_{\omega \in \Omega} d_j^{t\omega}/\ell^t \right\rfloor,$$

$\chi^-(t) = \min\{\chi(t-1), \chi(t)\}$, and $\chi^+(t) = \max\{\chi(t) - \chi(t-1), 0\}$.

- $|\mathcal{N}_c| = 2$ (two types of users) and $|\mathcal{N}_m| =$5, 10, and 20.
- Set $T = 24$ hours.
- Average energy consumption of Off, Idle, Processing, and Booting for a 3.0 Ghz server to be, respectively, 0W, 150W, 250W, and 250W (i.e., $v_i = 100W$, $f_i = 150W$).

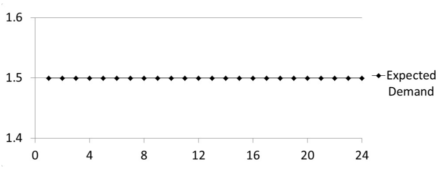
$\mathcal{N}_m$	$\mathcal{I} = 10\%$		$\mathcal{I} = 30\%$		$\mathcal{I} = 50\%$	
	$E[\sum_{t=1}^{T} \tilde{d}^t]$ (hours)	Bk (kWh)	$E[\sum_{t=1}^{T} \tilde{d}^t]$ (hours)	Bk (kWh)	$E[\sum_{t=1}^{T} \tilde{d}^t]$ (hours)	Bk (kWh)
5	12	19.2	36	21.6	60	24
10	24	38.4	72	43.2	120	48
20	48	76.8	144	86.4	240	96

- "$\mathcal{I}$": computational intensity
- $E[\sum_{t=1}^{T} \tilde{d}^t] = \mathcal{I} * |\mathcal{N}_m| * 24$ (hours)
- **Bk**: gives benchmark energy consumption (objective) by having servers first "on" then "idle."

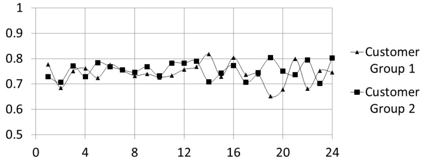
$\mathcal{N}_m$	$\mathcal{I} = 10\%$ $E[\sum_{t=1}^{T} \widetilde{d}^t]$ (hours)	Bk (kWh)	$\mathcal{I} = 30\%$ $E[\sum_{t=1}^{T} \widetilde{d}^t]$ (hours)	Bk (kWh)	$\mathcal{I} = 50\%$ $E[\sum_{t=1}^{T} \widetilde{d}^t]$ (hours)	Bk (kWh)
5	12	19.2	36	21.6	60	24
10	24	38.4	72	43.2	120	48
20	48	76.8	144	86.4	240	96

- "$\mathcal{I}$": computational intensity
- $E[\sum_{t=1}^{T} \widetilde{d}^t] = \mathcal{I} * |\mathcal{N}_m| * 24$ (hours)
- **Bk**: gives benchmark energy consumption (objective) by having servers first "on" then "idle."

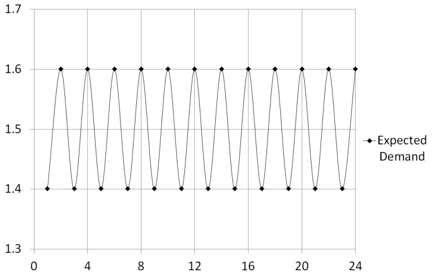
Demand Patterns



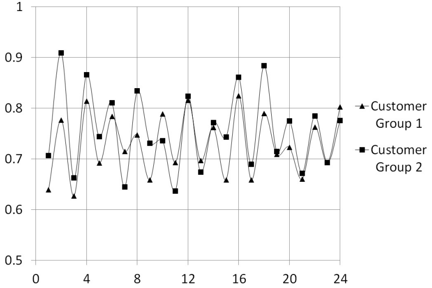
(a) Type 0 Demand Curve



(b) Type 0 Job Demand Sample

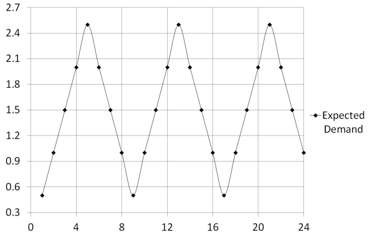


(c) Type 1 Demand Curve

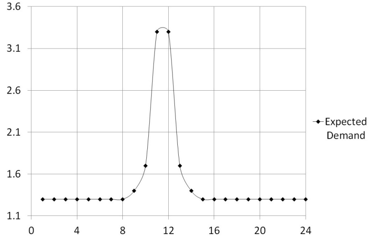


(d) Type 1 Job Demand Sample

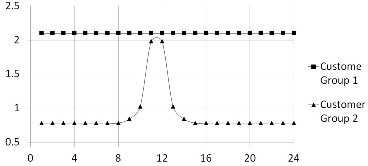
Demand Patterns



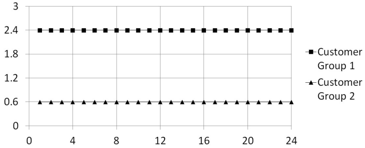
(e) Type 2 Demand Curve



(f) Type 3 Demand Curve



(g) Type 4 Demand Curve



(h) Type 5 Demand Curve

Types $0 \sim 3$: Homogeneous. Types 4 & 5: Heterogeneous.

Computational Design

- CPLEX 12.4 via ILOG Concert Technology with C++
- HP Workstation Z210 with CPU 3.20 GHz and 8GB memory
- CPU time limits =1800 seconds for each instance
- Test five instances for each parameter combination

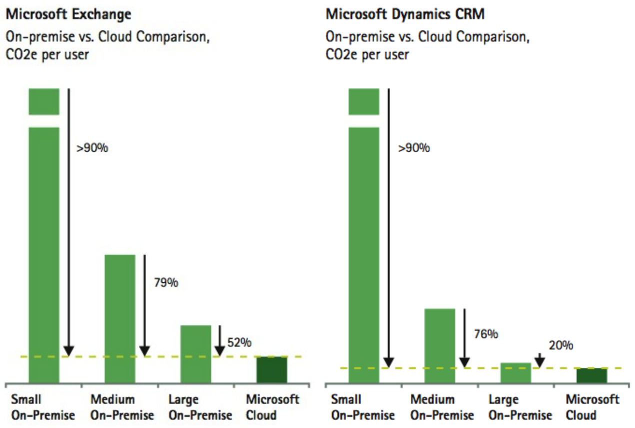
Results of Model 1

		$\mathcal{I} = 10\%$			$\mathcal{I} = 30\%$			$\mathcal{I} = 50\%$		
$\mathcal{N}_m$	Type	Bk	Oper	Save	Bk	Oper	Save	Bk	Oper	Save
5	T0	19.2	4.9	75%	21.6	11.0	49%	24	17.1	29%
	T1	19.2	4.9	75%	21.6	11.0	49%	24	17.1	29%
	T2	19.2	4.9	74%	21.6	12.0	44%	24	17.3	28%
	T3	19.2	5.2	73%	21.6	11.7	46%	24	-	-
10	T0	38.4	9.7	75%	43.2	22.0	49%	48	34.2	29%
	T1	38.4	8.2	79%	43.2	20.4	53%	48	32.7	32%
	T2	38.4	8.5	78%	43.2	22.2	49%	48	34.4	28%
	T3	38.4	7.3	81%	43.2	20.7	52%	48	-	-
20	T0	76.8	15.9	79%	86.4	40.3	53%	96	64.9	32%
	T1	76.8	14.3	81%	86.4	38.8	55%	96	65.1	32%
	T2	76.8	14.8	81%	86.4	41.3	52%	96	67.4	30%
	T3	76.8	13.9	82%	86.4	40.9	53%	96	-	-

Results of Model 1

		$\mathcal{I}=10\%$			$\mathcal{I}=30\%$			$\mathcal{I}=50\%$		
$\mathcal{N}_m$	Type	Bk	Oper	Save	Bk	Oper	Save	Bk	Oper	Save
5	T0	19.2	4.9	75%	21.6	11.0	49%	24	17.1	29%
	T1	19.2	4.9	75%	21.6	11.0	49%	24	17.1	29%
	T2	19.2	4.9	74%	21.6	12.0	44%	24	17.3	28%
	T3	19.2	5.2	73%	21.6	11.7	46%	24	-	-
10	T0	38.4	9.7	75%	43.2	22.0	49%	48	34.2	29%
	T1	38.4	8.2	79%	43.2	20.4	53%	48	32.7	32%
	T2	38.4	8.5	78%	43.2	22.2	49%	48	34.4	28%
	T3	38.4	7.3	81%	43.2	20.7	52%	48	-	-
20	T0	76.8	15.9	79%	86.4	40.3	53%	96	64.9	32%
	T1	76.8	14.3	81%	86.4	38.8	55%	96	65.1	32%
	T2	76.8	14.8	81%	86.4	41.3	52%	96	67.4	30%
	T3	76.8	13.9	82%	86.4	40.9	53%	96	-	-
	Avg.			80%			50%			30%

Recall...


Microsoft Exchange
On-premise vs. Cloud Comparison, CO2e per user
>90%
79%
52%
Small On-Premise
Medium On-Premise
Large On-Premise
Microsoft Cloud
Microsoft Dynamics CRM
On-premise vs. Cloud Comparison, CO2e per user
>90%
76%
20%
Small On-Premise
Medium On-Premise
Large On-Premise
Microsoft Cloud

Results of Model 1

		$\mathcal{I} = 10\%$			$\mathcal{I} = 30\%$			$\mathcal{I} = 50\%$		
$\mathcal{N}_m$	Type	Bk	Oper	Save	Bk	Oper	Save	Bk	Oper	Save
5	T0	19.2	4.9	75%	21.6	11.0	49%	24	17.1	29%
	T1	19.2	4.9	75%	21.6	11.0	49%	24	17.1	29%
	T2	19.2	4.9	74%	21.6	12.0	44%	24	17.3	28%
	T3	19.2	5.2	73%	21.6	11.7	46%	24	-	-
10	T0	38.4	9.7	75%	43.2	22.0	49%	48	34.2	29%
	T1	38.4	8.2	79%	43.2	20.4	53%	48	32.7	32%
	T2	38.4	8.5	78%	43.2	22.2	49%	48	34.4	28%
	T3	38.4	7.3	81%	43.2	20.7	52%	48	-	-
20	T0	76.8	15.9	79%	86.4	40.3	53%	96	64.9	32%
	T1	76.8	14.3	81%	86.4	38.8	55%	96	65.1	32%
	T2	76.8	14.8	81%	86.4	41.3	52%	96	67.4	30%
	T3	76.8	13.9	82%	86.4	40.9	53%	96	-	-

Results of Model 1

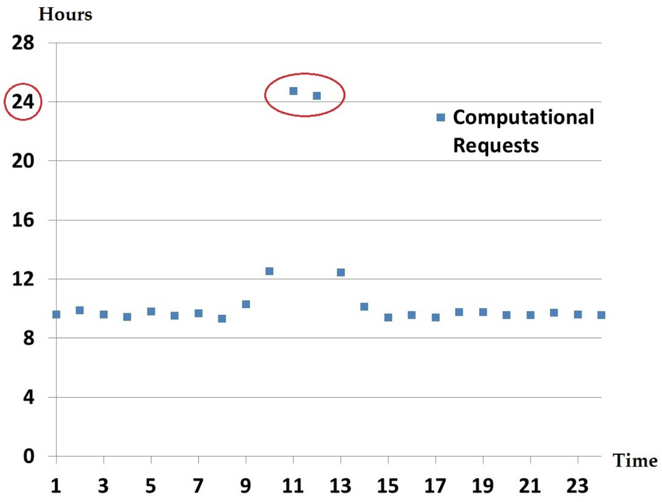


Figure: $\mathcal{N}_m = 20$, $\mathcal{I} = 50\%$, Type 3 Demand

A Revisit of Models

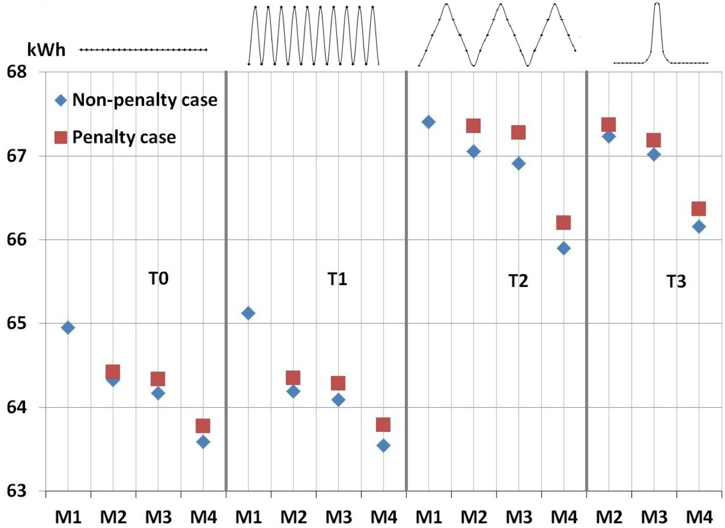
Model (1): $$\sum_{i\in\mathcal{N}_m} e_i \ell^t x_i^t \geq \max_{\omega\in\Omega} d^{t\omega} \quad \forall 1 \leq t \leq T.$$

Model (2): $$\sum_{k\in B_1(t)} \sum_{i\in\mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \max_{\omega\in\Omega} d_j^{t\omega} \quad \forall 1 \leq t \leq T.$$

Model (3): $$\mathbb{P}\left(\sum_{k\in B_1(t)} \sum_{i\in\mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \widetilde{d}_j^t, \forall j \in \mathcal{N}_c,\ 1 \leq t \leq T \right) \geq \alpha.$$

Model 4: $$\mathbb{P}\left(\sum_{k\in B_1(t)} \sum_{i\in\mathcal{N}_m} e_i \ell^k u_{ji}^{tk} \geq \widetilde{d}_j^t \right) \geq \alpha_j^t \quad \forall j \in \mathcal{N}_c,\ 1 \leq t \leq T.$$

Energy Use in Models 1-4 ($\mathcal{N}_m = 20$, $\mathcal{I} = 50\%$)



Unit penalty $p_j^{tk} = 100$ for penalty case, $\forall\, j \in \mathcal{N}_c$, $1 \le t \le T$, and $k \in B_1(t)$.

Solution Approach Comparison

		Model 1					Model 2		
Type	No.	C-Total	B-Time	B-Total	H-Time	H-Total	C-Total	H-Time	H-Total
	1	64.24	17.10	64.24	2.37	64.42	64.56	43.51	64.65
	2	64.21	10.67	64.21	2.36	64.39	64.52	11.84	64.53
T1	3	64.24	949.92	64.24	2.57	64.42	64.57	123.40	64.67
	4	64.41	1827.52	64.41	2.40	64.43	64.62	22.17	64.69
	5	64.21	28.88	64.21	2.59	64.38	64.50	14.35	64.55

Table: $\mathcal{N}_m = 20$, $\mathcal{I} = 50\%$, and Five Instances

"C-", solving Model (2) by directly solving its MIP.

"B-", employing Benders decomposition.

"H-", using the approximation approach.

Solution Approach Comparison

		Model 1					Model 2		
Type	No.	C-Total	B-Time	B-Total	H-Time	H-Total	C-Total	H-Time	H-Total
	1	64.24	17.10	64.24	2.37	64.42	64.56	43.51	64.65
	2	64.21	10.67	64.21	2.36	64.39	64.52	11.84	64.53
T1	3	64.24	949.92	64.24	2.57	64.42	64.57	123.40	64.67
	4	64.41	1827.52	64.41	2.40	64.43	64.62	22.17	64.69
	5	64.21	28.88	64.21	2.59	64.38	64.50	14.35	64.55

Table: $\mathcal{N}_m = 20$, $\mathcal{I} = 50\%$, and Five Instances

The performance of the Benders approach varies among instances and is unstable.

Solution Approach Comparison

		Model 1					Model 2		
Type	No.	C-Total	B-Time	B-Total	H-Time	H-Total	C-Total	H-Time	H-Total
T1	1	64.24	17.10	64.24	2.37	64.42	64.56	43.51	64.65
	2	64.21	10.67	64.21	2.36	64.39	64.52	11.84	64.53
	3	64.24	949.92	64.24	2.57	64.42	64.57	123.40	64.67
	4	64.41	1827.52	64.41	2.40	64.43	64.62	22.17	64.69
	5	64.21	28.88	64.21	2.59	64.38	64.50	14.35	64.55

Table: $\mathcal{N}_m = 20$, $\mathcal{I} = 50\%$, and Five Instances

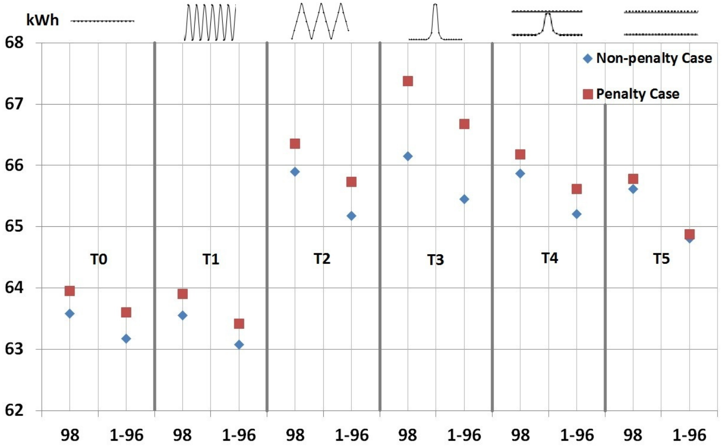
For Model 2, the differences between H-Total and C-Total are within 0.3% gaps for all instances.

Solution Approach Comparison

		Model 1					Model 2		
Type	No.	C-Total	B-Time	B-Total	H-Time	H-Total	C-Total	H-Time	H-Total
T1	1	64.24	17.10	64.24	2.37	64.42	64.56	43.51	64.65
	2	64.21	10.67	64.21	2.36	64.39	64.52	11.84	64.53
	3	64.24	949.92	64.24	2.57	64.42	64.57	123.40	64.67
	4	64.41	1827.52	64.41	2.40	64.43	64.62	22.17	64.69
	5	64.21	28.88	64.21	2.59	64.38	64.50	14.35	64.55

Table: $\mathcal{N}_m = 20$, $\mathcal{I} = 50\%$, and Five Instances

Effects of Use Prioritization (Model 4)



- **98**: $\alpha_j^t = 98\%, \forall j \in \mathcal{N}_c$; **1-96**: $\alpha_0^t = 100\%$, $\alpha_1^t = 96\%, \forall 1 \leq t \leq T$.

- Effectively managing energy footprints and QoS via stochastic optimization models.
- Yield respective 80%, 50%, and 30% of energy savings for 10%, 30%, and 50% demand intensity regardless of demand patterns.
- Backlogging and chance constraints provide additional flexibility in server scheduling and reduce energy use.
- The Benders decomposition and the heuristic approach are faster and yield good results.
- User prioritization via multiple chance constraints can effectively reduce consumed energy.

Thank You!

Questions?