Distributionally Robust Approaches for Optimal Power Flow with Uncertain Reserves from Load Control

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Introduction

- Why Chance-Constrained Optimal Power Flow (CC-OPF) Problem?
- How to Solve CC-OPF?
- Notation
- Joint and Individual CC-OPF Models
- Solution Approaches
  - Mixed-integer Linear programming (MILP) Approach (A1)
  - Gaussian Approximation Approach (A2)
  - Scenario Approximation Approach (A3)
  - Distributionally Robust Optimization Approach (A4)
- Computational Results
  - IEEE 9-Bus System
  - IEEE 39-Bus System

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Why Chance-Constrained Optimal Power Flow (CC-OPF) Problem?

- The Optimal Power Flow (OPF): minimize system-wide energy and reserve costs subject to the physical constraints of the system.
- More reserve needed: an increase in intermittent and uncertain power generation, i.e., wind and solar capacity
- Large amount of uncertainty in power systems motivates stochastic optimization approaches, i.e., CC-OPF.
    - Past work: Focused on managing uncertainty stemming from renewable energy production and load consumption
    - Our work: also the **uncertain balancing reserves** provided by load control

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How to Solve CC-OPF?

- A robust reformulation of the scenario approach
  - requires no knowledge of uncertain distributions
  - but significant number of "uncertain scenarios" – data!
- Such data may be unavailable in practice.
  - our goal: investigate the performance of a variety of methods to solve CC-OPF problems given **limited information of uncertain distribution.**

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Notation

- Decision variables:
  - energy production at generators P_G
  - generators' up- and down-reserve capacities $\overline{R}_G$, $\underline{R}_G$
  - loads' up- and down-reserve capacities $\overline{R}_L, \underline{R}_L$
  - "distribution vectors" $\overline{d}_G, \underline{d}_G$ and $\overline{d}_L, \underline{d}_L$
- Other variables:
  - actual generator reserves R_G and load reserves R_L
  - real-time supply/demand mismatch P_m
- Cost parameters:
  - $c = [c_0, c_1, c_2, \overline{c}_G, \underline{c}_G, \overline{c}_L, \underline{c}_L]^\mathsf{T}$
- Given data:
  - loads forecast P_L^f and wind forecast P_W^f
  - actual wind power $\widetilde{P}_W$, actual load $\widetilde{P}_L$
  - actual minimum and maximum load $[\underline{\widetilde{P}}_L, \overline{\widetilde{P}}_L]$
  - min/max generator production $\underline{P}_G, \overline{P}_G$

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Joint and Individual CC-OPF Models

- **[J-CC-OPF]:**

$$\begin{aligned}
\min \quad & c^{\mathsf{T}}[\mathbf{1}, P_G, P_G^2, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L] && (1)\\
\text{s.t.} \quad & P_m = \sum_{i=1}^{N_W}(\widetilde{P}_{W,i} - P^f_{W,i}) - \sum_{i=1}^{N_L}(\widetilde{P}_{L,i} - P^f_{L,i}) && (2)\\
& \sum_{i=1}^{N_G} \underline{d}_{G,i} + \sum_{i=1}^{N_L} \overline{d}_{L,i} = 1 && (3)\\
& \sum_{i=1}^{N_G} \overline{d}_{G,i} + \sum_{i=1}^{N_L} \underline{d}_{L,i} = 1 && (4)\\
& R_G = \overline{d}_G \max\{-P_m, 0\} - \underline{d}_G \max\{P_m, 0\} && (5)\\
& R_L = \overline{d}_L \max\{P_m, 0\} - \underline{d}_L \max\{-P_m, 0\} && (6)\\
& \mathbb{P}\left(\widetilde{A}x \geq \widetilde{b}\right) \geq 1 - \epsilon && (7)\\
& x = [P_G, \overline{R}_G, \underline{R}_G, \overline{R}_L, \underline{R}_L, \underline{d}_G, \overline{d}_G, \underline{d}_L, \overline{d}_L] \geq \mathbf{0}. && (8)
\end{aligned}$$

Joint and Individual CC-OPF Models

- Constraints inside (7)

$$\begin{aligned}\widetilde{A}x \geq \widetilde{b} = \{&\underline{P}_G \leq P_G + R_G \leq \overline{P}_G,\\ &\underline{\widetilde{P}}_L \leq \widetilde{P}_L + R_L \leq \overline{\widetilde{P}}_L,\\ -&\underline{R}_G \leq R_G \leq \overline{R}_G,\\ -&\underline{R}_L \leq R_L \leq \overline{R}_L,\\ -&P_{\text{line}} \leq B_{\text{flow}} \begin{bmatrix} 0 \\ B_{\text{bus}}^{-1}\hat{P}_{\text{inj}} \end{bmatrix} \leq P_{\text{line}}\}. \end{aligned} \tag{9}$$

- **[I-CC-OPF]:**

$$\begin{aligned} \min \quad & (1) \\ \text{s.t.} \quad & (2)\text{–}(6),\ (8) \\ & \mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m. \end{aligned} \tag{10}$$

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Solution Approaches: Mixed-Integer Linear programming (MILP) Approach (A1)

- Known as Sample Average Approximation (SAA) approach
- Reformulate individual chance constraints (10) $\mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m$ as

$$A_i^s x \geq b_i^s - M y_s^i \ \forall s \in \Omega, \ i = 1, \ldots, m \tag{11}$$

$$\sum\nolimits_{s \in \Omega} p^s y_s^i \leq \epsilon_i, \ \forall i, \ \text{and} \ y_s^i \in \{0, 1\} \ \forall s, \ i, \tag{12}$$

  where M is a large scalar coefficient.
- Associate each $s \in \Omega$ with a binary logic variable y_s^i such that
  - $y_s^i = 0$ indicates that $A_i^s x \geq b_i^s$.
  - $y_s^i = 1$ indicates that $A_i^s x < b_i^s$.

Solution Approaches: Gaussian Approximation Approach (A2)

- Consider an equivalent of individual chance constraints (10) $\mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m$

$$\mathbb{P}\left(\widetilde{A}_i' \bar{x} \leq b_i'\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m, \tag{13}$$

- Assume the uncertainty is Gaussian distributed:

$$\widetilde{A}_i' \sim N(\mu_i, \Sigma_i).$$

Then,

$$\widetilde{A}_i' \bar{x} - b_i' \sim N(\mu_i^\mathsf{T} \bar{x} - b', \bar{x}^\mathsf{T} \Sigma_i \bar{x}).$$

- We rewrite (13) as

$$b_i' - \mu_i^\mathsf{T} \bar{x} \geq \Phi^{-1}(1 - \epsilon_i) \sqrt{\bar{x}^\mathsf{T} \Sigma_i \bar{x}} \quad i = 1, \ldots, m. \tag{14}$$

The above are second-order cone constraints if $\Phi^{-1}(1 - \epsilon_i) \geq 0$, i.e., $1 - \epsilon_i \geq 0.5$.

Solution Approaches: Scenario Approximation Approach (A3)

- Replace each chance constraint in (10)
  $\mathbb{P}\left(\widetilde{A}_i x \geq \widetilde{b}_i\right) \geq 1 - \epsilon_i \quad i = 1, \ldots, m$ with

$$A_i^s x \geq b_i^s \ \forall s \in \Omega_{\text{ap}}. \tag{15}$$

- Both A1 and A2 require full distributional knowledge, while A3 requires large sample sizes and significant computation.

Solution Approaches: Distributionally Robust Optimization Approach (A4)

- The DR variant of (10):

$$\inf_{f(\xi)\in\mathcal{D}} \mathbb{P}_\xi(\widetilde{A}_i^\xi x \geq \widetilde{b}_i^\xi) \geq 1-\epsilon_i \;\forall i=1,\ldots,m. \tag{16}$$

- The confidence set (description in a general way)
Given samples $\{\xi^i\}_{i=1}^N$ of ξ, we first calculate the empirical mean and covariance matrix as $\mu_0 = \frac{1}{N}\sum_{i=1}^N \xi^i$ and $\Sigma_0 = \frac{1}{N}\sum_{i=1}^N(\xi-\mu_0^i)(\xi-\mu_0^i)^\top$, and then build a confidence set

$$\mathcal{D} = \left\{ f(\xi): \quad \begin{array}{l} \int_{\xi\in\mathcal{S}} f(\xi)d\xi = 1 \\ (\mathbb{E}[\xi]-\mu_0)^\top(\Sigma_0)^{-1}(\mathbb{E}[\xi]-\mu_0) \leq \gamma_1 \\ \mathbb{E}[(\xi-\mu_0)(\xi-\mu_0)^\top] \preceq \gamma_2\Sigma_0 \end{array} \right\}.$$

Solution Approaches: Distributionally Robust Optimization Approach (A4)

- (Duality theory) Let r_i, $\begin{bmatrix} H_i & p_i \\ p_i^\top & q_i \end{bmatrix}$, and G_i be the dual variables associated with the three constraints in the above confidence set $\mathcal{D}$, respectively. The individual chance constraints (16) are equivalent to

$$\gamma_2 \Sigma_0 \cdot G_i + 1 - r_i + \Sigma_0 \cdot H_i + \gamma_1 q_i \leq \epsilon_i y_i \tag{17}$$

$$\begin{bmatrix} G_i & -p_i \\ -p_i^\top & 1 - r_i \end{bmatrix} \succeq \begin{bmatrix} 0 & \frac{1}{2}\bar{A}_i^x \\ \frac{1}{2}(\bar{A}_i^x)^\top & y_i + (\bar{A}_i^x)^\top \mu_0 - \bar{b}_i^x \end{bmatrix} \tag{18}$$

$$\begin{bmatrix} G_i & -p_i \\ -p_i^\top & 1 - r_i \end{bmatrix} \succeq 0, \begin{bmatrix} H_i & p_i \\ p_i^\top & q_i \end{bmatrix} \succeq 0, y_i \geq 0, \ i = 1, \ldots, m, \tag{19}$$

  where operator "$\cdot$" in constraint (17) represents Frobenius inner product of two matrices (i.e., $A \cdot B = \mathrm{tr}(A^\top B)$). This is a semi-definite program and can be solved by commercial solvers.

- Importantly, note that the above approaches for bounding the unknown $f(\xi)$ are general and allow the uncertainty ξ to be **time-varying, correlated, and endogenous**.

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Computational Results: IEEE 9-Bus System

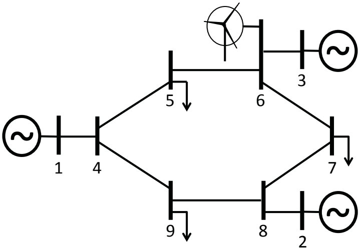

Figure: IEEE 9-bus system, with added wind generation.

Computational Results: IEEE 9-Bus System

Table: Results to IEEE 9-Bus system with $1 - \epsilon_i = 95\%$

		Obj.			Rel(%)			CPU		
		avg	min	max	avg	min	max	avg	min	max
A1	J-CC-OPF	1349	1328	1363	77	8	95	2	1	4
	I-CC-OPF	1346	1336	1357	72	46	90	5876	131	32817
A2	I-CC-OPF	1349	1340	1358	82	65	94	1	1	1
A3	I-CC-OPF	1408	1371	1525	100	99	100	55	54	57
A4	I-CC-OPF	1393	1365	1458	100	98	100	5	4	6

Computational Results: IEEE 9-Bus System

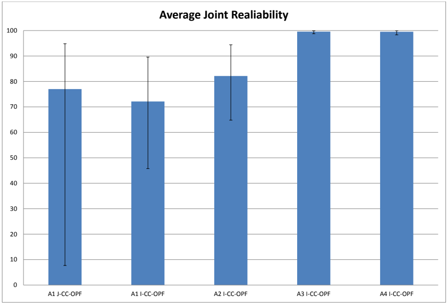

Figure: Average reliability to IEEE 9-Bus system with $1 - \epsilon_i = 95\%$

The highest/lowest value of the err bar is the largest/smallest realized probability.

Computational Results: IEEE 9-Bus System

Table: Results of I-CC-OPF solved by the DR approach A4

		$1-\epsilon_i =$	95.00%	90.00%	85.00%
Individual	Objective cost	avg	1392.64	1369.23	1359.97
		min	1352.46	1346.62	1346.62
		max	1457.81	1385.24	1372.75
	Reliability (%)	avg	99.50	97.97	94.51
		min	91.40	91.40	83.29
		max	99.96	99.70	99.18
	CPU seconds	avg	6.63	6.98	6.95
		min	6.13	4.73	6.27
		max	8.19	8.44	7.83

Computational Results: IEEE 9-Bus System

Table: Solutions from A1–A4 of I-CC-OPF with $1 - \epsilon_i = 95\%$

	$(P_G)_1$	$(P_G)_2$	$(P_G)_3$	$(\overline{R}_G)_1$	$(\overline{R}_G)_2$	$(\overline{R}_G)_3$	$(\underline{R}_G)_1$	$(\underline{R}_G)_2$	$(\underline{R}_G)_3$	$(\overline{R}_L)_1$	$(\overline{R}_L)_2$
A1	10.00	28.84	20.94	0.00	0.00	0.00	0.00	0.00	0.00	**4.44**	**1.21**
A2	10.00	28.89	20.97	0.00	0.00	0.00	0.00	0.00	0.00	3.88	**1.88**
A3	10.03	29.32	21.27	0.03	**2.35**	0.00	0.03	**2.79**	0.00	10.49	9.73
A4	10.00	29.22	21.20	0.00	**0.25**	0.00	0.00	**0.34**	0.00	10.97	7.34

	$(\overline{R}_L)_3$	$(\underline{R}_L)_1$	$(\underline{R}_L)_2$	$(\underline{R}_L)_3$	$(d_G)_1$	$(d_G)_2$	$(d_G)_3$	$(d_L)_1$	$(d_L)_2$	$(d_L)_3$
A1	8.05	**1.86**	**0.63**	3.41	0.00	0.00	0.00	**0.32**	**0.09**	0.58
A2	9.45	2.03	**1.08**	4.21	0.00	0.00	0.00	0.25	**0.12**	0.62
A3	4.74	8.55	7.85	4.00	0.00	0.10	0.00	0.38	0.35	0.17
A4	15.17	8.46	5.68	11.59	0.00	0.01	0.00	0.32	0.21	0.46

Computational Results: IEEE 9-Bus System

Table: Realization Results to the 9-bus system

		R_G	R_G	R_G	R_L	R_L	R_L	Active lines
	95%	0.00	0.00	0.00	0.00	0.76	3.22	2.0E-04
A1	90%	0.00	0.00	0.00	3.00	0.98	0.00	2.0E-04
	85%	0.00	0.00	0.00	0.00	0.00	3.98	1.0E-04
	95%	0.00	0.00	0.00	0.00	0.00	0.00	0.0E+00
A2	90%	0.00	0.00	0.00	0.86	0.09	3.03	0.0E+00
	85%	0.00	0.00	0.00	0.00	0.00	3.98	0.0E+00
	95%	0.00	0.00	0.00	2.00	1.51	0.47	1.0E-04
A3	90%	0.00	0.00	0.00	1.57	1.19	1.22	1.0E-04
	85%	0.00	0.00	0.00	1.78	1.54	0.66	1.0E-04
	95%	0.00	0.00	0.00	1.39	0.81	1.78	0.0E+00
A4	90%	0.00	0.00	0.00	1.18	0.71	2.10	0.0E+00
	85%	0.00	0.00	0.00	1.16	0.61	2.22	0.0E+00

Computational Results: IEEE 39-Bus System

Table: Simulated satisfaction rate (%) with $1 - \epsilon_i = 95\%,\ \forall i$

Constraint	1	2	3	4	5	6	7	8	9	10
A5	95.89	88.88	31.57	**95.29**	**94.98**	**94.97**	**94.86**	**99.87**	**99.79**	**94.62**
A2	**96.01**	**89.12**	**91.60**	91.60	91.60	91.60	91.60	91.60	91.60	91.60
A3	99.97	99.91	100	100	100	100	100	100	100	100
Constraint	11	12	13	14	15	16	17	18	19	20
A5	**98.38**	**94.85**	**94.56**	**94.56**	**99.46**	**94.58**	**92.06**	**93.12**	**93.66**	**93.05**
A2	91.60	91.60	91.60	91.60	91.60	91.60	91.60	91.60	91.60	91.60
A3	100	100	100	100	100	100	100	100	100	100
Constraint	21	22	23	24	25	26	27	28	29	30
A5	**92.99**	88.35	**97.68**	**97.50**	**97.50**	**97.46**	**99.91**	**99.86**	**97.31**	**99.15**
A2	91.60	**95.85**	95.85	95.85	95.85	95.85	95.85	95.85	95.85	95.85
A3	100	99.98	99.98	99.98	99.98	99.98	99.98	99.98	99.98	99.98
Constraint	31	32	33	34	35	36	37	38	39	40
A5	**97.44**	**97.25**	**97.25**	**99.65**	**97.27**	**96.11**	**96.68**	**96.91**	**96.56**	**96.54**
A2	95.85	95.85	95.85	95.85	95.85	95.85	95.85	95.85	95.85	95.85
A3	99.98	99.98	99.98	99.98	99.98	99.98	100	99.98	99.98	99.98

Computational Results: IEEE 39-Bus System

Table: Average Realization Results to the 39-bus system

	R_G		R_L		# of Active line
	avg	std	avg	std	
A5	-0.02	0.00	1.12	2.15	0.0000%
A2	0.00	0.00	1.41	1.59	0.0000%
A3	0.00	0.00	1.41	1.92	0.0300%

the negativeness of R_G in A5 is due to the inaccuracy of our results.

Computational Results: IEEE 39-Bus System

Table: Average performance (out of 37 Constraints) to IEEE 39-Bus system with $1 - \epsilon_i = 95\%$

	CPU seconds	Objective cost	Reliability (%)
A5	3015.98	25670.07	96.47
A2	4.10	25632.72	93.79
A3	6893.96	26129.16	99.99

Computational Results: IEEE 39-Bus System

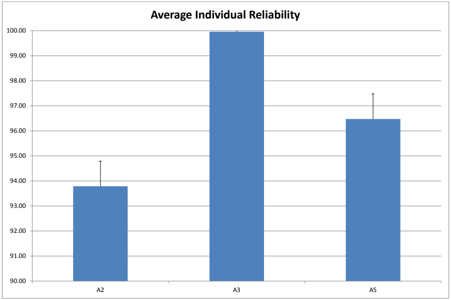

Figure: Average reliability (out of 37 Constraints) to IEEE 39-Bus system with $1 - \epsilon_i = 95\%$