Real Credit Cycles[*]

Pedro Bordalo[†]
University of Oxford

Nicola Gennaioli[‡]
Università Bocconi

Andrei Shleifer[§]
Harvard University

Stephen J. Terry[¶]
University of Michigan

April 2024

(First draft July 2019)

Abstract

We embed diagnostic expectations in a workhorse neoclassical model with heterogeneous firms and risky debt. A realistic degree of overreaction estimated from US firms' earnings forecasts generates realistic credit cycles. Good times produce economic and financial fragility, predicting future disappointment of expectations, low bond returns, and investment declines. To generate the size of spread increases observed during 2007-9, the model requires only moderate negative shocks. Diagnostic expectations offer a realistic, parsimonious way to produce financial reversals in business cycle models.

Keywords: diagnostic expectations, overreaction, firm heterogeneity

[*]We thank audiences at FRB Minneapolis, Yale Finance, Harvard Business School Finance, NBER Behavioral Macro, Northwestern Kellogg Finance, Stanford SITE Uncertainty, NHH, the European Central Bank, Boston University, Harvard Economics, University of Oregon, University of Virginia Darden, NBER Impulse and Propagation Mechanisms, FRB Chicago, AEAs, Queen Mary University of London, BI Norwegian Finance, Wharton Finance, Econometric Society, Green Line Macro Meeting, Dartmouth Macro Mini-Conference, CREi-UPF, Cornell University, University of Michigan, University of Notre Dame, University of Lausanne Finance, Wisconsin Finance, Carnegie Mellon Finance, and the Macro Finance Society, our discussants Alp Simsek, Cosmin Ilut, Ryan Chahrour, Ding Dong, Hanno Lustig, and Stefan Niemann, as well as Spencer Kwon, for valuable comments and suggestions.

[†]pedro.bordalo@sbs.ox.ac.uk
[‡]nicola.gennaioli@unibocconi.it
[§]shleifer@fas.harvard.edu
[¶]sjterry@umich.edu

1 Introduction

Developed economies experience recurrent boom-bust cycles in real and financial activity. Periods in which credit spreads are low, and investment, output, and leverage are high, tend to be followed by periods in which spreads rise, and investment, output growth, and leverage decline (Schularick and Taylor, 2012; López-Salido et al., 2017). In most accounts, instability reflects the amplification of fundamental shocks through financial frictions, fire sales, or demand externalities (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Eggertsson and Krugman, 2012). Because crises often occur absent fundamental shocks, it is sometimes assumed that the shock is "financial" such as a spike in investors' required returns (Korinek and Simsek, 2016), uncertainty (Arellano et al., 2019), or a bank run (Diamond and Dybvig, 1983). Critically, though, such financial shocks remain a black box.[1]

In this paper, we offer an alternative approach, in which boom-bust instability is due to non-rational expectations. We offer modern microfoundations to previous informal arguments (Minsky, 1977; Kindleberger, 1978) by building on the psychologically founded model of diagnostic expectations (Bordalo et al., 2018), which we embed in a workhorse neoclassical model with defaultable debt. This modification introduces a single new parameter capturing the overreaction of beliefs to total factor productivity (TFP) news. We discipline non-rationality by using data on managers' expectations about their firms' profitability. An empirically plausible degree of overreaction produces realistic boom-bust cycles. During good times investors are too optimistic, so credit and investment overexpand. When beliefs cool off, credit markets tighten, real activity declines, and default rates rise, consistent with the evidence in (Bordalo et al., 2018, 2019). This mechanism yields sharp reversals even in the absence of large negative TFP shocks, and matches the nontargeted predictability of realized bond returns at the firm and aggregate levels, without assuming exotic risk preferences.

In our setting the diagnostic expectations (DE) model (Bordalo et al., 2018) gives rise to the expectations formula:

$$\mathbb{E}_t^\theta \left(A_{t+1}\right) = \mathbb{E}_t \left(A_{t+1}\right) + \theta \left[\mathbb{E}_t \left(A_{t+1}\right) - \mathbb{E}_{t-1} \left(A_{t+1}\right)\right], \tag{1}$$

where A_{t+1} is future TFP, $\mathbb{E}_t(\cdot)$ is the rational expectation at time t, and $\theta \geq 0$ is a diagnosticity parameter. Equation (1) is founded in the psychology of selective recall: When $\theta > 0$ agents overreact, so recent good news does not just increase the true likelihood of good future outcomes. It also causes such outcomes to be top of mind and thus overweighted in beliefs. DE have two advantages relative to models of adaptive expectations (Cagan, 1956) or of exaggerated TFP persistence (Greenwood and Shleifer, 2014; Angeletos et al., 2020).

[1] Shleifer and Vishny (1992) offer the original treatment of fire sales. This amplification is also in Lorenzoni (2008), Stein (2012), and Dávila and Korinek (2017). Demand externalities are studied by Farhi and Werning (2020), Guerrieri and Lorenzoni (2017), Korinek and Simsek (2016), and Rognlie et al. (2018).

First, memory based belief distortions depend on the data generating process experienced by the agent. Thus, DE are not vulnerable to the Lucas (1976) critique. Second, memory implies that excess optimism swiftly cools off when good news recede into the past, even in the absence of bad shocks. As a result, small shocks cause significant boom-bust instability, which does not arise when - for instance - agents merely exaggerate persistence. Another advantage of DE is that they modify RE by a single parameter, θ, so they are portable across contexts. This has allowed researchers to estimate θ in different datasets, including the forecasts of financial (Bordalo et al., 2019) and macroeconomic analysts (Bordalo et al., 2020). These estimates discipline our results.

The paper proceeds as follows. In Section 2 we present novel evidence on boom-bust cycles in expectations, bond returns, and investment at the firm level. We show that managers' expectations of their firms' profits overreact: they are too optimistic in good times and too pessimistic in bad times. In turn, excess optimism predicts a one year-ahead decline in the firm's bond return as well as lower investment growth. We then investigate whether these micro level boom-bust dynamics: i) can be explained by a disciplined departure from rationality and whether ii) they can produce aggregate instability.

Section 3 introduces the DE model, discussing its foundations and implications. Sections 4 and 5 introduce DE about firms' TFP in a model in which consumers are risk neutral and firms hire labor, invest, and issue equity as well as risky debt subject to idiosyncratic and aggregate TFP shocks. In our main analysis the wage and the interest rate are fixed (deep pocketed lenders provide credit), but we endogenize an equilibrium wage in Section 7.2. We structurally estimate the model by matching firm-level moments on profitability, spreads, debt, investment, and forecast errors. The estimated degree of diagnosticity $\theta \approx 1$ is in the ballpark of previous estimates (Bordalo et al., 2019; Pflueger et al., 2020).

The model yields two key results. First, under DE, but not under an estimated RE version of the model, the reaction of aggregate investment to TFP shocks is highly nonlinear. In good times, even a small negative TFP shock causes a large drop in aggregate investment, while the same shock has a muted effect in normal times. Second, in the DE model large increases in credit spreads occur even after only moderately negative TFP shocks. Under RE, by contrast, spreads rise little even after much larger negative TFP shocks. Thanks to their overreaction and sudden reversal, DE offer a theory of amplification through "financial shocks": waning of excess optimism causes a sudden inward shift in the supply of capital, causing rising credit spreads (Jermann and Quadrini, 2012; Gilchrist and Zakrajšek, 2012).

In Section 6 we assess the model's ability to match untargeted facts. We first look at macro comovements. The DE model overall outperforms the RE model among key financial dimensions, producing volatile and countercyclical credit spreads. These are central features of credit cycles, and they are again due to the boom-bust nature of DE. Overeaction produces an outward shift in the supply of capital during good times, followed by an inward shift due to belief reversal.

Crucially, we then go back to the firm-level boom-bust cycles in expectations, investment, and bond returns documented in Section 2. We show that the DE (but not the RE) model offers a good account of them. We also show that the estimated DE model can produce realistic cycles at the sectoral level, in which good times are followed by disappointment and lower credit spreads. These patterns are also fully untargeted in our calibration.

As a final exercise, we ask how large a TFP shock is required in the DE model to produce the massive increase in credit spreads observed during 2007-09 and investigate the macroeconomic implications of such a shock. We find that a moderate decline in TFP is sufficient to generate the observed increase in credit spreads. The RE model is entirely incapable of matching this performance, with little movement in spreads in response to the identical shock. The DE model also produces larger declines in aggregate investment and earnings expectations, explaining a larger fraction of the observed declines in each variable.

The final Section 7 performs two robustness exercises. First, we endogenize the real wage in general equilibrium by adding a household labor choice with convex effort cost. As expected, this modification dampens volatility but does not eliminate sizable fragility after good times, even with flexible wages. Second, we perform sensitivity analysis by varying a range of model parameters. Again, our key results appear to be robust.

Our paper contributes to two strands of work in macroeconomics. Work on macro-financial instability maintains rational expectations but produces time-varying risk attitudes through habits (Campbell and Cochrane, 1999), long-run risk (Bansal and Yaron, 2004), or disaster risk (Barro, 2006). In these accounts, time-varying risk premia are not directly measured and predictions regarding survey expectations are counterfactual. Habit formation predicts countercyclical expected returns while survey expectations of returns are procyclical (Greenwood and Shleifer, 2014). Investors expects low returns when they expect a crash, contrary to the predictions of disaster risk models (Giglio et al., 2021). Here we assume standard preferences, in particular constant required returns, but relax belief rationality in a manner disciplined by micro data. Future work may combine the two ingredients.

A second strand of work studies departures from full information rational expectations, rational inattention in particular (Sims, 2003; Mankiw and Reis, 2002). Coibion and Gorodnichenko (2015) study inflation forecasts using information rigidities as in Woodford (2003). Kohlhas and Walther (2021) model asymmetric attention paid to distinct macroeconomic variables. Kozlowski et al. (2020) links belief dynamics to the persistence of the Great Recession. Angeletos et al. (2020) models dispersed information and overextrapolation of macroeconomic outcomes. This work does not produce excess volatility in beliefs and economic outcomes. A different approach emphasizes ambiguity aversion (Hansen and Sargent, 2001; Ilut and Schneider, 2014; Bianchi et al., 2018; Bhandari et al., Forthcoming). This mechanism can generate excess volatility in beliefs, but based on an underlying pessimism that is difficult to square with the observed excess optimism in beliefs data.[2]

Our key innovation is to allow for belief overreaction, discipline it using micro data, and study its macro-financial implications. We show that overreacting beliefs create significant excess macro-financial volatility and systematic reversals. Recent work introduces DE into neoclassical and New Keynesian models (Bianchi et al., 2023; L'Huillier et al., 2023). These papers emphasize that belief overreaction from DE generates quantitatively meaningful amplification and propagation of traditional business cycle shocks. In their setting, but not in ours, expectation formation concerns endogenous variables. Most important, they do not study financial fluctuations (their representative agent models do not allow for credit spreads), and they do not use micro expectations data to estimate their model. Enriching our setup with realistic ingredients such as bank runs, shifts in aggregate demand (Farhi and Werning, 2020), time-varying capacity utilization (King and Rebelo, 1999), or sharp cuts to firm-level employment in bad times (Ilut et al., 2018) may offer a more realistic model of macroeconomic volatility. Along these lines Maxted (2023) introduces DE into He and Krishnamurthy (2019)'s model. Krishnamurthy and Li (2024) also study beliefs and intermediation in a DE framework. Falato and Xiao (2024) studies overreaction in bank expectations.

2 Data

We use survey data to connect belief overreaction to boom-bust cycles in investment and bond returns at the firm level, even controlling for macro shocks.

We use micro data on firm-level forecasts from the IBES manager guidance database, which records - for an individual firm-fiscal year - a manager's forecast for their company's profits or earnings over the next year. We exploit forecasts made concurrently with the release of the year's financials, in a sample spanning the 1999-2018 period. We link this data to Compustat, which provides standard financial information. We also use the Mergent Fixed Income Securities Database (FISD), which contains issuance information including spreads on individual securities, and the FINRA's Trade Reporting and Compliance Engine (TRACE) dataset, which contains data on secondary market transactions from bond dealers and hence allows us to compute realized bond returns. The FISD-TRACE sample covers the years 2003-2018. Appendix B provides more information on data sources and sample construction, as well as descriptive statistics.

To assess whether expectations overreact, we regress a firm's next year's forecast errors,

[2]See also Falato and Xiao (2020) and Schaal and Taschereau-Dumouchel (2023) on dispersed information. Jaimovich and Rebelo (2007) study the impact of other belief distortions in macro models without financial frictions. Caballero and Simsek (2020) study demand externalities in models with extrapolative beliefs. Behavioral finance has studied credit cycles Greenwood et al. (2023), including under DE (Bordalo et al., 2018), without however assessing the quantitative performance of these accounts.

defined as realized minus predicted profits, on current-year firm-level profits, investment, debt issuance, and profit forecasts, controlling for time and firm effects. Under rational expectations, the manager's forecast errors should be unpredictable based on any information available to them when the forecast is made. If beliefs overreact, displaying overoptimism during good times and pessimism during bad times, then future forecast errors should be negatively correlated with the firm's current fundamentals, proxied by investment and debt issuance. Table 1 reports the results, that are consistent with overreaction: firms earning more, investing more, issuing more debt, or forecasting higher future profits on average experience more negative earnings surprises in the future.

Table 1: Predictable Forecast Errors

	(1)	(2)	(3)	(4)
		Forecast Error$_{t+1}$		
Estimation Method:	OLS	OLS	OLS	OLS
Profits$_t$	-0.044^{**}			
	(0.021)			
Investment$_t$		-0.457^{***}		
		(0.065)		
Debt$_t$			-0.326^{***}	
			(0.063)	
Forecast$_t$				-0.244^{***}
				(0.031)
Firm Effects	X	X	X	X
Year Effects	X	X	X	X
Years	1999-2018	1999-2018	1999-2018	1999-2018
Firm-Years	9666	9666	9666	9666

Notes: The table reports panel OLS estimates from the merged Compustat-IBES sample of the coefficients of a regression of forecast errors on the indicated variable. The standard errors are clustered at the firm level. All variables are measured at the firm-fiscal year level. Forecast errors in $t+1$ are realized earnings in $t+1$ minus firm forecasts in t, scaled by firm tangible assets. Investment in t is capital expenditures, scaled by firm tangible assets. Debt is long-term and short-term liabilities at the end of t, scaled by firm total assets. Forecasts are manager guidance in the year t earnings call about earnings in year $t+1$, scaled by firm tangible assets. $* = 10\%$ level, $** = 5\%$ level, and $*** = 1\%$ level. The standard deviations of each variable are 0.784 (forecast errors), 1.031 (profits), 0.233 (investment), 0.238 (debt), and 0.926 (forecasts) where $0.01 = 1\%$ relative to the firm's assets.

This evidence confirms the violations of rationality found by Gennaioli et al. (2016) and Barrero (2022) in different datasets and contexts. It is comforting that overreaction is a robust feature of firm forecasts across different samples of firms - private and publicly listed - with differing strategic incentives - with some data confidential and other publicly disclosed.[3] The magnitudes are meaningful. In column (2) a one-standard deviation higher investment rate (about 23 percentage points) on average predicts $23 \times 0.457 \approx 11$ percentage points stronger disappointment in earnings next year.

These results are robust to alternative specifications, as reported in Appendix Table B.3. We exclude the Great Recession, to avoid the influence of outliers, exclude firms with less than five years of earnings guidance (which are more likely to manipulate earnings forecasts Bertomeu et al. (2021)), and exclude firms with high-yield debt, to verify that overreaction is not restricted to risky firms. We continue to uncover evidence of forecast overreaction. We also show that the predictability of forecast errors works symmetrically in good and in bad times, defined as periods with above or below average investment or debt.

These patterns also do not reflect simple constant heterogeneity in optimism across firms, since each column controls for firm fixed effects. But one might be concerned that firm fixed effects generate the downward dynamic panel bias of Nickell (1981) in finite samples.[4] So in Appendix B we develop a GMM estimator based on covariances of the *growth* of outcomes within firms up to the current period with future forecast error growth. The estimator is asymptotically equivalent to the OLS specifications in Table 1 but not subject to the same dynamic panel bias. As Table B.2 shows our baseline overreaction patterns go through and, if anything, strengthen in magnitude using this alternative estimator.

If overreaction produces boom-bust cycles, belief disappointment should be correlated with reversals in firm-level outcomes. To assess this prediction, Table 2 performs a two-stage regression exercise. In the first stage, see column (1), we regress future forecast errors on current investment, analogous to Table 1 for this sample, which detects excess optimism (pessimism) as predictably negative (positive) forecast errors.

In the second stage, reported in columns (2) and (3), we regress future bond returns and investment growth on predicted forecast errors, controlling for time effects. Belief-driven cycles imply that overoptimism (negative predicted errors) and overpessimism (positive predicted errors) are followed by reversals in bond returns and investment growth.[5] This test is not causal, and investment reversals may be produced by "rational" mean reversion in

[3] Recent work documents overreaction by correlating forecast errors with forecast revisions (Bordalo et al., 2020, 2019; Coibion and Gorodnichenko, 2015). Here we do not have enough data to perform this type of revision-based analysis. Bouchaud et al. (2019) study equity analysts' forecasts. They document that these forecasts, while correlated with managerial forecasts, display a form of underreaction.

[4] Intuitively, the forecast errors on the left hand side of these regressions include profit innovations which in finite samples may influence the mean forecast errors subtracted by the OLS fixed effects estimator. If firm financial outcomes on the right hand side are correlated with these profit innovations, there will be a mechanical negative correlation resulting in downward bias of the OLS fixed effect estimator.

[5] In Appendix B we also check that these firm-level cycles are not driven by risky firms, which may display more cyclicality. We repeat the exercise in Table B.5 for the subset of investment grade firms, excluding all firms with high-yield debt. Our results are confirmed.

Table 2: Linking Forecast Errors and Firm Reversals

	(1)	(2)	(3)
	Forecast Error_{t+1}	Return_{t+1}	Δ Investment_{t+1}
IV Stage:	First	Second	Second
Forecast Error_{t+1}		0.007^{*}	0.483^{***}
		(0.004)	(0.061)
Investment_t	-0.565^{***}		
	(0.104)		
Year Effects	X	X	X
Years	2003-2018	2003-2018	2003-2018
Firm-Years	2852	2852	2852
First Stage F	29		

Notes: The table reports estimates of specifications on the merged Compustat - IBES - FISD/TRACE sample at the firm-fiscal year level. Column (1) reports IV first-stage estimates, and the remaining columns report IV second-stage estimates. Column (3) controls for current profits in the second stage. Standard errors are clustered at the firm level. $*$ = 10% level, $**$ = 5% level, and $***$ = 1% level. The standard deviation of the bond return is 0.014, the standard deviation of investment growth is 0.090, the standard deviation of the forecast error is 0.439, and the standard deviation of lagged investment is 0.133. For all series, 0.01 = 1% relative to a firm's tangible capital stock.

fundamentals, which correlates with "irrational" disappointment. To account for this possibility, the investment regression controls for current profits.[6] However, the results do suggest that departures of rationality arise not only for managers (as shown in column (1) and in Table 1) but also for bond market investors. More rigorously, simulated data from our model will later reveal that belief overreaction – in particular, overreaction is needed to account for these firm-level patterns.

The results are consistent with belief driven boom-bust cycles. Good times with high investment are systematically followed by disappointment (column 1), low future bond returns (column 2), and low future investment growth (column 3). The opposite occurs after bad times. In column (2) a firm predictably disappointed with a one standard deviation lower forecast error sees its bond returns fall by $0.007 \times 0.439 \approx 0.3$ percentage points on average, a sizable decline relative to the standard deviation of around 1.4 percentage points for bond returns in our sample. Models without overoptimistic pricing of debt on the part of investors cannot generate this sort of micro-level return predictability, regardless of the belief structure at work within firms. Column (3) reveals that the investment rate declines by $0.483 \times 0.439 \approx 21$ percentage points at the same firm. The rest of the paper draws out the aggregate consequences of these firm-level patterns by introducing diagnostic expectations to a standard neoclassical model.

[6] Column (1), like Table 1, provides evidence of non-rationality of managers' profit expectations, a result that is also robust to removing the control for current profits. See Table B.6 in the empirical Appendix B.

3 Diagnostic Expectations and Neglected Risk

3.1 Diagnostic Expectations

The Diagnostic Expectations (DE) model accounts for a broad range of well documented departures from rationality, starting from the work of Kahneman and Tversky in the 1970s. DE were developed by Bordalo et al. (2018) building on Gennaioli and Shleifer (2010). To see how they work, consider an agent forecasting a future variable X_{t+1}, say TFP, on the basis of its history. The true distribution is Markovian, denoted by $f(X_{t+1}|X_t)$. Under DE, the agent's beliefs follow the distorted distribution:

$$f^{\theta}(X_{t+1}|X_t) \propto f(X_{t+1}|X_t)\left[\frac{f(X_{t+1}|X_t)}{f(X_{t+1}|\mathbb{E}_{t-J}(X_t))}\right]^{\theta} \tag{2}$$

where $\theta \geq 0$ and $\mathbb{E}_{t-J}(X_t)$ is the rational expectation of X_t conditional on information at $t-J$.[7] Equation (2) captures the idea that beliefs overweight future outcomes that have become more likely on the basis of the last J periods' news. Overweighting is captured by the likelihood ratio on the right hand side, which measures the current increase in the probability of X_{t+1} relative to the case of J periods of neutral news, $X_t = \mathbb{E}_{t-J}(X_t)$. θ captures the strength of overweighting. When $\theta = 0$ expectations are rational. When $\theta > 0$ beliefs overreact.

Equation (2) is founded on the psychology of selective recall. The data generating process, captured by the true densities $f(X_{t+1}|X_n)$, $n \leq t$, is stored in the agent's memory database. The agent selectively recalls outcomes that follow from the last J periods' news, in the sense that they historically became more likely after such news were observed, and fails to recall other outcomes. Thus, judgments overreact because they overweight the future outcomes that news cause to be top of mind. Bordalo et al. (2021a) offer a model of selective memory that provides a foundation for overreaction in Equation (2) and find support for it in two experiments. See also Bordalo et al. (2023a) for related evidence on memory and probability assessments.

To see the implications of Equation (2), suppose that TFP follows an AR(1) process, $\ln A_t = \rho \ln A_{t-1} + \varepsilon_t$, where $\rho \in (0,1)$ and ε_t is Gaussian with mean zero and variance σ^2. At time t the diagnostic distribution of next period productivity is also Gaussian, with mean:

$$\mathbb{E}_t^{\theta}(\ln A_{t+1}) = \rho \ln A_t + \theta\rho \sum_{r=0}^{J-1} \rho^r \varepsilon_{t-r}, \tag{3}$$

[7]Equation (2) assumes that X_t is perfectly observable. Bordalo et al. (2019, 2020, 2023a) allow for information frictions such as dispersed information about X_t, in which case DE take the form of a distorted Kalman Filter model. In Equation (2) as we later discuss, the agent's reliance on the true data generating process, and hence the dependence of overreacting beliefs on the "rational expectation" is a byproduct of selective memory retrieval from an unbiased memory database, not of the agent's awareness of the rational expectation.

and variance σ^2. DE are too optimistic after J periods of on average good news and too pessimistic after J periods of on average bad news. Future forecast errors are negatively predicted by current TFP:

$$Cov\left[\ln A_{t+1} - \mathbb{E}_t^\theta\left(\ln A_{t+1}\right), \ln A_t\right] \propto -\theta\rho\sigma^2\left(\frac{1-\rho^{2J}}{1-\rho^2}\right) < 0,$$

which allows DE to account for the error predictability in Table 1. This cannot occur under RE, with $\theta = 0$, which implies a zero covariance above.

We shall allow overreaction to current news only, i.e. $J = 1$. This assumption is common in applications of DE (Bordalo et al., 2018), and we adopt it to minimize degrees of freedom. Some applications allow for $J > 1$ showing that in this case DE produce slowly brewing asset price bubbles (Bordalo et al., 2019; Maxted, 2023; Bordalo et al., 2021b). In macroeconomics, Bianchi et al. (2023) show that longer streaks of overreaction can generate hump-shaped dynamics in New Keynesian models.

DE offer two advantages relative to other models of non-rational expectations used in macroeconomics and finance. First, they are founded on the psychology of memory (Kahana, 2012; Bordalo et al., 2023a), which highlights two key ingredients: the database of experiences, which is produced by the data generating process, and the salience of recent news as memory cues. These ingredients, absent from mechanical models, have substantive implications. Reliance on the database implies that belief distortions depend on the data generating process through its persistence ρ. Thus, consistent with the evidence in (Bordalo et al., 2019, 2020; Afrouzi et al., 2023; Azeredo da Silveira et al., 2020), belief revisions are indeed stronger for more persistent variables, which renders DE immune to the Lucas critique (Lucas, 1976).[8] The role of news as memory cues implies that DE exhibit sharp reversals. As news recede into the past, distortions quickly wane and beliefs revert to rationality.

To see these aspects, consider an alternative formulation in which the agent inflates persistence believing that TFP follows $\ln A_t = \widehat{\rho}\,\ln A_{t-1} + \varepsilon_t$, where $\widehat{\rho} > \rho$, as proposed by Greenwood and Hanson (2013) and Angeletos et al. (2020). The parameter $\widehat{\rho}$ is not linked to features of the environment; if the true persistence of TFP changes, beliefs do not change, displaying a strong violation of forward looking behavior. In addition, beliefs stay overly optimistic as long as TFP is above its long run mean of $\ln A = 0$, so there are no sharp reversals in the absence of bad news. The same problems arise in other models of

[8]With dispersed information, DE allow us to reconcile individual-level overreaction with the sluggishness of consensus forecasts (Bordalo et al., 2020; Coibion and Gorodnichenko, 2015).

mechanical extrapolation (Cagan, 1956).[9,10,11]

The second advantage of DE is that departures from rationality are determined by a single parameter, the degree of diagnosticity θ. [12] θ has been estimated in various settings ranging from firm-level earnings forecasts by financial analysts (Bordalo et al., 2019), to professional forecasters predicting macroeconomic variables (Bordalo et al., 2020), to stock price volatility (Pflueger et al., 2020). These estimates point to a value of θ between 0.5 and 1.5, providing a valuable external benchmark to discipline our quantitative exercise and assess its reliability. We view Equation (3) as a microfounded, tractable, and disciplined way to capture the observed belief overreaction of managers' beliefs in Section 2. Future work should refine the model, in particular by allowing for the possibility of belief underreaction, which is also observed in some contexts (Gabaix, 2019; Ma et al., 2024).

At this point, one could proceed by introducing DE in an standard closed economy neoclassical model with time to build or adjustment costs. In this setting, DE can produce excessive investment booms in good times and subsequent investment busts when beliefs revert, amplifying macroeconomic volatility. However, because we are interested in macro-financial volatility, and in particular in the genesis of financial reversals such as sudden increases in credit spreads, we insert DE into an incomplete market setting with defaultable debt. We next present a stylized example to illustrate how in such a setting DE affect the credit spread demanded by capital supplier. We also discuss the differences between DE and other proposed forms of excess volatility.

3.2 Diagnostic Expectations, Belief Volatility, and Supply-Driven Bond Pricing: A Toy Model

A firm seeks to roll over one-period defaultable bonds. Deep-pocket risk-neutral lenders demand a constant expected return $R = 1$. If the probability of default perceived by lenders

[9]Another problem of this model is that it may produce unstable dynamics, which is indeed the case in our setting. In Appendix Section B.6, we show that matching our estimates of belief overreaction from Section 5 requires $\widehat{\rho} > 1$, which is incompatible with dynamic optimization. DE instead maintains stationarity, transforming beliefs from a stationary AR(1) to a stationary ARMA(1,1) process.

[10]The DE model can produce exaggeration of persistence, and link it to the data generating process, as a special case. When $J \to \infty$, diagnostic beliefs are $\mathbb{E}_t^\theta\left(\ln A_{t+1}\right) = \rho\left(1+\theta\right)\ln A_t$, so that $\widehat{\rho} = \rho\left(1+\theta\right)$.

[11]Kohlhas and Walther (2021) show that overreaction may be produced if forecasters are overconfident. This model, however, cannot produce overreaction of consensus forecasts, which is instead documented in Bordalo et al. (2022). Kohlhas and Walther (2021) show that a form of overreaction can obtain under a form of rational inattention. This formulation is however unable to capture the overreaction of individual level forecasts documented in Bordalo et al. (2020).

[12]This parameter, θ, is also the only extra degree of freedom under our assumption $J = 1$, and further $J = 1$ is the only horizon choice delivering time-consistency of the resulting DE belief system. Appendix Table B.4 provides empirical evidence on the persistence of forecast overreaction up to lag $J = 2$ but finds no effect at lags $J > 2$. In light of these two considerations, we focus on $J = 1$ as our baseline case for model analysis in the rest of the paper.

for next period is δ_t^θ, the interest rate is $\widehat{R}_t = \frac{1}{1-\delta_t^\theta}$, so the credit spread is:

$$\widehat{R}_t - 1 \equiv S_t = \frac{\delta_t^\theta}{1-\delta_t^\theta}, \tag{4}$$

which increases in the default risk δ_t^θ perceived by lenders.

Suppose that the firm defaults at time t if and only if its productivity is lower than a threshold, $A_t < A^*$. The perceived probability of default is then given by:

$$\delta_t^\theta = \Phi\left[\frac{\ln A^* - \mathbb{E}_t^\theta\left(\ln A_{t+1}\right)}{\sigma}\right], \tag{5}$$

where $\Phi(\cdot)$ is the standardized Gaussian CDF. Due to DE, perceived risk is too low after good news and too high after bad news.

Equation (5) has implications for the path of credit spreads. Substituting δ_t^θ in (4) and linearizing with respect to $\mathbb{E}_t^\theta\left(\ln A_{t+1}\right)$ around its long run mean yields:

$$S_t \approx S_\infty - S\mathbb{E}_t^\theta\left(\ln A_{t+1}\right), \tag{6}$$

where $S_\infty > 0$ is the long run spread and S is a positive scalar. The spread falls when creditors are more optimistic about future productivity. Inserting (3) into (6) we obtain:

$$S_t \approx S_\infty\left(1-\rho\right) + \rho S_{t-1} - S\rho\left(1+\theta\right)\varepsilon_t + S\theta\rho^2\varepsilon_{t-1}. \tag{7}$$

Under RE, with $\theta = 0$, spreads mirror TFP and follow an AR(1) process with persistence ρ. Under DE, with $\theta > 0$, there are two differences. First, current TFP shocks are amplified, as captured by the term $-S\rho\left(1+\theta\right)\varepsilon_t$. This is a source of excess volatility: after a positive TFP shock, beliefs become too optimistic and the spread declines too much. Second, there is reversal of past TFP shocks, as captured by the term $S\theta\rho^2\varepsilon_{t-1}$. This is a source of boom-bust dynamics: in the future, current optimism wanes and the spread increases.

Due to the latter mechanism, DE generate predictable bond returns, which arise because the current spread S_t can differ from its RE counterpart S_t^*. Equation (7) implies:

$$S_t - S_t^* \approx -S\rho\theta\varepsilon_t. \tag{8}$$

When current expectations are overly optimistic, $\theta\varepsilon_t > 0$, bonds are overpriced. In the future, bond payouts are systematically disappointing, causing low returns. The converse holds when expectations are pessimistic. In sum, DE account for: i) overoptimism in good times, ii) excess shifts in the supply of capital and hence in spreads, which iii) then revert together with bond returns. Existing business cycle research has sought to produce such "excess" macroeconomic volatility under RE by introducing ingredients such as news shocks (Barsky and Sims, 2011; Bianchi, 2011), ambiguity (Bianchi and Melosi, 2016), and uncertainty (Bianchi et al., 2018; Bloom et al., 2018; Arellano et al., 2019) in complete or incomplete market neoclassical models.

DE have two advantages compared to these mechanisms. First and foremost, unlike the previous rational expectations mechanism DE can jointly match market outcomes and beliefs data. Expectations play a key role in macroeconomic transmission mechanisms, and building models based on systematically counterfactual expectations is problematic. Moreover, targeting our model to the size of firm-level overreaction helps isolate the role of belief overreaction by itself for aggregate macro-financial fluctuations while still leaving room for alternative channels such as ambiguity. Second, even when it comes to market outcomes, DE yield new implications compared to these alternative mechanisms. Under RE, a positive news shock, for instance, produces an investment boom but not a concurrent output boom, yielding a counterfactual negative correlation between consumption and investment. This is not true under DE, because excess optimism is produced by a current TFP boom, so current consumption can also go up. A sudden reduction in ambiguity or uncertainty can on the other hand cause an investment and consumption boom today. Compared to DE, however, these models produce neither an excessive boom today, nor a sharp reversal in the future. More generally, under any RE mechanism macrofinancial volatility reflects the volatility of TFP, which appears limited in the data (Fernald, 2014). DE allows to obtain excess volatility from small TFP shocks.

4 Neoclassical Model with Diagnostic Firms and Lenders

A unit mass of firms with different and persistent productivities decides whether to default, hire labor, invest, issue equity, and borrow subject to capital adjustment costs. Credit is supplied by a continuum of risk-neutral lenders. The only difference from a workhorse neoclassical model with firm heterogeneity and risky debt (Khan and Thomas, 2008; Arellano et al., 2019; Gilchrist et al., 2014) is that firms and lenders form expectations diagnostically.

Our main analysis is in partial equilibrium: we take the risk-free rate R and the wage rate W as given. The exogenous risk-free rate reflects a deliberate methodological choice. In conventional macro models the risk-free rate is endogenized by adding curvature to the utility function. Often, time varying risk premia are also allowed for, by introducing changes in risk aversion or long run risk. We abstract from these ingredients for two reasons. First, in these models the risk free rate is procyclical, which is counterfactual (Winberry, 2021; Cooper and Willis, 2015; Bachmann et al., 2013). Furthermore, even though early quantitative work suggested that an endogenous interest rate would smooth out micro nonlinearities present in partial equilibrium (Khan and Thomas, 2008), a recently building consensus suggests that business cycle nonlinearities survive in heterogeneous firms models in the presence of empirically plausible (low) investment elasticities to interest rates (Koby and Wolf, 2020). This implies that adding consumption curvature is unlikely to overturn our key conclusions about the impact of belief overreaction.[13]

Second, there is no systematic measurement of time-varying risk preferences or time varying risks, and the predictions of rational time-varying risk premia models are inconsistent with measured expectations of returns. Rational models predict high expected returns in bad times, but measured expectations of returns are actually low (Greenwood and Shleifer, 2014). Furthermore, recent evidence shows that over-reacting beliefs about future earnings growth can account for longstanding stock market puzzles (Bordalo et al., 2024), including return predictability, and shed light on macro-financial fluctuations (Bordalo et al., 2023b), even in the absence of time variation in required returns. Our focus on a fixed required return and on overreacting beliefs about future earnings growth is consistent with this approach.

The exogeneity of the wage rate is merely a simplifying assumption. We relax it in Section 7, where we allow W to be endogenous by introducing elastic labor supply. This is an informative exercise: we find that in good times overoptimistic beliefs boost labor demand and the real wage. In principle, this effect could dampen the beliefs-driven cycle. As we show, however, DE continue to create significant nonlinearity in this case as well.

4.1 Firms

Time is discrete. We use $'$ to denote future values and $_{-1}$ to indicate lagged values. Uppercase letters refer to macro or common values, lowercase letters refer to idiosyncratic objects. The generic firm has micro-level TFP z and is subject to macro level TFP A. It uses capital k and labor n as inputs to produce output according to a decreasing returns technology

$$y = Azk^{\alpha}n^{\nu}, \qquad \alpha + \nu < 1.$$

Capital evolves based on investment i, which entails a one-period time to build

$$k' = i + (1-\delta)k, \qquad 0 < \delta < 1.$$

Investment entails quadratic adjustment costs $AC(i,k) = \frac{\eta_k}{2}\left(\frac{i}{k}\right)^2 k$, where $\eta_k > 0$. The log of micro TFP follows the AR(1) process

$$\log z' = \rho_z \log z + \varepsilon_z', \quad \varepsilon_z' \sim N(0, \sigma_z^2), \quad 0 < \rho_z < 1\ , \tag{9}$$

[13] Specifically, our use of quadratic adjustment costs, together with an estimation strategy detailed below linked to the observed covariance matrix of investment and profitability, results in realistically moderate elasticities of investment to price changes along the lines of those suggested by Koby and Wolf (2020).

while the log of macro TFP follows

$$\log A' = \rho_A \log A + \varepsilon'_A, \quad \varepsilon'_A \sim N(0, \sigma_A^2), \quad 0 < \rho_A < 1 \,. \tag{10}$$

Firms and lenders form diagnostic beliefs about a firm's future productivity. Given the AR(1) processes (9) and (10), and given Equation (2), diagnostic beliefs over micro and macro TFP are described by the lognormal processes:

$$\log z' | (\log z, \varepsilon_z) \sim N\left[\rho_z(\log z + \theta \varepsilon_z), \sigma_z^2\right] \tag{11}$$

$$\log A' | (\log A, \varepsilon_A) \sim N\left[\rho_A(\log A + \theta \varepsilon_A), \sigma_A^2\right] . \tag{12}$$

When $\theta > 0$ the agent forecasts future productivity by overweighting current news, as if the true productivity process follows an ARMA (1,1).[14] $\theta > 0$ is the only difference between our model and a workhorse heterogeneous firms macro-financial model of fluctuations. The parsimonious single-parameter deviation of our overreactive DE belief system from the workhorse driving process in neoclassical models provides considerable transparency during our structural estimation exercise in Section 5 below.

Firms act competitively. In each period, the timing of events is as follows. First, each firm decides whether to default on its debt. If a firm defaults, its assets net of deadweight default costs are recovered by lenders, and the firm restarts with zero capital and debt after one period. If a firm repays, it also hires labor at wage W, chooses how much to invest, how much one-period debt to issue, and finally pays a dividend to its shareholders or issues equity. Firms maximize the expected discounted sum of current and future payouts, where the discount rate $(R)^{-1} < 1$ reflects the risk-free rate R.

Consider the firm's problem, starting from the last stage. The firm's dividend d is given by the standard formula

$$d = (1 - \tau)\left[y - Wn - AC(i, k) - \phi\right] + q^\theta(s, k', b')b' - i - b + \tau(R - 1 + \delta k), \tag{13}$$

which includes profits, given by the firm's output minus the wage bill, the adjustment cost, and a fixed production cost $\phi > 0$, net of the corporate income tax rate $\tau \in (0, 1)$. It also includes the resources raised by issuing new debt b' priced by the schedule q^θ, minus the investment cost i and debt repayment b. Finally, it includes the tax rebates for capital depreciation and interest expenses on debt.[15]

If dividends are negative, $d < 0$, the firm issues equity. Following Gomes (2001), this

[14] Another approach to capture extrapolation is Fuster et al. (2010)'s Natural Expectations, in which long lags in the data generating process are neglected by agents who end up overestimating short-term persistence in processes with long-term mean reversion.

[15] For computational simplicity, we assume the rebate is on average equal to the risk-free rate $R - 1$.

entails a cost $IC(d) = I(d<0)(\eta_f + \eta_d |d|)$, where $\eta_f > 0$ is the fixed and $\eta_d > 0$ is the variable cost of issuance.

The firm makes its other decisions considering four state variables: its current micro TFP z, macro TFP A, the micro shock ε_z and the macro shock ε_A. The exogenous news states are relevant for forming diagnostic expectations, and we collect all of the firm's exogenous states in the vector $s = (z, \varepsilon_z, A, \varepsilon_A)$. A firm is also identified by two endogenous states, its inherited capital stock k and debt b. Given an overall state (s, k, b), the firm's problem can be written recursively. Upon entering the current period, the value of the firm is given by:

$$V^\theta(s,k,b) = \max\left[V_D^\theta(s), V_{ND}^\theta(s,k,b)\right], \tag{14}$$

where $V_{ND}^\theta(s,k,b)$ is the continuation value from not defaulting and $V_D^\theta(s)$ the continuation from defaulting. Condition $V_{ND}^\theta(s,k,b) < V_D^\theta(s)$ identifies states in which the firm optimally defaults. The continuation value from not defaulting is recursively determined as:

$$V_{ND}^\theta(s,k,b) = \max_{k',b',n}\left\{d - IC(d) + \frac{1}{R}\mathbb{E}^\theta\left[V^\theta(s',k',b')|s\right]\right\}. \tag{15}$$

If the firm does not default, it hires labor n, capital k' and sets debt b' to maximize its current dividend plus its diagnostically expected discounted future value $V^\theta(s',k',b')$.[16] The labor choice n is statically optimized, leaving only the intertemporal choices of k' and b'.

If the firm defaults, its assets k net of deadweight costs are claimed by lenders during a period of reorganization with no production. The firm restarts with zero debt and assets:

$$V_D^\theta(s) = \left\{0 + \frac{1}{R}\mathbb{E}^\theta\left[V(s',0,0)|s\right]\right\}. \tag{16}$$

After defaulting, a firm must borrow to invest. Equations (14), (15), and (16) determine the optimal default policy by $df^\theta(s,k,b)$ and, for non defaulting firms having $df^\theta(s,k,b) = 0$, the policies for endogenous states $k'^\theta(s,k,b)$ and $b'^\theta(s,k,b)$.

[16] We apply DE to the recursive formulation of the problem, Equation (14). The diagnostic agent believes that productivity follows an ARMA(1,1) and correctly thinks that they will continue to believe the same in the future. The recursive problem is equivalent to an optimal control problem in which the probability distribution of A_{t+s} at time t is the product $\Pi_{j=1}^{s} f^\theta(A_{t+j}|A_{t+j-1}, \varepsilon_{A,t+j-1})$ of the conditional distributions between times t and $t+s-1$. This distribution has the same mean as the time t diagnostic distribution $f^\theta(A_{t+s}|A_t, \epsilon_{At})$ but has larger variance. This is due to overreaction to news (which are zero on average) in the intermediate periods.

4.2 Lenders

Firms borrow from risk-neutral deep-pocket lenders whose required expected return is the risk-free rate R. If a firm (s,k,b) defaults on debt b, the lender receives the recovery rate

$$\mathcal{R}(k,b) = (1-\tau)\,\gamma\frac{(1-\delta)k}{b}$$

which reflects, net of tax, an exogenous fraction γ of the liquidation value $(1-\delta)\,k$ of the firm's capital stock. The remaining fraction $1-\gamma$ is a deadweight loss.

The price of debt $q^{\theta}(s,k',b')$ adjusts endogenously so that the diagnostically expected bond return is equal to the risk free rate R :

$$q^{\theta}(s,k',b') = \frac{1}{R}\mathbb{E}^{\theta}\left[1 + df^{\theta}(s',k',b')\left(\mathcal{R}(k',b')-1\right)|s\right]. \tag{17}$$

To equalize expected bond returns across firms, riskier firms promise a higher interest rate.[17] Thus, the firm's interest rate spread relative to the risk-free rate is given by:

$$S^{\theta}(s,k',b') = \frac{1}{q^{\theta}(s,k',b')} - R.$$

These equations illustrate how diagnosticity affects spreads. On the demand side, diagnosticity affects the firm's default $df^{\theta}(s,k,b)$, debt $b'^{\theta}(s,k,b)$, and investment $k'^{\theta}(s,k,b)$ policies. On the supply side, diagnosticity affects the probability of default perceived by lenders, as captured by the operator $\mathbb{E}^{\theta}(\cdot)$ in (17). The interaction between demand and supply plays a key role in producing macroeconomic fluctuations.

4.3 Solving the Model

A partial equilibrium is a collection of i) firm policies b'^{θ}, k'^{θ}, and df^{θ}, ii) firm values V^{θ}_{ND}, V^{θ}_{D}, and V^{θ}, and iii) a lender debt price schedule q^{θ} such that iv) taking as given debt prices q^{θ}, firm policies and values satisfy Equations (14), (15), and (16), v) taking as given firm policies, debt prices satisfy the zero-profit condition in Equation (17), and vi) expectations in the firm and lender problems $\mathbb{E}^{\theta}$ reflect the dynamics in Equations (11) and (12).

We solve the model numerically. In addition to the set of Bellman equations for V^{θ}, V^{θ}_{ND}, and V^{θ}_{D}, the model features an equilibrium fixed point between firm default policies df^{θ} and credit prices q^{θ} in Equation (17). We employ an iterative approach detailed in Appendix A. First, we guess a firm default rule df^{θ}, computing the implied debt price schedule q^{θ} according to the lenders' zero-profit condition. Then, we solve the Bellman equations for V^{θ}, V^{θ}_{D}, and V^{θ}_{ND} using discretization and policy iteration. If the implied default states, i.e., those with

[17] The realized firm bond return is given by $R^{firm}(s,s',k',b') = \frac{1+df^{\theta}(s',k',b')\left(\mathcal{R}(k',b')-1\right)}{q^{\theta}(s,k',b')}$.

Table 3: Externally Fixed Parameters

	Parameter	Role	Value	Source
1	δ	Depreciation rate	0.10	Annual Solution
2	R	Risk-free rate	1.04	Annual Solution
3	α	Capital elasticity	0.25	Bloom et al. (2018)
4	ν	Labor elasticity	0.50	Bloom et al. (2018)
5	ρ_A	Macro TFP autocorrelation	0.95	Bloom et al. (2018)
6	τ	Corporate income tax	0.20	Effective rates, CBO (2017)

Notes: The table reports the parameter symbol, numerical value, a description, and source information for each of the externally fixed parameters.

$V^{\theta}_{ND} < V^{\theta}_{D}$, match the set of initial guesses, then the iteration is complete. Otherwise, we compute the newly implied default states and repeat the process. The algorithm we employ is standard within the literature solving quantitative dynamic corporate finance models and follows the implementation in Strebulaev and Whited (2012).[18]

DE create a wedge between the value functions of two firms reaching the same current productivity via different news: the firm experiencing good recent news displays overoptimism and hence an inflated continuation value relative to a firm experiencing bad recent news (see Appendix Figure A.1). This effect on the demand side for credit, combined with diagnostic shifts in the supply of capital by lenders, leads to aggregate effects.

5 Estimating the Model & Inspecting the Mechanism

5.1 Estimation

Our model has sixteen parameters, listed in Tables 3 and 4. The six parameters in Table 3 are set to conventional values for a model like ours. The remaining ten parameters are: ρ_z and σ_z, which govern the micro-level TFP process, η_k and ϕ capture the adjustment and operating costs, γ the lenders' recovery rate, σ_A encodes macro volatility, η_f and η_d the equity issuance costs. θ is the key diagnosticity parameter. We also allow for and estimate the volatility σ_π of iid measurement noise in profits, which captures spurious variability in measured profits due to accounting conventions.

Using the simulated method of moments (SMM) we structurally estimate these ten parameters using sixteen statistics. Three of them are macro moments: the average credit

[18] Our numerical approach here is highly computationally intensive, given the presence of four exogenous states, two endogenous states, an endogenous default rule, and a debt-pricing fixed point. However, judicious application of parallelization and an economical approach to storage of micro-level outcomes following Young (2010) and Terry (2017) allow for solution of the model in several minutes on a desktop computer.

spread, the average frequency of default, and GDP growth volatility. Macro data on default and spreads encode information about the fixed cost ϕ and recovery rate γ, GDP growth volatility is informative about macro shocks σ_A. The remaining thirteen moments are at the firm level. One set of firm level moments comes from the variance-covariance matrix of profits, investment rates, spreads, and debt issuance. Profits and their correlations yield information about firm-level productivity and noise, helping to identify σ_z, ρ_z, and σ_π. Firm investment also reflects adjustment costs, helping to identify η_k. Debt issuance, together with credit spreads, aid in the identification of equity issuance costs η_f and η_d.

Three additional moments use data on forecast errors, whose use is the key innovation in our estimation, directly disciplining the degree of overreaction θ. One moment is the standard deviation of forecast errors. The other two moments are the correlation of the change in future forecast errors with current changes in the investment rate and debt issuance. Moments based on within-firm changes allow for systematic heterogeneity across firms, i.e., firm fixed effects, while still naturally linking to overreaction. Suppose that investment i_t, debt issuance b'_t and profits π_t are linearly increasing in expected log TFP, with positive slope coefficients respectively given by a_i, a_b, and a_π. Using Equation (3) the covariance between the future change in forecast errors and the current change in investment and debt is then given by:

$$cov\left[\left(\pi_{t+2} - E^{\theta}_{t+1}\pi_{t+2}\right) - \left(\pi_{t+1} - E^{\theta}_{t}\pi_{t+1}\right), x_t - x_{t-1}\right] = a_\pi a_x \rho\theta\left(1+\theta\right), \qquad x = i,\, b. \quad (18)$$

Equation (18) is positive if and only if expectations are diagnostic, $\theta > 0$. If beliefs overreact, good news leads to systematic disappointment next period, but no disappointment two periods from now, causing forecast errors to grow between $t+1$ and $t+2$.[19] The RE model is instead unable - because of its more general failure to generate forecast error predictability - to generate any comovement between future forecast error growth and investment or debt changes of the firm.

Table 5 reports the moments we target in the Data column. We obtain these moments using our combined datasets with firm financials, earnings forecasts, and credit spreads described in Section 2. Just as in Section 2 we focus on idiosyncratic variation, residualizing the underlying investment, debt, and spread series with respect to firm and time effects. As seen in Section 2, the micro data shows overreaction, as reflected in the positive forecast error growth covariances. To estimate the model parameters, we minimize the deviation of the empirical moments in Table 5 from those computed in a comparable unconditional simulation of the model. We weight the moments optimally using the inverse of our estimate

[19] Appendix B offers a detailed analysis of this property and further derivations of moment conditions in closed form in the linear case discussed above. Note that while our quantitative model is clearly nonlinear, and the formula in Equation (18) is therefore approximate, the intuition still holds in the more general model and proves useful for identification of the parameter θ. In particular, the covariance moments grow with the value of θ in our DE simulations.

Table 4: Parameter Estimates

	Parameter	Role	DE Model	RE Model
1	θ	Diagnosticity	0.913(0.202)	–
2	ρ_z	Micro TFP autocorrelation	0.670(0.059)	0.924(0.042)
3	σ_z	Micro TFP volatility	0.251(0.039)	0.096(0.022)
4	η_k	Capital adjustment cost	4.023(0.622)	3.490(0.433)
5	ϕ	Fixed operating cost	0.155(0.116)	0.075(0.116)
6	γ	Recovery rate	0.233(0.065)	0.177(0.097)
7	σ_A	Macro TFP volatility	0.006(0.003)	0.008(0.003)
8	η_f	Equity fixed cost	0.012(0.066)	0.076(0.229)
9	η_d	Equity linear cost	0.109(0.208)	0.100(0.416)
10	σ_π	Earnings noise	0.678(0.153)	0.390(0.082)

Notes: The table reports point estimates and standard errors for each of the parameters in our SMM estimation of both the DE model and the RE model. The moment covariance matrix is based on firm-level clustering in the micro block and a stationary block bootstrap in the macro block. The moment Jacobian is computed numerically. In the SMM estimation, the weighting matrix is optimal, i.e., the inverse of the moment covariance matrix.

of the moment covariance matrix, implying an asymptotically efficient SMM estimator. See Appendix B for a more detailed description of the variable definitions, sample construction, and our approach to computing the SMM point estimates and standard errors. Given our use of ten parameters to target sixteen moments, this is a highly overidentified structural estimation of a nonlinear model. We can exploit a great deal of information but are not in general able to deliver an exact fit.

Table 4 reports the SMM point estimates and standard errors for our DE model. The diagnosticity parameter $\theta \approx 0.9$ is in the ballpark of the values found by Bordalo et al. (2018) using data on professional forecasts of credit spreads ($\theta = 0.9$), by Bordalo et al. (2019) using analyst expectations of US listed firms' long-term earnings growth ($\theta = 0.9$), by Pflueger et al. (2020) using stock price-derived measures of risk perception ($\theta = 1$), and by Bordalo et al. (2020) using professional forecasts of several macro series ($\theta = 0.5$). Intuitively, a value of θ near 1 means that forecast errors are roughly equal to the size of incoming news.

The estimated values governing physical factors such as micro TFP volatility σ_z and capital adjustment costs η_k are close to those from other work estimating firm-level shock processes with similar data (Gourio and Rudanko, 2014; Terry, 2023; Khan and Thomas, 2008). The parameters governing financial frictions indicate equity issuance costs η_f, η_d and recovery rates γ comparable to those in Hennessy and Whited (2007). The fixed operating costs ϕ in model units is linked to average default rates and spreads. Finally, the estimated measurement error in profits σ_π suggests that a high degree of noise in accounting conventions is needed to match the covariance of profits and other firm-level outcomes.

Table 5: Estimated Model Fit

	Moment	Data	DE Model	RE Model
Panel A: Micro Moments				
1	Cov(Δ Forecast Error$_{t+1}$,Δ Investment$_{t-1}$)	0.003	0.005	0.000
2	Cov(Δ Forecast Error$_{t+1}$,Δ Debt$_{t-1}$)	0.004	0.008	0.001
3	Std. Dev(Forecast Error$_{t+1}$)	0.305	0.204	0.220
4	Std. Dev(Profit$_t$)	0.262	0.180	0.221
5	Corr(Profit$_t$, Investment$_t$)	0.257	0.398	0.097
6	Corr(Profit$_t$, Debt$_t$)	0.120	0.455	0.077
7	Corr(Profit$_t$, Spread$_t$)	-0.159	-0.028	0.071
8	Std. Dev(Investment$_t$)	0.067	0.065	0.041
9	Corr(Investment$_t$,Debt$_t$)	0.104	0.812	0.450
10	Corr(Investment$_t$,Spread$_t$)	-0.057	-0.079	0.114
11	Std. Dev(Debt$_t$)	0.112	0.107	0.092
12	Corr(Debt$_t$,Spread$_t$)	-0.036	-0.011	0.097
13	Std. Dev(Spread$_t$)	0.011	0.011	0.007
Panel B: Macro Moments				
14	$\mathbb{E}$(Spread$_t$)	0.029	0.021	0.018
15	$\mathbb{E}$(Default$_t$)	0.003	0.006	0.002
16	Std. Dev(Δ GDP$_t$)	0.015	0.013	0.016

Notes: The moments were computed on a sample combining information from the Compustat, IBES Manager Guidance, and FISD/Trace Bond databases from 2002-2017, with 4,697 firm-years spanning 493 firms. For the micro moments, the forecast error, profit, and investment, series are expressed relative to firm tangible capital stocks, while the debt series is scaled by total assets and the spread is in proportional units. For the macro moments, the mean spread is the average across years of the mean spread across firms in the FISD/Trace Bond-Compustat merged database, mean default is the average across years of the mean default rate across firms in the FISD/Trace-Compustat merged database. The GDP series in annual GDP in chained 2012 dollars.

Using the same micro and macro moments, we also conduct a constrained SMM estimation exercise to pin down the nine parameters of a RE model in which we set $\theta = 0$, eliminating diagnosticity in beliefs. We do not expect this model to succeed, for it abstracts from many dimensions such as the absence of financial intermediaries, time varying risk premia, etc. However, the RE model allows us to quantify the improvement obtained when adding only the overreaction parameter θ. The SMM point estimates and standard errors for the RE model are also reported in Table 4. Comparing parameter estimates, in the DE model we estimate a substantially lower persistence ρ_z of micro TFP shocks, and a lower volatility σ_A of aggregate TFP compared to the RE model. After good news, the DE manager becomes overoptimistic about the future, which boosts investment "as if" the current shocks were more persistent. DE is also a source of "shocks," which reduces the estimated σ_A: when optimism wanes, investment is cut "as if" a negative shock hit. In addition, DE also inflates perceived volatility. Because production function-based payoffs are convex, this implies that a firm's average incentive to default are also lower. To match the observed default frequency, the DE model estimates a higher fixed operating cost ϕ.

Table 5 reports our fit with simulated moment values in both the estimated DE and RE models. The DE model offers an excellent match of the micro moments, especially considering its strong nonlinearity and the overidentified SMM estimation. It accounts for the predictability of forecast errors and for the negative correlation of investment and profits with the spread. The RE version fails to do so. Under RE, managers and lenders do not become overoptimistic after good news, and their forecast errors are unpredictable. Furthermore, lenders do not become much more willing to lend after good news, causing the spread to rise in good times at odds with countercyclical spreads. The DE model also delivers a higher correlation between investment and profits than the RE model, again due to overreaction and more in line with the data. The main discrepancy of the DE model with the data is that the former produces a counterfactually high correlation between debt and investment. We speculate that an extended version of our model allowing for empirically realistic costs of debt financing, such as restrictive contractual covenants, would moderate debt movements and further improve the DE model's fit.

These differences between the DE and RE models prove critical to account for boom-bust cycles, both at the aggregate and firm levels. We next examine the qualitative mechanisms of the estimated DE model, with particular emphasis on nonlinearities in the macro investment response to a TFP shock.

5.2 Model Mechanisms

5.2.1 Nonlinearity in the Investment Response to TFP shocks

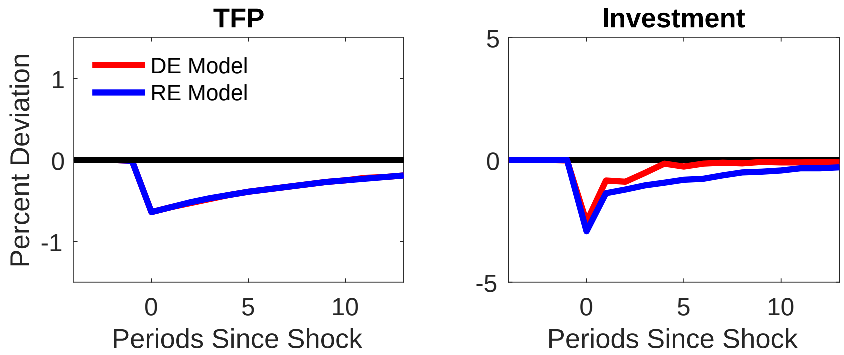
To study the real implications of DE, we simulate the contemporaneous response of macro investment to a negative macro TFP shock. We compute this response for different initial conditions, captured by TFP shocks of varying magnitudes in the previous period. This exercise highlights one important consequence of DE: fragility after good times. This nonlinearity is critical in our framework to account for boom-bust credit cycles.

Figure 1 plots the average impulse response of investment to a negative shock to macro TFP in the DE model (red line) and RE model (blue line).[20] These responses reflect "typical times," based on simulated reactions to negative TFP shocks across 10,000 experiments following the methodology of Koop et al. (1996). In normal times, the investment responses to TFP shocks differ only slightly across the DE and RE models. As usual in neoclassical models, a negative shock to TFP causes a larger decline in investment.

We next simulate the impact of the same negative shock to macro TFP varying the

[20]The shock is identically sized in the two experiments as a one standard deviation shock to macro TFP using the parameter estimate $\hat{\sigma}_A$ from the DE model.

Figure 1: Investment Response to a Negative TFP Shock



Notes: The figure plots the simulated generalized impulse responses of investment (right panel) to a one-standard deviation negative shock to macro TFP (left panel) occurring at period 0. The DE model (red) and RE model (blue) paths are based on identical shocks to TFP over 10,000 simulated experiments following the Koop et al. (1996) methodology.

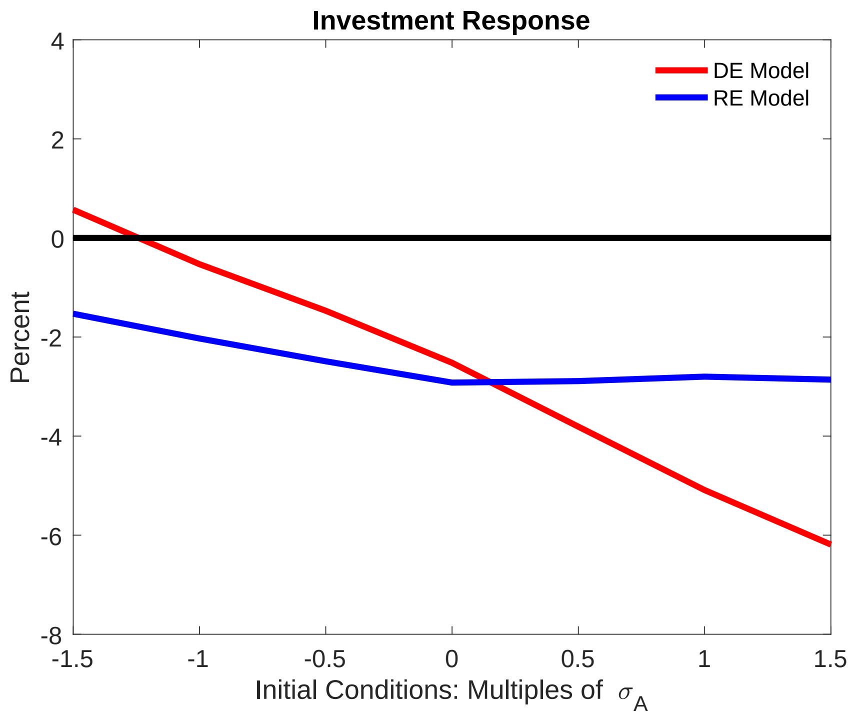
initial conditions. Figure 2 reports the investment response in the DE model (vertical axis, red line) as a function of the magnitude of the previous period's macro TFP shock (horizontal axis). There is a strong nonlinearity taking the form of "fragility in good times:" the same adverse TFP shock is much more damaging for aggregate investment when it occurs in good times. Figure 2 adds to the same diagram the response of investment in the RE model (blue line). The latter is almost the same across different initial conditions, with little nonlinearity or excess fragility in good times. Recent work on investment dynamics over the business cycle (Bachmann et al., 2013; Winberry, 2021; Bloom et al., 2018) suggests that investment exhibits more sensitivity to shocks during booms than during normal times. Figure 2 shows that overreacting beliefs may play a key role in producing this feature.

The investment response in Figure 2 combines two forces: the demand for capital reflecting managers' expectations of future profits, and the supply of capital reflecting lenders' expectations of default. In good times, diagnostic managers are overly optimistic, so they borrow and invest more than under RE. Diagnostic lenders are also overly optimistic, and hence are more willing to lend than under RE. Under DE, then, both the demand and the supply of credit overreact in good times, which causes a leverage and investment boom. When a negative shock hits, managers and lenders become too pessimistic, both the demand and the supply of credit contract, and firms drastically cut investment.[21] The effect is drastic because, starting from a position of overoptimism, firms are overindebted to begin with.[22]

[21] Interestingly, state-dependence in Figure 2 also generates a positive response during bad times under DE, when even a moderate negative shock to TFP can be considered "good news" relative to overpessimistic beliefs. Under RE, the response curve is uniformly negative.

[22] One can decompose the DE nonlinearity in Figure 2 into contributions from direct belief reversals versus endogenous increases in the riskiness of overoptimistic firms' states k and b. Appendix Figure A.4 reveals

Figure 2: Investment Nonlinearity



Notes: The vertical axis in the figure reports the simulated first-period impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. Both the DE model (red line) and RE model (blue line) are reported on the figure.

One question arising here is whether credit supply vs credit demand play a larger role in creating real fragility after good times under DE. We thus consider what happens if we relax lenders' irrationality. In particular, we impose that lenders are fully rational, they have $\theta = 0$, while θ is unconstrained for firms. Reestimating the model for this case, with parameter estimates in Appendix Table A.2, we simulate a set of "rational pricing" responses of investment to a negative TFP shock comparable to those in Figure 2. Qualitatively, the response of investment displays the nonlinearity typical of the DE model. Quantitatively, though, shutting down the overreaction of credit supply sharply reduces the magnitude of the effect as displayed in Appendix Figure A.2. Overreaction by lenders is an important force in our model, as we also show below.[23] We conclude that under DE shifts in the supply of credit play an important role in creating fragility during booms, especially due to overreaction to aggregate shocks, a pattern which is also evident in the next exercise.

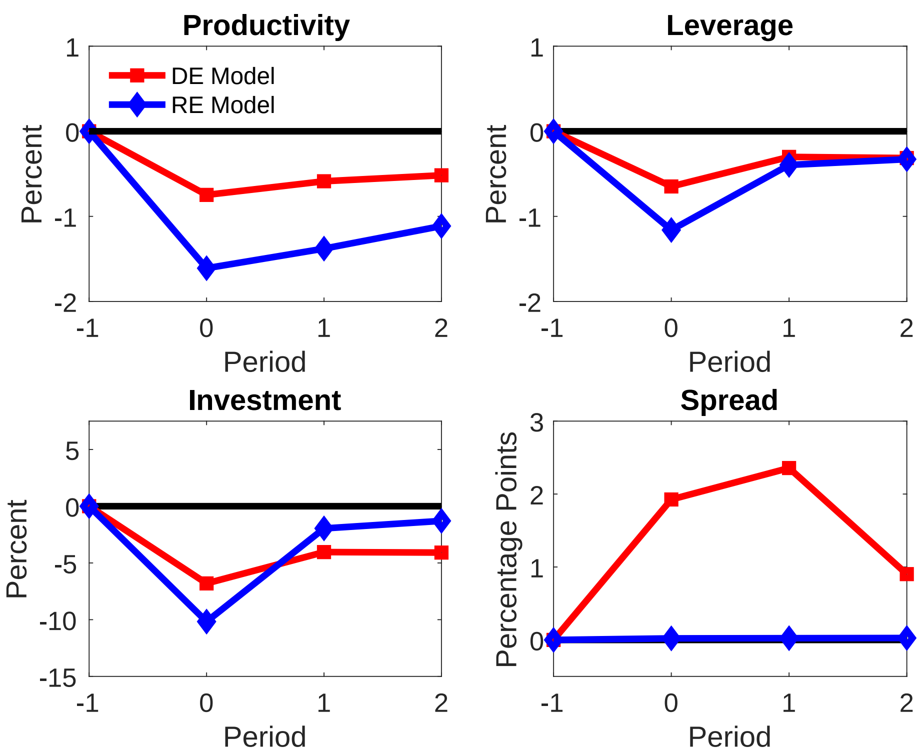
that both channels play a meaningful quantitative role.

[23] In Appendix Figure A.3 we reproduce investment responses for another special case, one with lender DE over aggregate shocks and RE over firm-level shocks. The resulting aggregate investment nonlinearity is similar to our baseline DE case.

5.2.2 Financial Reversals

A second related implication is that DE can produce strong macro-financial reversals without large negative TFP shocks. We simulate the model unconditionally for 250,000 periods and consider large financial reversals, defined as periods in which spread growth is in the top 10% of the distribution and investment growth is in the bottom 10%. We extract the average dynamics of TFP, leverage, investment and spreads around these episodes (labelled as period "0") for both the DE and the RE model and report them in Figure 3.

Figure 3: Financial Disruptions on Average



Notes: The figure plots the average path of macro TFP A (top left), macro leverage B'/K (top right), macro investment I (bottom left), and the macro spread $\mathbb{E}$ Spread (bottom right) around periods of financial disruption from an unconditional simulation of the DE model (red line) versus the RE model (blue line). The variables are plotted in percent (TFP, credit, and investment) or percentage point (spread) deviations from the pre-disruption period. A financial disruption, dated at period t on the horizontal axis, is a period with spread growth in the top 10% and investment growth in the bottom 10% of realizations in the simulated data for each model.

Consistent with existing evidence, in the DE model (red lines) reversals arise after only a mild decline in TFP (top left panel) relative to the RE case (blue lines). Despite a relatively small fundamental TFP disruption the DE model generates declines in leverage (top right panel) and investment (bottom left panel) similar to the RE model. And, unlike the RE model which generates hardly any movement in credit spreads during financial disruptions (bottom right panel), the DE model exhibits large increases. Two forces are at work. First, overreaction leads pessimistic firms under DE to cut back severely on investment and financing even in the face of only small changes in TFP. Second, under DE but not under RE, credit supply also shifts downwards in an overly pessimistic fashion, boosting spreads even in the face of a decline in aggregate leverage. This confirms the importance of overreacting lenders to account for shifts in the supply of capital and hence in credit spreads.

6 Quantitative Implications

We examine the model's ability to quantitatively match two sets of untargeted facts. The first set concerns our motivating evidence on: i) macro credit spread fluctuations, ii) the boom-bust dynamics of beliefs, returns, and investment documented in Section 2. The second set of facts concerns movements of credit spreads and other aggregates around the financial crisis of 2008.

6.1 Volatile Credit Spreads

Consider the ability of our model to reproduce macro credit spread fluctuations, which are entirely untargeted in our calibration procedure. Table 6 reports both the volatility of the mean credit spread (column 1) and its cyclicality or correlation with output (column 2) in the data, the DE model, and the RE model. The DE and RE model moments are computed based on an identical set of exogenous shocks, i.e., we feed the same sample paths for TFP into the two models. In the data, we see meaningful credit spread volatility with a standard deviation of about 1% annually. Unlike the RE model, which is unable to produce meaningful volatility of credit spreads with a standard deviation of around only 0.1%, an order of magnitude lower than the data value, the DE model generates substantial fluctuations in line with the data with a standard deviation of around 1%.[24] The DE model's meaningful credit spread fluctuations are countercyclical (column 2), also in the line with the data and natural in a model featuring strong procyclical shifts in both credit supply and demand due to overreacting beliefs. The RE model also produces spread countercyclicality, but with too little volatility to be quantitatively meaningful.

This result shows that DE is a promising way to introduce macrofinancial volatility (and in particular the countercyclicality of spreads) in standard business cycle analysis. Of course, DE cannot alone generate a perfect fit of entirely untargeted business cycle moments, which

[24] The failure of RE to generate meaningful aggregate volatility is even further exaggerated when comparing the estimated DE model to a counterfactual rational expectations model in which we set $\theta = 0$ but otherwise keep the parameters fixed at their DE estimates. In this case aggregate spreads feature a standard deviation of only 0.00005%.

Table 6: Macro Credit Spread Fluctuations

	(1) Standard Deviation	(2) Corr. with Output
Data	0.011	−0.237
DE Model	0.010	−0.313
RE Model	0.001	−0.623

Notes: The table reports the standard deviation (column 1) and the correlation with aggregate output (column 2) of the aggregate credit spread. The data sample is 2003-18 at annual frequency. Empirically, output is real GDP from the US NIPA accounts, and the credit spread is the mean across firms in a given year in the merged Compustat - IBES - FISD/TRACE sample. Empirically, output is HP-filtered in logs with smoothing parameter 100. The model moments are computed from an unconditional simulation of 50,000 years for the DE model (middle row) and the RE model (bottom row), with an identical set of aggregate TFP shock draws for both model versions. Model quantities refer to the total value (output) or average value (spread) computed from the distribution of firms. Units are in proportional terms, i.e., 0.01 = 1%.

also reflect factors (aggregate demand, price rigidities, etc.) from which we abstract here. It is however instructive to consider what a "DE block" would contribute to a standard business cycle model using the language of "wedges," in the spirit of Chari et al. (2007). Appendix Section A.7 analytically characterizes this issue in a simplified version of our model. We find that belief overreaction generates countercyclical fluctuations in the investment wedge.[25] Intuitively, overoptimism during good times causes a DE firm to act as if its investment price was counterfactually low from the perspective of a business cycle accountant imposing belief rationality, and the reverse during good times. The fact that DE map to an investment wedge rather than, say, efficiency or labor wedges, matters for two reasons. First, investment wedge fluctuations are often the target of macro-financial models (Kiyotaki and Moore, 1997; Justiniano et al., 2011; Brinca et al., 2016), and our analysis shows that DE can produce realistic fluctuations in macro-financial outcomes as well as in beliefs. Second, the analysis shows that our model is particularly useful for understanding a particular set of conditional facts, i.e., large, predictable macro-financial reversals after good times. Substantial room remains for other mechanisms, including those generating labor and efficiency wedge volatility. These can call for alternative mechanisms such as uncertainty shocks and nominal rigidities (Christiano et al., 2014), and an important issue is whether a combination of these shocks with DE can improve our understanding of unconditional business cycle variation, as some early analyses seem to show (Bianchi et al., 2023; L'Huillier et al., 2023).

[25] Unsurprisingly, in our model with frictionless static labor demand, DE does not cause fluctuations in labor or efficiency wedges in the analysis in Appendix Section A.7.

6.2 Boom-Bust Cycles in Financial and Real Activity

We next assess the explanatory power of the model for the firm-level boom-bust cycles documented in Section 2 but also address its ability to account for aggregate (i.e., sector-level) cycles. Consider firm-level cycles first. We simulate the DE and RE models for many periods and then use the simulated data to re-estimate the two-stage regressions of Table 2 in Section 2. In the first stage, we regress future profit forecast errors on current investment. In the second stage, we use these fitted errors as an explanatory variable for future bond returns on the financial side and investment growth on the real side. Table 7 reports the results. In line with the estimates of Table 2, in the DE model high investment predicts future negative forecast errors in column (1), indicating that expectations are overly optimistic in good times. Of course, the RE model (unreported) fails completely, by construction, to generate forecast error predictability of the type seen in column (1).

Table 7: Linking Forecast Errors and Micro Reversals in the Model

	(1)	(2)	(3)
	Fcst. Error$_{t+1}$	Return$_{t+1}$	ΔInvestment$_{t+1}$
IV Stage:	First	Second	Second
Forecast Error$_{t+1}$		0.012***	1.134***
		(0.001)	(0.004)
Investment$_t$	-0.842^{***}		
	(0.006)		
Year Effects	X	X	X
Firms	1000	1000	1000
Firm-Years	250000	250000	250000
First Stage F	17534		

Notes: The table reports first- and second-stage estimates based on simulated firm-level data from the DE model. Column (1) reports the first stage, and columns (2)-(3) report second-stage regressions. Standard errors are clustered at the firm level. $*$ = 10% level, $**$ = 5% level, and $***$ = 1% level. Forecast error is realized minus expected profits normalized by the firm's capital stock. Investment is the investment rate, i.e., capital expenditures normalized by the firm's capital stock. The bond return is the realized bond return. Column (3) controls for current profits. For all series, 0.01 = 1%.

In columns (2) and (3) the DE model also captures the positive correlation between predicted forecast errors, bond returns, and investment growth: when borrowers and lenders are overly optimistic (their profit forecasts are systematically above reality), realized bond returns and investment growth are lower. Quantitatively, comparing Table 7 from simulated data to the estimates from Table 2, the DE model reasonably reproduces the completely untargeted size of the first- and second-stage coefficients.[26] We conclude that firm-level

[26]While the first-stage coefficient in Table 7's column (1) is formally untargeted, our SMM estimation

reversals in real and financial conditions after good times are plausibly sized in the DE model while being completely absent in the RE model.

Table 8: Linking Forecast Errors and Aggregate Reversals

	(1)	(2)	(3)
	Forecast Error_{t+1}	Return_{t+1}	Δ Investment_{t+1}
IV Stage:	First	Second	Second
Panel A: Industry Data			
Forecast Error_{t+1}		0.047^{***}	1.221^{***}
		(0.014)	(0.050)
Investment_t	-0.476^{***}		
	(0.153)		
Industry Effects	X	X	X
Industries	111	111	111
Industry-Years	1291	1291	1291
First Stage F	10		
Panel B: DE Model			
Forecast Error_{t+1}		0.048^{***}	2.230^{***}
		(0.002)	(0.013)
Investment_t	-0.513^{***}		
	(0.004)		
Industry Effects	X	X	X
Years	50000	50000	50000
First Stage F	16575		

Notes: The table reports first- and second-stage estimates based on industry data from the Compustat-IBES-FISD/TRACE sample at the SIC3 $\times$ fiscal year level (Panel A) and simulated aggregate data from the estimated DE model (Panel B). Forecast error is average realized minus expected profits normalized by capital stocks. Investment is the average investment rate, i.e., capital expenditures normalized by the capital stock. Return is the average realized bond return, and spread is the average realized bond spread relative to the risk-free rate. Column (1) reports the first stage, and columns (2)-(3) report second-stage regressions. Standard errors are clustered at the industry level. For the purposes of rough comparison with industry data in Panel A, simulated aggregate data in Panel B is grouped into sequential blocks or industries with the same average length as in the data. So in Panel B "industry effects" refers to fixed effects for these groupings. $* = 10\%$ level, $** = 5\%$ level, and $*** = 1\%$ level. Columns (2)-(3) control for the lagged spread and current profits relative to the capital stock. For all series, $0.01 = 1\%$.

Firm-level reversals need not translate into aggregate reversals. First, firm managers may be particularly attentive to firm-level news, not aggregate news, and hence overreact only to the former. Second, and related, the effect of overoptimism may be amplified during extreme conditions such as a firm's financial distress, but the aggregate-level consequences may be muted due to the paucity of defaulting firms. Investigating these patterns at a more

procedure does target a set of closely linked firm-level covariances. In light of this, a comparison of the also untargeted second-stage coefficients in columns (2)-(3) to their empirical counterparts in Table 2 provides a tighter test.

aggregated level therefore offers an important test.

Studying belief dynamics at the macroeconomic level is difficult due to the relatively short post-2000 time window for our annual manager expectations dataset. As an alternative, we implement our two-stage empirical approach at the 3-digit sector level, which offers meaningful aggregation while also allowing us to exploit cross-sectoral variation. Panel A of Table 8 reports the results. In column (1), our first stage reveals that periods of high average investment in a sector predict lower average forecast errors in the subsequent year, i.e., good times beget excess optimism. In columns (2)-(3), we see that periods of excess optimism are followed by lower bond returns and lower investment growth in the same sector. An industry in which overoptimistic firms are *predictably* disappointed by one standard deviation more on average (0.32) sees average bond returns decline by -0.32 $\times$ 0.049 $\approx$ 1.5 percentage points (column 2) and average investment rate declines of -0.32 $\times$ 1.221 $\approx$ 39 percentage points (column 3).[27] There are significant boom-bust sectoral cycles in the data.

Even though our model lacks an explicit industry structure, we assess its ability to produce sector-level cycles by grouping simulated aggregate data from our model into nonoverlapping sequential blocks or "industries." We then run panel two-stage regressions, carefully maintaining symmetry between our simulated data and empirical work by: i) including industry effects and ii) only considering blocks or industries with average length at just above 10 years as in the data. Panel B reports the estimates based on simulated data from our estimated DE model, since the unreported RE model is unable, by construction, to produce forecast error predictability. We see that the DE model reproduces the aggregate link between good times (in which investment is high) and negative future forecast errors, reversals in realized bond returns, and reversals in investment growth. Quantitatively, the DE model decently matches the estimated coefficients in Panel A, which is remarkable given the entirely untargeted nature of these aggregate boom-bust patterns.

We conclude that the DE model offers a good account of systematic or aggregated sources of boom-bust cycles in real and financial activity at the firm and aggregate levels. Targeting beliefs data improves quantitative macro models' match to the data in a parsimonious way.

6.3 The Financial Crisis of 2007-09

Lastly, we study the ability of the DE model to generate the increase in credit spreads associated with the Lehman crisis in September 2008. Our model misses some important elements of large crises such as gradual and persistent asset price inflation and runs on financial intermediaries. Asset price bubbles can be obtained under DE by introducing

[27]The empirical results in Panel A of Table 8 are quite robust. The exact level of aggregation turns out to not be crucial for our results. Appendix Table B.7 reports similar results based on industry data aggregated to the 2-digit level. In columns (2)-(3) we control for lagged spreads and profits, to maintain comparability with our firm-level analysis, but this is not essential as shown in Appendix Table B.8. Including industry effects is also not essential, as shown in Appendix Table B.9.

information frictions as in Bordalo et al. (2021b), but here we abstract from them to isolate the role of overreaction in the sharpest way. For the same reason, we abstract from financial intermediaries considered in Maxted (2023) and Krishnamurthy and Li (2024).

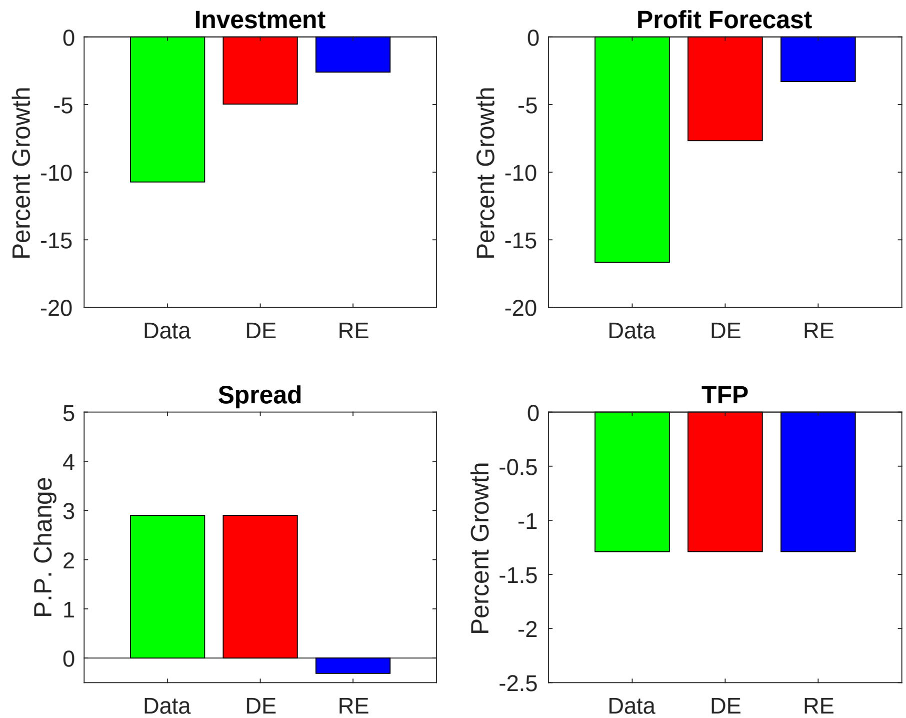
What type of TFP shock does the DE model need to produce the jump in spreads in the late 2000's? What are the macro consequences of such shocks? To answer these questions, we perform a "crisis decomposition" exercise. We separate the 2004-2009 period into a "pre-crisis" period, 2004 - 2007, and a "crisis" period, 2008 - 2009 (as mentioned before, our model yields neither a gradual path of asset price inflation nor the gradual spread reduction during 2004-2007). Across these periods, US credit spreads jumped from an average of 1.6% to 4.5%.

We use the simulated DE model to reverse engineer the TFP path needed to produce this jump. Figure 4 reports this TFP path along with its implications for the growth of investment, corporate profit forecasts, and credit spreads. We reports results for both the DE and the RE model. The red (blue) bars show the difference between crisis and pre-crisis outcomes produced by the DE (RE) model, while the green lines are data. In the bottom left panel, the DE model by construction perfectly matches the pre-crisis and crisis spreads. Remarkably, though, the bottom right panel shows that the DE model explains the credit spreads increase with a path for TFP growth that is very similar to measured TFP growth. This fact is noteworthy because the TFP dynamics are untargeted. Feeding in the same TFP path, the RE model does not produce any meaningful increase in the spread. Clearly, the limited ability of the RE model to provide overall spread volatility seen in Table 6 also prevents the model from generating realistic credit market fluctuations in response to this TFP shock. In the top row, we see that belief dynamics are key drivers of the different behavior of the two models. The decline in investment (top left) is larger in the DE model and moves together with a larger drop in profit forecasts (top right) than in the RE case. So DE beliefs appear critical to quantitatively account for sudden reversals in macro and financial activity occurring with modest fundamental shocks.

Such reversals are due to fragility built during good times. This fragility entails realistic macro consequences: a large disruption in credit markets and investment, together with a strong reduction in profit forecasts. These macroeconomic changes in the DE model are not identical to those in the data, but the quantitative fit is noteworthy considering the simplicity of our model. Even with the same TFP shock, the RE model produces substantially more modest dynamics. As evident from the impulse response analysis of Section 5.2, fragility during good times under DE offers a powerful amplifier of nonlinearities that can help quantitatively account for large crisis episodes even in our simple model.

This exercise shows that DE help account for the salient features of boom-bust cycles in macro-financial aggregates. The realism of the analysis and its fit can be improved by adding ingredients that have likely played a role, such as the housing bubble, intermediary leverage,

Figure 4: The Financial Crisis of 2007-09



Notes: Each panel plots a macro series from our crisis decomposition exercise, with the data (green), DE model (red), and RE model (blue) included. The panels plot changes from the pre-crisis to crisis periods. In the data, all empirical values are averages drawn from the pre-crisis (2004-07) or crisis (2008-09) periods. Spread is the average spread across firms in our Mergent FISD-TRACE-Compustat sample, private nonresidential fixed investment is from NIPA, the macro TFP level is from John Fernald's website, and profit forecasts are the sum of predicted earnings across all firms in our Compustat-IBES guidance data. In the DE model, we choose the TFP growth series in the bottom right panel in order to exactly match the empirical spread values in the bottom left panel. We feed the resulting TFP growth series into the RE model to produce the RE bar in each panel.

and the link between household debt and consumption. Boom-bust belief dynamics, however, seem a realistic and parsimonious ingredient to generate overexpansion in good times, and a sharp financial and real contraction after modest TFP shocks.

7 Robustness

We conclude by reporting the results of two robustness exercises. First, we explore the robustness of our investment nonlinearity result to a range of alternative parameterizations. Second, we endogenize wages in general equilibrium, which can dampen volatility, and again assess the contemporaneous response of macro investment to TFP shocks.

7.1 Robustness to Alternative Parameterization

In Appendix Figure A.8 we explore the robustness of our investment nonlinearity result to alternative parameterizations of the model. Starting from the DE parameter estimates in Table 4, a parameterization which we label Baseline, we vary each of the parameters to round higher and lower values, recomputing the sensitivity of investment to a negative shock over the full range of initial conditions. Nonlinearity remains meaningful in each of these robustness checks, with a downward-sloping response indicating more fragility during good times.

7.2 General Equilibrium

Anticipated procyclical price movements may dampen changes in the anticipated marginal product of capital and hence push against volatility or nonlinearity in investment. To assess the importance of this mechanism, we extend the model to general equilibrium. This exercise poses a Krusell and Smith (1998)-style challenge, and we develop a novel solution technique for overcoming this challenge in Appendix A. Although our solution method proves accurate and preserves the nonlinearity of the model, it is highly computationally intensive with a nested inner loop/outer loop alternation that makes structural estimation of the general equilibrium model infeasible.

For a representative household we model period utility as $U(C, N) = C - \frac{\omega}{1+\frac{1}{\lambda}} N^{1+\frac{1}{\lambda}}$, where C is consumption, the disutility of labor is governed by $\omega > 0$, and the elasticity of labor supply is given by $\lambda > 0$. See Appendix A for a full definition of general equilibrium, involving optimal labor supply and savings decisions by the household, optimal lending choices by a competitive group of lenders, optimal capital investment, labor demand, and risky borrowing decisions by a distribution of nonfinancial firms, and various market clearing and consistency conditions.

One aspect of the equilibrium is straightforward to characterize. The real interest rate is constant and pinned down by the inverse of the household's subjective discount factor $0 < \beta < 1$, i.e., $R = \frac{1}{\beta}$. However, as usual in this class of heterogeneous firms models with anticipated macro shocks, general equilibrium presents two computational challenges (Krusell and Smith, 1998). In particular, let $\mu(s, k, b)$ be the cross-sectional distribution of micro states $s = (z, \varepsilon_z)$, capital k, and debt b. The macro state is (μ, A, ε_A). The first challenge is that the macro state (μ, A, ε_A) is intractable because μ is a distribution. The second challenge is that the mapping $W(\mu, A, \varepsilon_A)$ is a complicated implicit object which must be consistent with the firm-level decisions embedded in market clearing through

$$\left(\frac{W(\mu, A, \varepsilon_A)}{\omega}\right)^{\lambda} = \int n(s, k, b, \mu, A, \varepsilon_A | W) d\mu(s, k, b), \tag{19}$$

where the left hand side is the household's closed-form labor supply and the right hand side reflects labor demand generated by the current cross-sectional distribution of firms μ.

We follow a novel computational approach tailored to our problem and detailed in Appendix A. Our approach is to replace the macro state $(\mu_t, A_t, \varepsilon_{At})$ with a truncated history of macro shocks $(A_t, A_{t-1}, ..., A_{t-K})$, nonparametrically storing predictions of the wage W_t given each shock history. We then follow an outer loop/inner loop approach, guessing a wage mapping, solving and simulating the model, and updating the wage predictions until convergence. Our solution technique proves tractable and accurate in practice. We parameterize the model based on the estimated values from Table 4. We further assume a conservative Frisch elasticity of labor supply of $\lambda = 0.5$ and choose β to deliver the same fixed 4% annual real interest rate as considered above.

Wages in this equilibrium structure reflect an intratemporal mapping from the macro states to prices, so it might not be immediately clear how explicitly intertemporal DE interact with general equilibrium. In Appendix Figure A.5, we plot expected future wage growth as a function of current productivity growth in the DE and RE models. In the DE model, a positive productivity shock increases perceptions of the future demand for capital and labor and hence future wage growth relative to the RE model. These anticipated wage increases dampen investment today, a force that is stronger under DE than RE. Our general equilibrium solution investigates whether the investment nonlinearities highlighted above under DE survive these price movements.

Appendix Figure A.7, the general equilibrium analog of Figure 2, reports the response of investment to a negative TFP shock for different initial conditions. Endogenous wages moderate the magnitude of the investment response relative to partial equilibrium. But the investment response continues to display substantial nonlinearity under DE (red line): the negative shock exerts a much larger negative impact in good times. In the RE model (blue line), by contrast, the general equilibrium feedback working through wages eliminates the already very slight state dependence of investment almost entirely. In sum, even after allowing for fully flexible wages in general equilibrium, the DE economy proves more fragile than the RE economy in good times and hence more responsive to negative shocks due to the boom-bust mechanism created by overreacting beliefs.

8 Conclusion

The financial crisis of 2008 renewed economists' interest in financial instability. One key challenge is to understand where such instability comes from. We showed that non-rational beliefs, and in particular overreaction to news, can generate realistic credit cycles without relying on financial shocks. These boom-bust dynamics exhibit predictability in line with the evidence on cyclical movements in credit spreads (López-Salido et al., 2017) and on large financial crises (Greenwood et al., 2022). Critically, our results are obtained in a neoclassical model in which a single new parameter, the degree of belief overreaction, is estimated using microdata on managers' errors in forecasts of the earnings growth of their firms. Realistic micro-level belief distortions can, once aggregated, generate realistic credit cycles with financial overexpansion in good times, fragility, and sharp reversals as small adverse news arrives.

Future work can enrich our approach. A key factor is the role of financial intermediaries, which recent work has already begun to investigate (Maxted, 2023; Krishnamurthy and Li, 2024). Another important aspect is household debt, which has been shown to be a key determinant of drops in aggregate demand during financial tightenings (Mian et al., 2017; Mian and Sufi, 2009). Prolonged asset price bubbles may also be important, especially to account for large crises Greenwood et al. (2022). These factors open exciting avenues for studying the transmission of beliefs to the real economy. As an initial step, we show that realistic departures from rationality disciplined by expectations data can be introduced into standard macroeconomic models and significantly improve their explanatory power.

References

Afrouzi, Hassan, Spencer Kwon, Augustin Landier, Yueran Ma, and David Thesmar (2023), "Overreaction in expectations: Evidence and theory." *Quarterly Journal of Economics*, 138, 1713–1764.

Angeletos, George-Marios, Zhen Huo, and Karthik Sastry (2020), "Imperfect macroeconomic expectations: Evidence and theory." *NBER Macroeconomics Annual*, 35, 1–86.

Arellano, Cristina, Yan Bai, and Patrick Kehoe (2019), "Financial frictions and fluctuations in volatility." *Journal of Political Economy*, 127, 2049–2103.

Azeredo da Silveira, Rava, Yeji Sung, and Michael Woodford (2020), "Optimally imprecise memory and biased forecasts." Working paper.

Bachmann, Rüdiger, Ricardo Caballero, and Eduardo Engel (2013), "Aggregate implications of lumpy investment: new evidence and a DSGE model." *American Economic Journal: Macroeconomics*, 5, 29–67.

Bansal, Ravi and Amir Yaron (2004), "Risks for the long run: A potential resolution of asset pricing puzzles." *Journal of Finance*, 59, 1481–1509.

Barrero, Jose (2022), "The micro and macro of manager beliefs." *Journal of Financial Economics*, 143, 640–667.

Barro, Robert (2006), "Rare disasters and asset markets in the twentieth century." *Quarterly Journal of Economics*, 121, 823–866.

Barsky, Robert B. and Eric R. Sims (2011), "News shocks and business cycle." *Journal of Monetary Economics*, 58.

Bernanke, Ben and Mark Gertler (1989), "Agency costs, net worth, and business fluctuations." *American Economic Review*, 79, 14–31.

Bertomeu, Jeremy, Iván Marinovic, Stephen Terry, and Felipe Varas (2021), "The dynamics of concealment." *Journal of Financial Economics*, 143, 227–246.

Bhandari, Anmol, Jaroslav Borovicka, and Paul Ho (Forthcoming), "Survey data and subjective beliefs in business cycle models." *Review of Economic Studies*.

Bianchi, Francesco, Cosmin Ilut, and Hikaru Saijo (2023), "Diagnostic business cycles." *Review of Economic Studies*, 91, 129–162.

Bianchi, Francesco, Cosmin Ilut, and Martin Schneider (2018), "Uncertainty shocks, asset supply and pricing over the business cycle." *Review of Economic Studies*, 85, 810–54.

Bianchi, Francesco and Leonardo Melosi (2016), "Modeling the evolution of expectations and uncertainty in general equilibrium." *International Economic Review*, 57, 717–756.

Bianchi, Javier (2011), "Overborrowing and systemic externalities in the business cycle." *American Economic Review*, 101, 3400–3426.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen Terry (2018), "Really uncertain business cycles." *Econometrica*, 86, 1031–1065.

Bordalo, Pedro, Katherine Coffman, Nicola Gennaioli, Frederik Schwerter, and Andrei Shleifer (2021a), "Memory and representativeness." *Psychological Review*, 128, 71–85.

Bordalo, Pedro, John Conlon, Nicola Gennaioli, Spencer Kwon, and Andrei Shleifer (2023a), "Memory and probability." *Quarterly Journal of Economics*, 138, 265–311.

Bordalo, Pedro, Nicola Gennaioli, Spencer Yongwook Kwon, and Andrei Shleifer (2021b), "Diagnostic bubbles." *Journal of Financial Economics*, 141, 1060–1077.

Bordalo, Pedro, Nicola Gennaioli, Rafael La Porta, and Andrei Shleifer (2024), "Belief Overreaction and Stock Market Puzzles." *Journal of Political Economy*. Forthcoming.

Bordalo, Pedro, Nicola Gennaioli, Rafael LaPorta, Matthew O'Brien, and Andrei Shleifer (2023b), "Long Term Expectations and Aggregate Fluctuations." *NBER Macroeconomics Annual*. Forthcoming.

Bordalo, Pedro, Nicola Gennaioli, Rafael LaPorta, and Andrei Shleifer (2019), "Diagnostic expectations and stock returns." *Journal of Finance*, 74, 2839–2874.

Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer (2020), "Overreaction in macroeconomic expectations." *American Economic Review*, 110, 2748–82.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer (2018), "Diagnostic expectations and credit cycles." *Journal of Finance*, 73, 199–227.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer (2022), "Overreaction and Diagnostic Expectations in Macroeconomics." *Journal of Economic Perspectives*, 36, 223–244.

Bouchaud, Jean-Philippe, Philipp Krueger, Augustin Landier, and David Thesmar (2019), "Sticky expectations and the profitability anomaly." *Journal of Finance*, 74, 639–674.

Brinca, Pedro, V.V. Chari, Patrick J. Kehoe, and Ellen R. McGrattan (2016), "Accounting for business cycles." *Handbook of Macroeconomics*, 1013–63.

Caballero, Ricardo and Alp Simsek (2020), "A risk-centric model of demand recessions and speculation." *Quarterly Journal of Economics*, 135, 1493–1566.

Cagan, Phillip (1956), "The Monetary Dynamics of Hyperinflation." In *Studies in the Quantity Theory of Money* (Milton Friedman, ed.), University of Chicago Press, Chicago.

Campbell, John and John Cochrane (1999), "By force of habit: A consumption-based explanation of aggregate stock market behavior." *Journal of Political Economy*, 107, 205–251.

CBO (2017), "International comparisons of corporate income tax rates." Exhibit 7.

Chari, V.V., Patrick J Kehoe, and Ellen R McGrattan (2007), "Business cycle accounting." *Econometrica*, 75, 781–836.

Christiano, Lawrence J, Roberto Motto, and Massimo Rostagno (2014), "Risk shocks." *American Economic Review*, 104, 27–65.

Coibion, Olivier and Yuriy Gorodnichenko (2015), "Information rigidity and the expectations formation process: A simply framework and new facts." *American Economic Review*, 105, 2644–78.

Cooper, Russell and Jonathan Willis (2015), "Discounting: Investment sensitivity and aggregate implications." Working paper.

Dávila, Eduardo and Anton Korinek (2017), "Pecuniary externalities in economies with financial frictions." *Review of Economic Studies*, 85, 352–395.

Den Haan, Wouter (2010), "Comparison of solutions to the incomplete markets model with aggregate uncertainty." *Journal of Economic Dynamics and Control*, 34, 4–27.

Diamond, Douglas and Philip Dybvig (1983), "Bank runs, deposit insurance, and liquidity." *Journal of Political Economy*, 91, 401–419.

Eggertsson, Gauti and Paul Krugman (2012), "Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach." *Quarterly Journal of Economics*, 127, 1469–1513.

Falato, Antonio and Jasmine Xiao (2020), "The information driven financial accelerator." Working paper.

Falato, Antonio and Jasmine Xiao (2024), "Expectations and credit slumps." Working paper.

Farhi, Emmanuel and Ivan Werning (2020), "Taming a Minsky cycle." Working paper.

Fernald, John (2014), "A quarterly, utilization-adjusted series on total factor productivity." Working paper.

Fuster, Andreas, David Laibson, and Brock Mendel (2010), "Natural expectations and macroeconomic fluctuations." *Journal of Economic Perspectives*, 24, 67–84.

Gabaix, Xavier (2019), "Behavioral inattention." In *Handbook of Behavioral Economics: Applications and Foundations* (B. Douglas Bernheim, Stefano DellaVigna, and David Laibson, eds.), volume 2, chapter 4, Elsevier.

Gennaioli, Nicola, Yueran Ma, and Andrei Shleifer (2016), "Expectations and investment." *NBER Macroeconomics Annual*, 30, 379–431.

Gennaioli, Nicola and Andrei Shleifer (2010), "What comes to mind." *Quarterly Journal of Economics*, 125, 1399–1433.

Giglio, Stefano, Matteo Maggiori, Johannes Stroebel, and Stephen Utkus (2021), "Five facts about beliefs and portfolios." *American Economic Review*, 111, 1481–1522.

Gilchrist, Simon and Egon Zakrajšek (2012), "Credit spreads and business cycle fluctuations." *American Economic Review*, 102, 1692–1720.

Gilchrist, Simon, Egon Zakrajšek, and Jae Sim (2014), "Uncertainty, financial frictions, and investment dynamics." Working paper.

Gomes, Joao F. (2001), "Financing investment." *American Economic Review*, 91, 1263–85.

Gourio, Francois and Leena Rudanko (2014), "Customer capital." *Review of Economic Studies*, 81, 1102–36.

Greenwood, Robin and Samuel Hanson (2013), "Issuer quality and corporate bond returns." *Review of Financial Studies*, 26, 1483–1525.

Greenwood, Robin, Samuel Hanson, and Lawrence Jin (2023), "Reflexivity in credit markets." Working paper.

Greenwood, Robin, Samuel Hanson, Andrei Shleifer, and Jakob Ahm Sørensen (2022), "Predictable financial crises." *Journal of Finance*, 77, 863–921.

Greenwood, Robin and Andrei Shleifer (2014), "Expectations of returns and expected returns." *Review of Financial Studies*, 27, 714–746.

Guerrieri, Veronica and Guido Lorenzoni (2017), "Credit crises, precautionary savings, and the liquidity trap." *Quarterly Journal of Economics*, 132, 1427–1467.

Hansen, Lars Peter and Thomas Sargent (2001), "Acknowledging misspecification in macroeconomic theory." *Review of Economic Dynamics*, 4, 519–535.

He, Zhiguo and Arvind Krishnamurthy (2019), "A macroeconomic framework for quantifying systemic risk." *American Economic Journal: Macroeconomics*, 11, 1–37.

Hennessy, Christopher and Toni Whited (2007), "How costly is external financing? evidence from a structural estimation." *Journal of Finance*, 62, 1705–1745.

Ilut, Cosmin, Matthias Kehrig, and Martin Schneider (2018), "Slow to hire, quick to fire: Employment dynamics with asymmetric responses to news." *Journal of Political Economy*, 126, 2011–71.

Ilut, Cosmin L and Martin Schneider (2014), "Ambiguous business cycles." *American Economic Review*, 104, 2368–99.

Jaimovich, Nir and Sergio Rebelo (2007), "Behavioral theories of the business cycle." *Journal of the European Economic Association*, 5, 361–68.

Jermann, Urban and Vincenzo Quadrini (2012), "Macroeconomic effects of financial shocks." *American Economic Review*, 102, 238–71.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti (2011), "Investment shocks and the relative price of investment." *Review of Economic Dynamics*, 24, 102–121.

Kahana, Michael Jacob (2012), *Foundations of Human Memory*. Oxford University Press.

Khan, Aubhik and Julia Thomas (2008), "Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics." *Econometrica*, 76, 395–436.

Kindleberger, Charles (1978), *Manias, Panics, and Crashes: A History of Financial Crises*. John Wiley & Sons.

King, Robert and Sergio Rebelo (1999), "Resuscitating real business cycles." *Handbook of Macroeconomics*, 1, 927–1007.

Kiyotaki, Nobuhiro and John Moore (1997), "Credit cycles." *Journal of Political Economy*, 105, 211–248.

Koby, Yann and Christian Wolf (2020), "Aggregation in heterogeneous-firm models: Theory and measurement." Working paper.

Kohlhas, Alexandre and Ansgar Walther (2021), "Asymmetric attention." *American Economic Review*, 111, 2879–2925.

Koop, Gary, Hashem Pesaran, and Simon Potter (1996), "Impulse response analysis in nonlinear multivariate models." *Journal of Econometrics*, 74, 119–147.

Korinek, Anton and Alp Simsek (2016), "Liquidity trap and excessive leverage." *American Economic Review*, 106, 699–738.

Kozlowski, Julian, Laura Veldkamp, and Venky Venkateswaran (2020), "The tail that wags the economy: Beliefs and persistent stagnation." *Journal of Political Economy*, 128, 2839–2879.

Krishnamurthy, Arvind and Wenhao Li (2024), "Dissecting mechanisms of financial crises: Intermediation and sentiment." Working paper.

Krusell, Per and Anthony Smith, Jr (1998), "Income and wealth heterogeneity in the macroeconomy." *Journal of Political Economy*, 106, 867–896.

L'Huillier, Jean-Paul, Sanjay Singh, and Donghoon Yoo (2023), "Incorporating diagnostic expectations into the new keynesian framework." *Review of Economic Studies*.

López-Salido, David, Jeremy Stein, and Egon Zakrajšek (2017), "Credit-market sentiment and the business cycle." *Quarterly Journal of Economics*, 132, 1373–1426.

Lorenzoni, Guido (2008), "Inefficient credit booms." *Review of Economic Studies*, 75, 809–833.

Lucas, Robert (1976), "Econometric policy evaluation: A critique." *Carnegie-Rochester Conference Series on Public Policy*, 1, 19–46.

Ma, Yueran, Tiziano Ropele, David Sraer, and David Thesmar (2024), "A quantitative analysis of distortions in managerial forecasts." Working paper.

Mankiw, Gregory and Ricardo Reis (2002), "Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve." *Quarterly Journal of Economics*, 117, 1295–1328.

Maxted, Peter (2023), "A macro-finance model with sentiment." *Review of Economic Studies*, 91, 438–475.

Mian, Atif and Amir Sufi (2009), "The consequences of mortgage credit expansion: evidence from the U.S. mortgage default crisis." *Quarterly Journal of Economics*, 124, 1449–96.

Mian, Atif, Amir Sufi, and Emil Verner (2017), "Household debt and business cycles worldwide." *Quarterly Journal of Economics*, 132, 1755–1817.

Minsky, Hyman (1977), "The financial instability hypothesis: An interpretation of Keynes and an alternative to standard theory." *Challenge*, 20, 20–27.

Nickell, Stephen (1981), "Biases in dynamic models with fixed effects." *Econometrica*, 49, 1417–1426.

Pflueger, Carolin, Emil Siriwardane, and Adi Sunderam (2020), "Financial market risk perceptions and the macroeconomy." *Quarterly Journal of Economics*, 135, 1443–1491.

Rognlie, Matthew, Andrei Shleifer, and Alp Simsek (2018), "Investment hangover and the great recession." *American Economic Journal: Macroeconomics*, 10, 113–53.

Schaal, Edouard and Mathieu Taschereau-Dumouchel (2023), "Herding through booms and busts." *Journal of Economic Theory*, 210.

Schularick, Moritz and Alan Taylor (2012), "Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008." *American Economic Review*, 102, 1029–61.

Shleifer, Andrei and Robert Vishny (1992), "Liquidation values and debt capacity: A market equilibrium approach." *Journal of Finance*, 47, 1343–1366.

Sims, Christopher (2003), "Implications of rational inattention." *Journal of Monetary Economics*, 50, 665–690.

Stein, Jeremy (2012), "Monetary policy as financial stability regulation." *Quarterly Journal of Economics*, 127, 57–95.

Strebulaev, Ilya and Toni Whited (2012), "Dynamic models and structural estimation in corporate finance." *Foundations and Trends in Finance*, 6, 1–163.

Tauchen, George (1986), "Finite state Markov-chain approximations to univariate and vector autoregressions." *Economics Letters*, 20, 177–181.

Terry, Stephen (2017), "Alternative methods for solving heterogeneous firm models." *Journal of Money, Credit and Banking*, 49, 1081–1111.

Terry, Stephen (2023), "The macro impact of short-termism." *Econometrica*, 91, 1881–1912.

Winberry, Thomas (2021), "Lumpy investment, business cycles, and stimulus policy." *American Economic Review*, 111, 364–396.

Woodford, Michael (2003), "Imperfect common knowledge and the effects of monetary policy." In *Knowledge, Information, and Expectations in Modern Macroeconomics* (Philippe Aghion, Roman Frydman, Joseph Stiglitz, and Michael Woodford, eds.), Princeton University Press.

Young, Eric (2010), "Solving the incomplete markets model with aggregate uncertainty using the krusell–smith algorithm and non-stochastic simulations." *Journal of Economic Dynamics and Control*, 34, 36–41.

Appendices for Online Publication Only

A Model

A.1 Solving the Model

The computational algorithm involves iteration on an outer loop (related to debt pricing) and an inner loop (related to firm policies). Before solving the model, we discretize the state space $(s, k, b) = (z, \varepsilon_z, A, \varepsilon_A, k, b)$ into $n_z \times n_z \times n_A \times n_A \times n_k \times n_b$ grid points. We then discretize the rational and perceived diagnostic transitions of the exogenous states according to Tauchen (1986). The computational algorithm - following Strebulaev and Whited (2012) - proceeds as follows:

Start outer loop.

1. Guess a default policy $df^\theta(s, k, b)$, and compute the implied debt prices $q^\theta(s, k, b)$ according to the lenders diagnostic zero-profit condition Equation (17).

   Start inner loop.

   (a) Given the debt prices $q^\theta(s, k, b)$ and default policy $df^\theta(s, k, b)$, solve the diagnostic firm's Bellman Equations (14), (15), and (16) for $V^\theta(s, k, b)$, $V^\theta_{ND}(s, k, b)$, and $V^\theta_D(s)$ as well as the implied optimal policies for investment and debt issuance $k'^\theta(s, k, b)$, $b'^\theta(s, k, b)$. Use standard discrete-state, discrete-policy dynamic programming policy iteration to do so.

2. Compute updated default policies $df^\theta(s, k, b)$ according to the default choice defining V^θ in Equation (14), i.e., $V^\theta_{ND}(s, k, b) < V^\theta_D(s)$.

3. Compute the ergodic distribution $\mu(s, k, b)$ implied by the firm policies for default, capital, and debt $df^\theta(s, k, b)$, $k'^\theta(s, k, b)$, and $b'^\theta(s, k, b)$.

4. Compute the mass of states in which the guessed default policy differs from the updated default policy. If this set of states has mass lower than some tolerance, exit. If not, then go to top and restart with the updated set of default states as your new guess.

We implement this computationally intensive algorithm in heavily parallelized Fortran on a 2017 iMac Pro, with runtimes around 250 seconds. Table A.1 reports the value of several dimensions used for the baseline solution of the model. Figure A.1 plots the perceived value function V^θ in our estimated DE model for a range of different capital and productivity news realizations, with overreaction generating more perceived value after goods news even conditional upon today's productivity state. The value function in the RE model, also plotted, displays no such overreaction in the direction of recent news.

Table A.1: Computational Choices

Quantity	Description	Value
T^{sim}	Simulated periods	6
T^{erg}	Initially discarded periods	50
N^{firm}	Number of firms	1000
N^{IRF}	Number of IRF economies	10000
T^{IRF}	Length of IRF economies	75
T^{decomp}	Length of historical decomposition	2
n_z	Micro productivity grid size	5
n_A	Macro productivity grid size	5
n_k	Capital grid size	30
n_b	Debt grid size	30

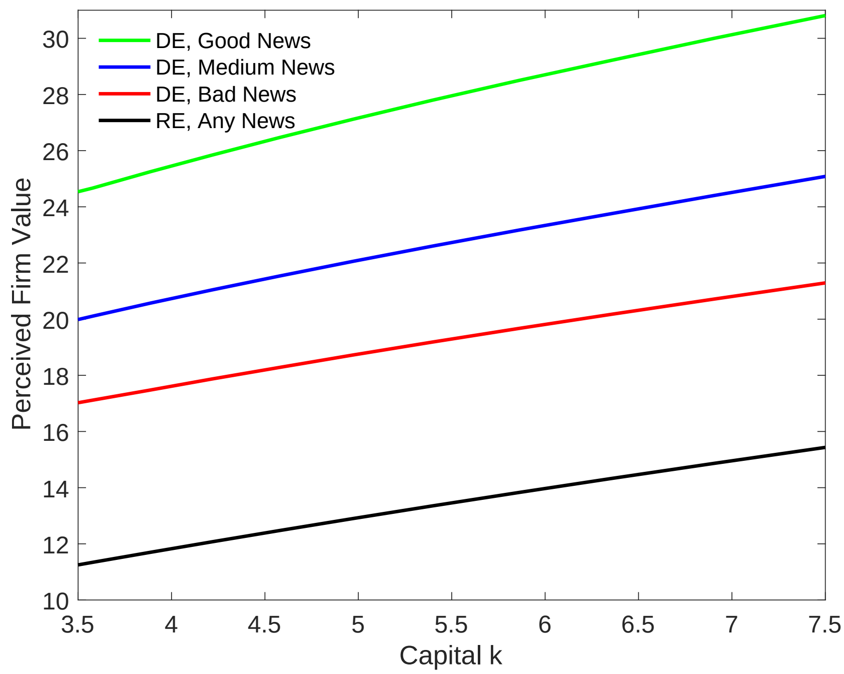
Notes: The table reports various computational values used in discretizing and solving the model.

A.2 Simulating the Model

After the model is solved, we unconditionally simulate the model by drawing exogenous uniform random shocks and combining this information with the transition matrix for macro TFP to simulate the macro process for A_t for some periods $t = 1, ..., T^{sim} + T^{erg}$. At the micro level, we simulate the model "non-stochastically" according to the method of Young (2010), i.e., we store the dynamics of the weight of the cross-sectional distribution at each discretized point in the state space (s, k, b) rather than simulating a large number of firms. Note that when simulating the model, all macro shocks and distributional dynamics are determined according to the *rational* or true representations of the driving process, even though debt pricing and firm polices may involve diagnostic expectations.

With the simulated distribution in hand for each period, macro series of interest are simply weighted sums of micro-level outcomes across this distribution, discarding the first T^{erg} periods to remove the influence of initial conditions. Note that we do in fact simulate a number of individual firms N^{firm} for the purpose of computing moments within our SMM estimation algorithm, but this is not a step required for the purpose of solving the model or simulating within-period business cycle aggregates.

Figure A.1: Firm Value and Diagnosticity



Notes: The figure plots the perceived value function V^θ as a function of firm capital k for the estimated DE model, together with the firm's value function V in the estimated RE model. All lines hold fixed micro TFP z, macro TFP A, macro news ε_A, and the firm's debt b at identical representative values. The four lines reflect different realizations of micro TFP news ε_z, with positive news in the DE model (green line), medium news in the DE model (blue line), bad news in the DE model (red line), and any news in the RE model (black line).

A.3 Computing Impulse Responses

Our approach to impulse response calculation in this nonlinear context follows Koop et al. (1996), i.e., we compute nonlinear generalized impulse responses. To understand the impact of a given sequence of shocks, we perform the following:

1. For a large number N^{irf} of economies of length T^{irf}, simulate two different versions of the simulation, the "shock" and "no shock" versions. For each economy and each version, we simulate the macro TFP process by first drawing T^{irf} uniform shocks for comparison with the macro TFP transition matrix. Then, simulate both versions unconditionally using identical macro TFP shocks until period $T^{shock} < T^{irf}$.

2. From period T^{shock} and continuing as long as the desired sequence of exogenous innovations you wish to impose lasts, impose a number of periods of certain pre-determined innovations in productivity for the "shock" case, while continuing to simulate the "no shock" economy unconditionally.

3. After the imposed shocks sequence is complete, simulate macro TFP in both economies as normal.

4. After the macro TFP process is determined for each pair of economies, compute the business cycle aggregates of interest in each economy, period, and version by using the simulation approach outlined above.

5. If business cycle aggregate $X_{i,t}^{shock}$ is series X in economy i in period t in the shock case, and $X_{i,t}^{noshock}$ is series X in economy i in period t in the no shock case, then define the impulse response to the predetermined sequence of innovations as

$$IRF_t^X = \frac{1}{N^{irf}} \sum_{i=1}^{N^{irf}} \frac{X_{i,t}^{shock} - X_{i,t}^{noshock}}{X_{i,t}^{noshock}}.$$

The main text's set of impulse response figures reports the series IRF^X for the indicated macro-financial aggregates. Note, however, that the impulse responses presented in the text are scaled to equal an exact shock size, while the productivity grid in the model varies discretely. We achieve this by imposing movements up or down by a single grid point, imposing Step 2 above only with a certain probability chosen in each period to deliver the desired average shock size.

A.4 Performing the Spread Matching Exercise

In a classic linear setting, performing historical decompositions such as the one used in Section 6 for the Great Financial Crisis is typically a trivial matter of inverting a data path using simple linear algebra. However, our nonlinear model with heterogeneity and a discretized productivity process poses some additional computational challenges. Given the empirical path across two period $t = 1, 2$ for macro credit spreads to match $(S_1, S_2, ..., S_T^{decomp})$, we proceed as follows.

First, we pick an initial period drawn from a representative location in the unconditional simulation of the model, fixing the associated simulated cross-sectional distribution of firm-level states μ_0 drawn from the simulation of the model. Call this period $t = 0$, and note that at the end of period 0 a cross-sectional distribution μ_1 is pre-determined. Then for each period $t = 1, ..., T^{decomp}$, do the following:

1. Guess a value for macro TFP A_t, and find the bracketing interval $[A_{i-1}, A_i]$ together with linear interpolation weights $\omega(A_t, i) = \frac{A_t - A_{i-1}}{A_i - A_{i-1}}$ for the guessed value of productivity.

2. Compute the implied policies of all firms in the cross-sectional distribution μ_t given a macro TFP level equal to A_i, together with the implied macro spread level $S(A_i)$. Repeat the process for macro TFP equal to A_{i-1} to obtain $S(A_{i-1})$.

3. Assume that firms play a "mixed strategy" over the two macro TFP grid points, in which case the resulting macro spread level is $(1 - \omega(A_t, i))S(A_{i-1}) + \omega(A_t, i)S(A_i)$.

4. If the implied macro spread level is not equal to the desired spread value S_t to within some tolerance, then update your guess for macro TFP A_t and return to Step 1. Otherwise proceed.

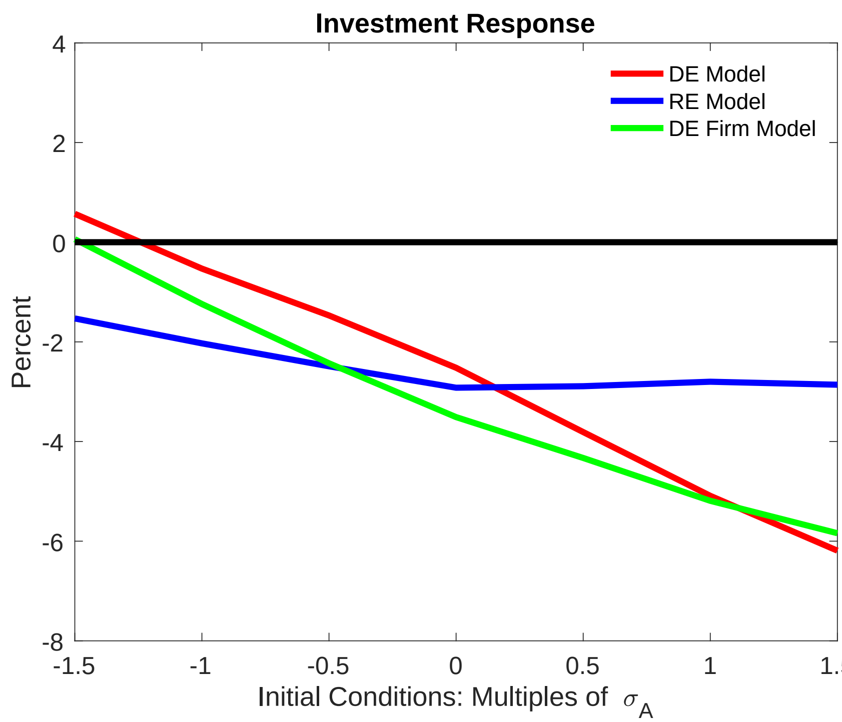
5. Given a productivity guess which delivers exactly the correct interpolated value of macro productivity in period t, compute the beginning-of-period distribution μ_{t+1} of firm-level states by pushing forward a fraction $\omega(A_t, i)$ of the distribution μ_t using firm policies associated with A_i and a fraction $1 - \omega(A_t, i)$ of the distribution μ_t using firm policies associated with A_{i-1}.

At the end of this process, you have determined a smooth value of productivity A_t which gives you an implied macro spread series exactly consistent with the target value in period t, and you have updated the cross-sectional distribution in an internally consistent fashion given the smooth value of productivity between grid points. Repeating this process for each period $t = 1, ..., T^{decomp}$ yields a productivity path A_t, as well as a set of cross-sectional distributions μ_t, which exactly match the target data path for spread. All other macro aggregates of interest can then be computed from the distributional and macro TFP path. Note that for the spread matching exercise for the Great Recession and financial crisis in Section 6, we set $T^{decomp} = 2$, with $t = 1$ being the "Pre-Crisis" period and $t = 2$ being the "Crisis" period.

A.5 DE Firms with RE Lenders

Table A.2 reports parameter estimates for a model in which firms exhibit DE with $\theta > 0$ but lenders exhibit RE with $\theta = 0$. Figure A.2 plots the equivalent of Figure 2, plotting investment responses to a negative TFP shock for a range of initial conditions, in this model.

Figure A.2: Investment Nonlinearity with Rational Pricing



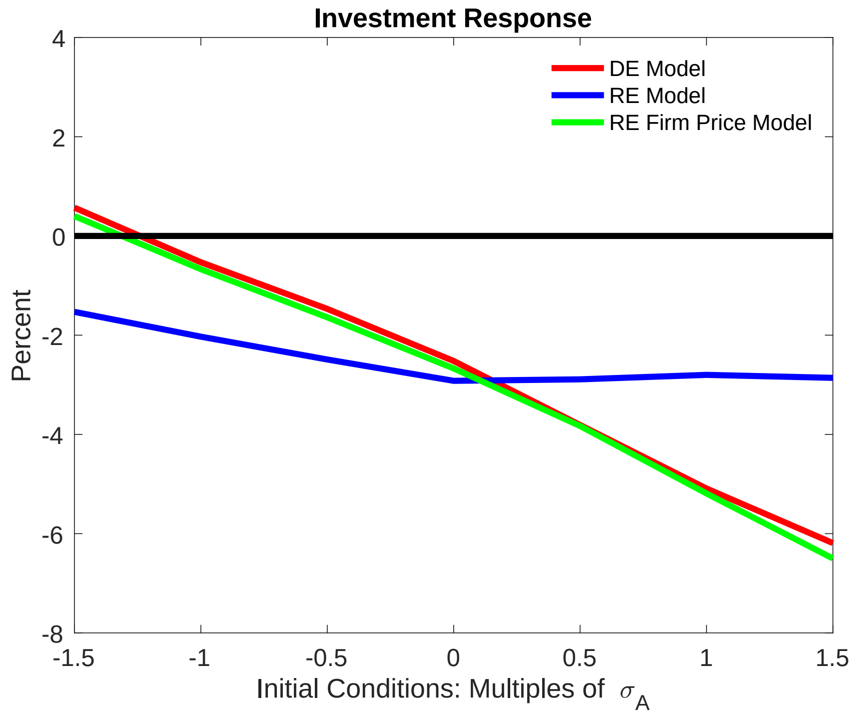
Notes: The vertical axis in the figure reports the simulated first-period impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. The DE model (red line), RE model (blue line), and DE model with diagnostic firms and rational lenders (green line) are reported on the figure.

Figure A.3 also plots investment responses to a negative aggregate TFP shock in a model in which lenders exhibit DE with respect to aggregate shocks but RE with respect to firm-level shocks.

A.6 Belief Updating versus Endogenous Sensitivity

The IRF nonlinearity revealed in the main text in Figure 2 can in principle stem from one of two not mutually exclusive sources: i) direct reversals of overoptimistic diagnostic beliefs after good news, or ii) higher riskiness in the distribution of overoptimistic firms' endogenous states b and k after a good news. To piece apart the direct contribution of beliefs versus

Figure A.3: Investment Nonlinearity with Rational Pricing at Firm Level



Notes: The vertical axis in the figure reports the simulated first-period impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. The DE model (red line), RE model (blue line), and model with DE firms, lender RE on firm-level shocks, and lender DE on aggregate shocks (green line) are reported on the figure.

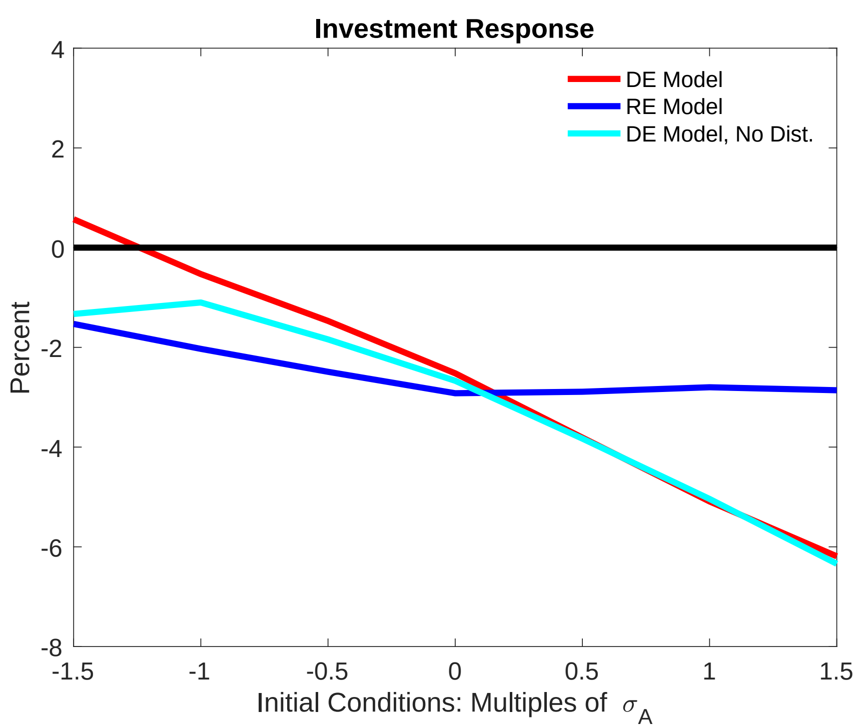
the indirect contribution of firm states, we compute the response of the diagnostic economy to a one-standard deviation negative shock to TFP after a range of different initial shocks, in each case holding the distribution of endogenous states equal to the one which would have obtained without the negative TFP shock. The light blue line in Figure A.4 reveals the resulting investment responses as a function of the initial conditions. Without the endogenously higher riskiness in capital and debt positions generated by firms, the light blue line continues to slope down but is not quite as strongly sloped as the red line. Therefore, both the direct belief reversal and endogenous distributional shifts due to diagnosticity are critical for generating the overall nonlinearity in Figure 2.

Table A.2: Parameter Estimates for RE Lenders, DE Firms Model

	Parameter	Role	Value
1	θ^{firms}	Diagnosticity	0.913
2	ρ_z	Micro TFP autocorrelation	0.653
3	σ_z	Micro TFP volatility	0.207
4	η_k	Capital adjustment cost	4.152
5	ϕ	Fixed operating cost	0.195
6	γ	Recovery rate	0.179
7	σ_A	Macro TFP volatility	0.006
8	η_f	Equity fixed cost	0.017
9	η_d	Equity linear cost	0.104
10	σ_π	Earnings noise	0.633

Notes: The table reports point estimates in our SMM estimation of the model with RE credit pricing by lenders and DE policies by firms. In the estimation, the weighting matrix for moments is optimal, i.e., the inverse of the moment covariance matrix.

Figure A.4: Investment Nonlinearity with No Distributional Dynamics



Notes: The vertical axis in the figure reports the simulated first-period impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. The DE model (red line), RE model (blue line), and DE model with no shift in the distribution of endogenous states (light blue line) are reported on the figure.

A.7 Investment Wedges

Consider a benchmark representative firm version of our PE model without investment frictions, i.e., without adjustment costs or financial frictions. In this case, the representative firm solves

$$\max_{K'} ... - K' + \frac{1}{R}\mathbb{E}^{firm}\left[\Pi(A', K') + (1-\delta)K',\right] + ...$$

where the profit function is defined as

$$\Pi(A, K) = \max_{N} AK^{\alpha}N^{\nu} - WN$$

and the expectations operator $\mathbb{E}^{firm}$ obeys whatever belief system the firm uses (RE, DE, etc...). Static labor optimization implies that

$$\Pi(A, K) = (1-\nu)\left(\frac{\nu}{W}\right)^{\frac{\nu}{1-\nu}} A^{\frac{1}{1-\nu}} K^{\frac{\alpha}{1-\nu}},$$

so the capital FOC for the frictionless representative economy is given by

$$R - 1 + \delta = \mathbb{E}^{firm}\frac{\partial}{\partial K'}\Pi(A', K'), \tag{20}$$

where again the expectations operator can be either DE or RE depending on the firm's belief system. FOC (20) together with the profit function expression lead to the capital choice

$$K' = \left(\frac{\alpha}{R-1+\delta}\right)^{\frac{1-\nu}{1-(\alpha+\nu)}} \left(\frac{\nu}{W}\right)^{\frac{\nu}{1-(\alpha+\nu)}} \left(\mathbb{E}^{firm}\left[A'^{\frac{1}{1-\nu}}\right]\right)^{\frac{1-\nu}{1-(\alpha+\nu)}}. \tag{21}$$

Ok, so now consider an investment wedge representation of a similar economy. In this case, the firm is assumed to solve the problem

$$\max_{K'} ... - \tau_I K' + \frac{1}{R}\mathbb{E}\left[\Pi(A', K') + (1-\delta)K'\tau_I',\right] + ... \tag{22}$$

where τ_I is typically assumed to be an investment wedge that follows an AR(1) with autocorrelation ρ_I and $\mathbb{E}$ is the RE operator. In this case, the FOC with the investment wedge is given by

$$\tau_I = \frac{1}{R}\mathbb{E}\left[\frac{\partial}{\partial K'}\Pi(A', K') + (1-\delta)\tau_I'\right]$$

$$\tau_I\left[R - \rho_I(1-\delta)\right] = \mathbb{E}\left[\frac{\partial}{\partial K'}\Pi(A', K')\right]. \tag{23}$$

Here, the expectation $\mathbb{E}$ on the RHS is the rational expectation, and K' is whatever capital choice is made by the firm. Two results are immediate.

First, if capital K' is chosen by a RE decisionmaker, then $\mathbb{E}^{firm} = \mathbb{E}$. And inspecting

Equations (20) and (23) reveals that only a constant $\tau_I = 1$ (and associated $\rho_I = 1$) are consistent with the model. In other words, the investment wedge does not move if there are no frictions and the representative decisionmaker has RE beliefs. This is not an interesting result, it's simply stating that in a RE model with no frictions, the investment wedge is trivially constant and equal to 1.

Second, if the decisionmaker chooses capital K' in equation (21) subject to DE beliefs, then the RE version of FOC (20) will not be satisfied and in general you must have movement in the investment wedge τ_I in Equation (23) which must be satisfied under RE. In particular, the investment wedge equation becomes

$$\tau_I \left[R - \rho_I(1-\delta)\right] = \mathbb{E}\left[\frac{\partial}{\partial K'}\Pi(A', K'^{\theta})\right], \tag{24}$$

where the expectation $\mathbb{E}$ is rational but the DE value K'^{θ} is given from (21) in this specialized case as

$$K'^{\theta} = \left(\frac{\alpha}{R-1+\delta}\right)^{\frac{1-\nu}{1-(\alpha+\nu)}} \left(\frac{\nu}{W}\right)^{\frac{\nu}{1-(\alpha+\nu)}} \left(\mathbb{E}^{\theta}\left[A'^{\frac{1}{1-\nu}}\right]\right)^{\frac{1-\nu}{1-(\alpha+\nu)}}. \tag{25}$$

Straightforward calculations, substituting (25) into (24), reveal that

$$\tau_I \propto \frac{\mathbb{E}A'^{\frac{1}{1-\nu}}}{\mathbb{E}^{\theta}A'^{\frac{1}{1-\nu}}}.$$

The numerator is RE, the denominator is DE. But these are both analytically tractable lognormal expectations under a Gaussian lognormal AR(1) assumption for A:

$$\mathbb{E}A'^{\frac{1}{1-\nu}} = e^{\frac{\rho_A}{1-\nu}\log A + \frac{\sigma_A^2}{2(1-\nu)^2}}$$

$$\mathbb{E}^{\theta}A'^{\frac{1}{1-\nu}} = e^{\frac{\rho_A}{1-\nu}(\log A + \theta\varepsilon_A) + \frac{\sigma_A^2}{2(1-\nu)^2}}.$$

We therefore obtain the formula

$$\tau_I \propto e^{-\frac{\rho_A}{1-\nu}\theta\varepsilon_A}, \tag{26}$$

This reveals that the investment wedge is indeed an AR(1) with a lognormal innovation and no persistence. In logs, as this is usually presented, we obtain that

$$\log \tau_I = \Omega - \frac{\rho_A}{1-\nu}\theta\varepsilon_A$$

up to some constant Ω. So, we have uncovered that in a PE frictionless representative agent version of our economy, DE does indeed generate investment wedge fluctuations. In this economy DE **only** creates investment wedge fluctuations, with efficiency, labor, and government consumption wedges constant trivially due to the representative firm's lack of financial frictions, static labor optimization, and lack of a government respectively. Intuitively, with good news, an economist doing business cycle accounting under RE perceives that the DE firm acts like its price of capital investment declines. With bad news, the RE economist doing business cycle accounting perceives that the DE firm acts like its price of capital investment increases.

A.8 General Equilibrium Definition

In Section 7.2 we consider an extended model with a representative household and endogenously flexible wages. Here, we outline the household, lender, and firm problems, define the equilibrium, and derive the two household first-order conditions used in the main text.

A.8.1 Household

A diagnostic representative household supplies labor N and saves in a risk-free bond B' in zero-net supply according to

$$H^{\theta}(\mu, A, \varepsilon_A, B) = \max_{B',N} \left[C - \frac{\omega}{1+\frac{1}{\lambda}} N^{1+\frac{1}{\lambda}} + \beta \mathbb{E}^{\theta} \left(H^{\theta}(\mu', A', \varepsilon'_A, B') | \mu, A, \varepsilon_A \right) \right]. \tag{27}$$

The household states include the distribution $\mu(s,k,b)$ across firm states $s = (z, \epsilon_z)$, k, and b, macro TFP A, macro news ε_A, and holdings of the bond B. The household's budget constraint is given by

$$C + B' = RB + WN + D^F + D^L + T.$$

Above D^F reflects net aggregate payouts from the firm sector, and D^L reflects aggregate payouts from lenders, both of whom are owned by the household. T reflects lump-sum transfers of corporate taxes from firms and lenders to the household. The household takes as given both the real interest rate R and the wage W.

A.8.2 Lenders

A continuum of diagnostic risk-neutral lenders funds itself via risk-free debt, taking as given the real interest rate R. Each lender participates in a range of competitive credit markets for firms with state s, future capital k', and a desired loan amount b'. If a borrower firm defaults on its debt, the lender receives the after-tax recovery rate

$$\mathcal{R}(k', b') = \gamma(1-\tau)(1-\delta)\frac{k'}{b'},$$

where $1-\gamma$ reflects a deadweight loss upon default. A lender's expected present discounted value of participating in the market for lending to firm (s, k', b') given the aggregate state

(μ, A, ε_A) is given by

$$\begin{array}{c} -q(s,k',b',\mu,A,\varepsilon_A)b' \\ +\frac{1}{R}\mathbb{E}^{\theta}\left[(1-df(s',k',b',\mu',A',\varepsilon_A'))b' + df(s',k',b',\mu',A',\varepsilon_A')b'\mathcal{R}(k',b')|s,\mu,A,\varepsilon_A\right] \end{array}. \tag{28}$$

A lender must choose to enter, or not to enter, each individual credit market, and any net profits are paid to the household owner.

A.8.3 Firms

A continuum of competitive diagnostic firms with idiosyncratic states (s, k, b) owned by the household takes as given the real interest rate R, the wage W, and a debt price schedule $q(s, k', b', \mu, A, \varepsilon_A)$, choosing investment for future capital k', debt issuance b', static labor demand n, and whether to default in order to maximize the expected present discounted value of its payouts according to

$$V^{\theta}(s,k,b,\mu,A,\varepsilon_A) = \max_{df\in\{0,1\}} \left[df V_D^{\theta}(s,\mu,A,\varepsilon_A) + (1-df)V_{ND}^{\theta}(s,k,b,\mu,A,\varepsilon_A)\right] \tag{29}$$

$$V_{ND}^{\theta}(s,k,b,\mu,A,\varepsilon_A) = \max_{k',b',n}\left\{d - IC(d) + \frac{1}{R}\mathbb{E}^{\theta}\left[V^{\theta}(s',k',b',\mu',A',\varepsilon_A')|s,\mu,A,\varepsilon_A\right]\right\} \tag{30}$$

$$d = \begin{array}{c} (1-\tau)\left[y - Wn - AC(i,k) - \phi\right] \\ +q(s,k',b',\mu,A,\varepsilon_A)b' - i - b + \tau(R-1+\delta k) \end{array}$$

$$V_D^{\theta}(s,\mu,A,\varepsilon_A) = \left\{0 + \frac{1}{R}\mathbb{E}^{\theta}\left[V(s',0,0,\mu',A',\varepsilon_A')|s,\mu,A,\varepsilon_A\right]\right\} \tag{31}$$

A.8.4 Definition

An equilibrium in this economy under diagnostic expectations is a collection including

- a wage function $W^{\theta}(\mu, A, \varepsilon_A)$,
- a fixed real interest rate R,
- a debt pricing function $q^{\theta}(s, k', b', \mu, A, \varepsilon_A)$,
- household value H^{θ} and policy functions N^{θ}, B'^{θ} (each functions of $(\mu, A, \varepsilon_A, B)$),
- lender participation decisions for each credit market (s, k', b'),

- firm value and policy functions V^θ, V^θ_{ND}, V^θ_D, k'^θ, b'^θ, n^θ and df^θ (each functions of $(s, k, b, \mu, A, \varepsilon_A)$), and
- a transition mapping $\mu' = \Gamma^\theta(\mu, A, \varepsilon_A)$ for the distribution $\mu(s, k, b)$ across periods

such that

- taking as given wages and the interest rate, household values and policies satisfy their dynamic problem (27) under DE beliefs from (12),
- taking as given debt prices and firm policies, lenders optimally whether to participate or not in each credit market given their payoffs (28) under DE beliefs from (12) and (11),
- taking as given wages, the interest rate, and debt prices, firm values and policies satisfy their dynamic problem defined by (29), (30), and (31) under DE beliefs from (12) and (11),
- labor markets clear with labor demand from firms equal to labor supply from households
$$\int n^\theta(s, k, b, \mu, A, \varepsilon_A) d\mu(s, k, b) = N^\theta(\mu, A, \varepsilon_A, B),$$
- individual firm credit markets clear with nonpositive expected payouts in (28) for each market (s, k', b') and strictly zero expected payouts when the market is active,
- the risk-free debt market clears with risk-free debt supply from the households equal to the total funding needs of the lenders as required by firm borrowing
$$B'^\theta(\mu, A, \varepsilon_A, B) = \int b'^\theta(s, k, b, \mu, A, \varepsilon_A) d\mu,$$
and
- the transition mapping Γ^θ accurately reflects transitions of states given firm policies and diagnostic expectations, i.e.,
$$\int_{\{(s',k',b')\in\mathcal{A}\}} d\mu'(s', k', b') = \int_{\mathcal{B}} f^\theta(s'|s) d\mu(s, k, b)$$
$$\mathcal{B}(\mathcal{A}, \mu, A, \varepsilon_A) = \{(s, k, b) | (s', k'^\theta, b'^\theta) \in \mathcal{A} \text{ for some } s'\}$$
for any well behaved set $\mathcal{A}$ whenever $\mu' = \Gamma^\theta(\mu, A, \varepsilon_A)$.

A.8.5 Characterization of Prices

Note that the linearity of household utility over consumption, together with optimal savings in risk-free debt, implies that a constant risk-free rate $R = \frac{1}{\beta}$ is the only one consistent with equilibrium. Discounted payoffs of firms and lenders at this interest rate are therefore equivalent to expected present discounted value maximization at the household's stochastic discount factor (in this case degenerate to β). Furthermore, a household's labor supply optimality condition is given by

$$\omega N^{\frac{1}{\lambda}} = W,$$

and since this expression doesn't depend upon consumption C it must also not depend upon household debt B. Therefore, the labor market clearing condition above can be rewritten

$$\left(\frac{W^\theta(\mu, A, \varepsilon_A)}{\omega}\right)^\lambda = \int n^\theta(s, k, b, \mu, A, \varepsilon_A) d\mu(s, k, b),$$

which is equivalent to the Equation (19) given in the main text. Also note that linearity of the household's preferences allows us to avoid specifying the nature of aggregate resource constraints or goods market clearing, since these details do not impact firm, lender, or household policies at the margin, although it is natural and harmless to assume that the structure of taxation clears a fiscal budget constraint each period and that the household absorbs any unexpectedly high or low aggregate payouts from diagnostic firms and lenders. Finally, note that for any market in which lending is active, the competitive nature of lending together yields zero diagnostically expected discounted profits in Equation (28) so that

$$q^\theta(s, k', b', \mu, A, \varepsilon_A) = \frac{1}{R}\mathbb{E}^\theta\left[(1 - df^\theta(s', k', b', \mu', A', \varepsilon'_A)) + df^\theta(s', k', b', \mu', A', \varepsilon'_A)\mathcal{R}(k', b')|s, \mu, A, \varepsilon_A\right]. \tag{32}$$

A.8.6 General Equilibrium Solution Algorithm

One might suspect that this problem poses more difficulty than the usual rational expectations model because of diagnostic expectations. But the characterization of prices above, dependent upon the careful assumption of linear consumption utility, actually implies that we face only the usual numerical challenges that arise in any model with a "Krusell Smith problem." In other words, there are two familiar computational challenges to numerically solving the general equilibrium model laid out above. First, the distributional state μ entering into the pricing function W^θ is intractable. Second, the transition mapping Γ^θ (implicitly appearing in the optimal debt pricing condition and firm value functions) and the wage mapping W^θ appearing in firm and household problems are also intractable and unavailable in closed form.

We implement a tractable approach to addressing both problems by replacing the distributional state μ with a truncated history of length K of the aggregate TFP shocks $(A_{t-K}, A_{t-K+1}, ..., A_{t-1}, A_t)$. For moderate history lengths K, the approximate state is more tractable than the distribution μ, and the mapping Γ^θ collapses to the simple diagnostic expectations of the evolution of the exogenous process which we have already exploited. With this simplification in hand, we then follow an outer loop/inner loop approach to solving the model.

1. Guess a mapping from a truncated history of macro states

$$(A_{t-K}, A_{t-K+1}, ..., A_{t-1}, A_t) \to W_t.$$

2. Solve the model conditional upon this tractable truncated history, where $(A_t, A_{t-1}, ..., A_{t-K})$ enters the firm's state vector and hence the Bellman equations determining investment, default, and debt issuance policies.

3. Simulate the model for a large number of periods $t = 1, ..., T$, clearing markets with W_t in each period t by numerically solving the nonlinear equation

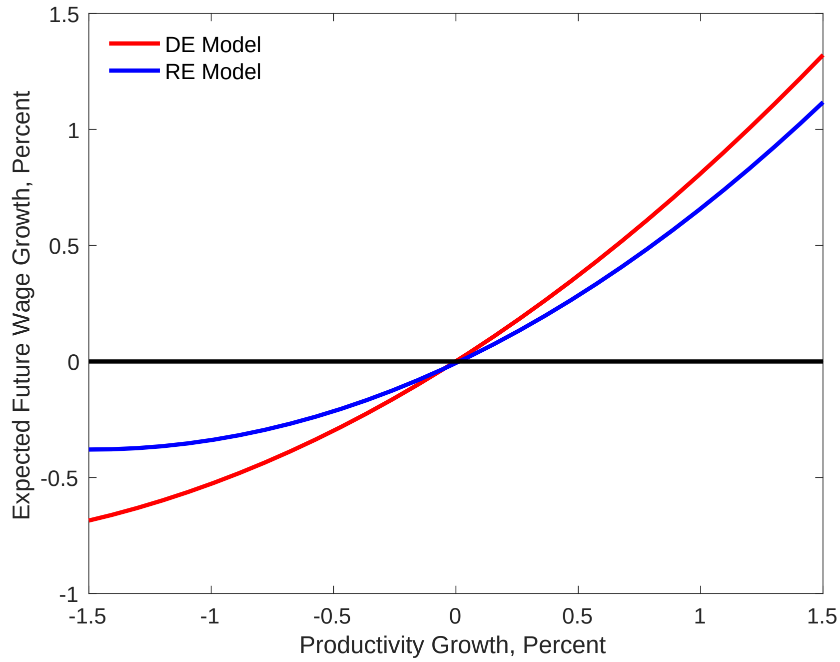
$$\left(\frac{W_t}{\omega}\right)^\lambda = \int n(s, k, b|W_t) d\mu_t(s, k, b)$$

   for each period t in the simulation. Note that this is a well behaved nonlinear equation in one variable. The analytically computable static labor policies $n(s, k, b|W_t)$ are strictly declining in W_t on the RHS and the function on the LHS is strictly increasing in W_t. In practice, markets can be cleared robustly using bisection or another similar algorithm.

4. Based on the simulated wage data, update your wage prediction mapping from Step 1. If the mapping has converged to within some tolerance, exit. If not update the mapping and return to Step 1.

A few practical comments are in order about this approach to solving the Krusell Smith problem, which to our knowledge is a novel, although conceptually straightforward, solution method. First, given the discretized macro TFP state space, we store the wage mapping nonparametrically as a matrix of mean wages conditional upon each combination of truncated macro TFP histories. After simulation, the wage mapping update step simply involves repeated calculations of mean wages within the appropriate subsamples of the simulated data. Figure A.5 plots the perceived wage mapping under DE and RE as a function of macro TFP growth in our converged GE solutions. Second, because the macro state is replaced with macro TFP shock histories rather than with an augmented endogenous macro

Figure A.5: Expected Wage Growth

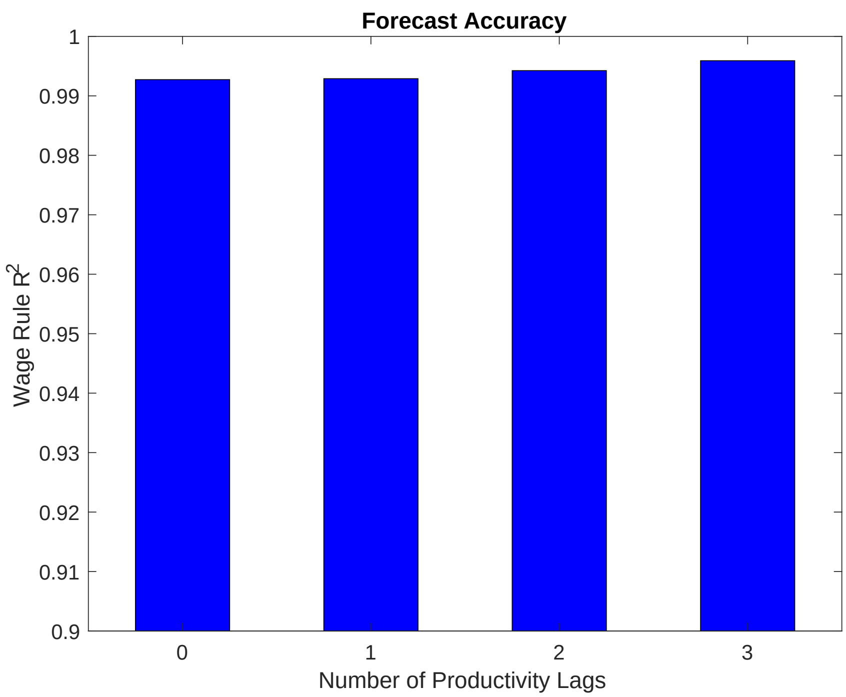


Notes: The figure plots the expected path of wage growth in the next period (vertical axis) given productivity growth today (horizontal axis) in the DE model (red) and RE model (blue).

moment, there is no need to create an approximate anticipated default rule used to price debt. Lenders simply price debt according to the usual no-arbitrage condition in Equation (32) above. Third, because no endogenous moments are forecasted in our solution method, there is no Den Haan (2010)-style distinction between static and dynamic forecasts of the wage. In other words, there is no room for forecast errors about endogenous macro moments to accumulate over time, since only exogenous shock histories are used for forecasts. So, unlike in typical adaptations of the Krusell and Smith (1998) method, the R^2 of the implicit wage forecast rule is in this case an appropriate metric of accuracy. With this in mind, Figure A.6 plots the estimated R^2 of regressions of the log wage on fully populated sets of dummies for macro TFP histories of up to a given lag length. Once a single lag is taken into account, incorporating information from yesterday's TFP level about the current distribution of capital and hence labor demand in the cross section, the R^2 measures stabilize. Our baseline case, which uses a single lag with $K = 1$ in the wage prediction rule, is therefore a parsimonious but apparently accurate choice.

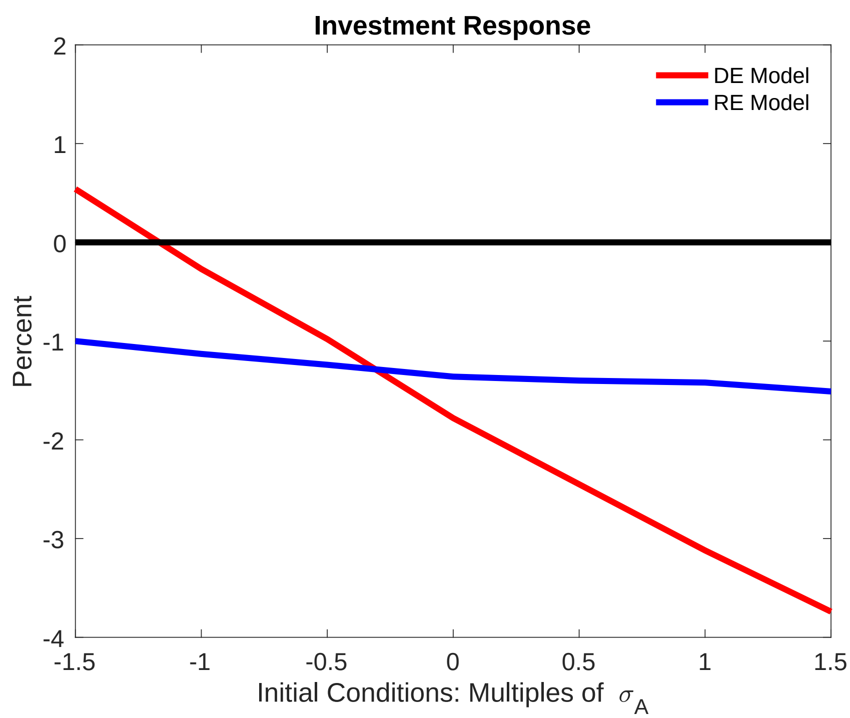
We use this solution to re-compute investment nonlinearities in Figure 2 for the GE case, with the results contained in Figure A.7.

Figure A.6: Wage Predictions and TFP Lags



Notes: The figure plots the R^2 of nonparametric regressions of log wages on discrete histories of macro TFP of increasing length.

Figure A.7: Investment Nonlinearity with General Equilibrium

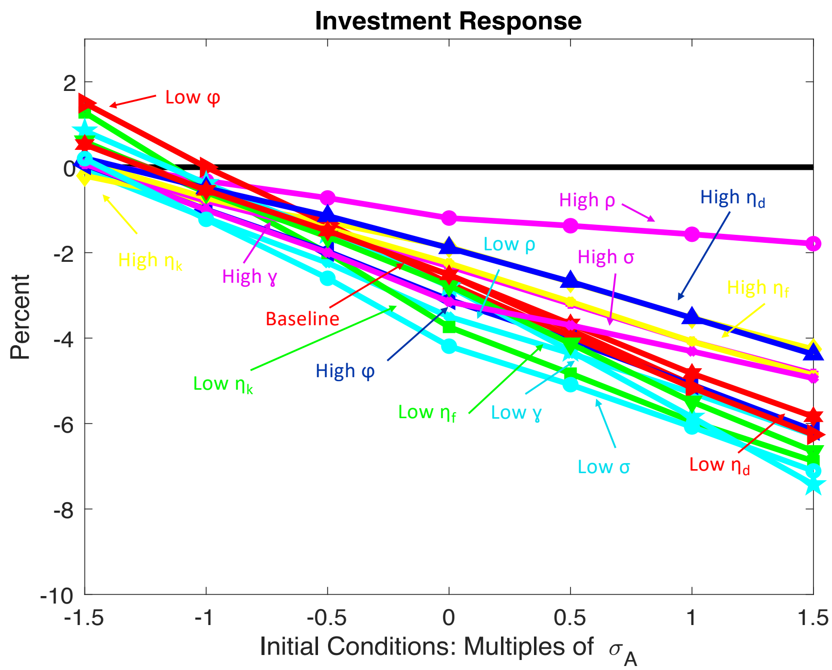


Notes: The vertical axis in the figure reports the simulated first-period impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. Both the DE model (red line) and RE model (blue line) are reported on the figure. These results are computed in the general equilibrium model with labor market clearing.

A.9 Parameter Robustness Checks for Investment Nonlinearity

In the baseline estimated DE model in partial equilibrium we recomputed investment the nonlinearity plot in Figure 2 for a range of parameter robustness checks, with the resulting responses of investment plotted in Figure A.8.

Figure A.8: Investment Nonlinearity with Alternative Parameters



Notes: The vertical axis in the figure reports the simulated impulse response of macro investment to a one-standard deviation negative shock to macro TFP, and the horizontal axis reports the initial conditions, i.e., the magnitude of the shock to macro TFP in the previous period. All models are perturbations of the estimated Baseline DE model, with a single parameter shifted to round higher or lower values bracketing the parameter point estimate. The autocorrelation of micro TFP ρ_z is set to 0.75 (high) and 0.6 (low). The volatility of micro TFP σ_z is set to 0.3 (high) and 0.2 (low). The capital adjustment cost η_k is set to 4.3 (high) and 3.7 (low). The fixed operating cost ϕ is set to 0.25 (high) and 0.05 (low). The recovery rate γ is set to 0.30 (high) and 0.15 (low). The equity fixed cost η_f is set to 0.025 (high) and 0.005 (low). The equity linear cost η_d is set to 0.15 (high) and 0.05 (low). Macro TFP volatility σ_A, not reported, is held constant in order to keep the shock experiment constant. The magnitude of diagnosticity θ, also held constant, is already varied in Figure 2 to extreme values. The noise in earnings σ_π, also not reported, affects only simulated micro moments.

B Data

B.1 Microdata on Firm Beliefs from Compustat and IBES-Guidance

In our analysis of firm financial and profit forecasts we use a combination of the Compustat Fundamentals Annual and IBES manager guidance databases. The combined sample for the Compustat-IBES data spans 1999-2017 for 9666 firm-fiscal years spanning 1391 firms. To construct our sample, we discard utilities and financials as well as any firm-years with negative values for assets, capital, employment or sales. Descriptive statistics for each variable from this sample used in our analysis, as well as firm revenues and capital, are reported in Table B.1.

Table B.1: Sample Descriptive Statistics

Quantity	Mean	Standard Deviation
Sales	6549.4	22231.5
Capital	1492.1	6185.2
Assets	6689.6	27242.4
Profit	0.4362	1.0310
Investment	0.3104	0.2331
Debt	0.2708	0.2377
Forecast Error	-0.2610	0.7842
Forecast	0.7994	0.9256

Notes: The table reports descriptive statistics for the sample of 1391 firms from 1999-2017 and 9666 firm-years in the combined Compustat-IBES database. The first three rows represent revenues and the book value of the capital stock and total assets, in $ millions. The remaining rows reflect the ratio of realized earnings to the book value of the capital stock, the capital expenditures investment rate, the ratio of end of period total liabilities to the asset stock, the next-period forecast error defined as realized future profits minus manager guidance scaled by firm capital, and the next period forecast defined as manager profit guidance scaled by firm capital. The sample was winsorized before computing the descriptive statistics above.

The variable definitions are given as follows, with both empirical and model information attached:

- **Earnings or profits** are equal to GAAP net income, Compustat `ib`. The model equivalent is $\pi = (1-\tau)(y - Wn - AC(i,k) - \phi) + \tau((R-1)b + \delta k) - \delta k$.
- **Capital** k is equal to the book value of plants, property, and equipment, Compustat `ppent`. The model equivalent is the state variable k.
- **Investment** i is equal to the total value of capital expenditures, Compustat `capxv`. The model equivalent is the policy variable $i = k' - (1-\delta)k$.
- **Debt** b is equal to the total net value of liabilities, Compustat $\texttt{dltt} + \texttt{dlc} - \texttt{che}$. The model equivalent is the state variable b.

- **Forecast error** fe is equal to the realized value of earnings π minus the forecast level of earnings π^f made from the previous fiscal year, where realized earnings are Compustat `ib` and forecast earnings are equal to manager guidance from the IBES database. The model equivalent is the earnings value π above, minus the forecast level implied by firm-level diagnostic expectations, the definition of π, and firm policies predetermined in the previous period.

We also use the merged Compustat-IBES guidance sample to run various robustness checks to the firm forecast error predictability regressions reported in the main text. Table B.3 shows similar forecast error predictability maintains after the Great Recession, in a sample of firms present for five or more years in the data, and after discarding all firms with high-yield debt as classified by Moody's ratings.

B.2 Microdata on Bonds from FISD-TRACE

We use the WRDS US Corporate Bond Return database, which merges the Mergent FISD and FINRA TRACE datasets with issuance and secondary market information on corporate bond issues, respectively. We consider only unsecured, unconvertible debentures and convert secondary market yields to spreads based on comparable Treasury rates, with a resulting dataset of around 80,000 issues from mid-2002 to late 2019. We link the bond return database to Compustat firm financials through the WRDS CRSP link, and we aggregate from the issue to firm level by computing average yields and bond returns for a firm in Q4 of a given year. The resulting dataset spans around 1,500 large US public firms. Linking this panel to the IBES-manager guidance data yields the sample used in Table 2 in the main text. Table B.5 replicates Table 2 conditioning only on investment grade bonds. Table B.6 replicates Table 2 but does not include current profit controls for the investment regressions nor lagged spread controls for the spread regressions.

B.3 Macro Data

At the macro level, we use a combination of information from the NIPA accounts, the BIS, Moody's. The following variables are relevant, all at annual frequency or converted to annual frequency by averaging.

- **Output** Y is real GDP from the NIPA accounts.
- **Investment** I is real nonresidential private investment from the NIPA accounts in the data.
- **Spreads** are the Moody's BAA spread relative to 10-year Treasury bonds, at an annualized rate.

- **TFP** is the annualized value of the series `dtfp` from John Fernald's website.
- **Profit Fcst** is the sum of earnings guidance across firms in a given year from our Compustat-IBES merged database.

In the empirical business cycle moments in Table 6, we reports moments from the HP-filtered values of output together with unadjusted spread levels. In the spread matching/Great Recession exercise in Figure 4, we report the average spread and the average growth of credit, output, investment, and profit forecasts in each subperiod.

B.4 SMM Estimation

Our SMM estimation exercise in Section 5.1 involves three steps: 1) moment and covariance matrix calculations, 2) model estimation, and 3) standard error calculation. We detail each of these steps in turn.

B.4.1 Moment and Covariance Matrix Calculation

Table 5 reports a set of target moments at the micro and macro levels for our SMM estimation exercise. The micro moments are a covariance matrix of the vector

$$X_{it} = (\text{Profit}_{it}, \text{Investment}_{it}, \text{Debt}_{it}, \text{Spread}_{it})'$$

for firm i in fiscal year t from our merged Compustat-IBES-FISD-TRACE sample. The micro moments also include the covariance of future forecast error growth with lagged investment and debt issuance growth at a two-period horizon, together with the variance of future forecast errors. The merged sample with all of the required variables available spans 493 firms and 4697 total observations. To compute the micro moments, we use the following procedure:

- Demean X_{it} by firm and year to obtain $\hat{X}_{it}$
- Compute the covariance matrix as the mean of $\hat{X}_{it}\hat{X}_{it}'$.
- Apply the standard formula for the clustered covariance of a mean vector to obtain the moment covariance matrix Ω_{Micro}, clustering across firms.

With the estimated micro moments and the estimated moment covariance matrix for the micro moments in hand, we then turn to the calculation of the macro moments and their covariance matrix. Note that the macro moments are the mean default rate, the mean spread, and the standard deviation of real GDP growth. We compute the mean default and spread series from our merged FISD-TRACE data on corporate debt, and we compute real

GDP growth from the NIPA data. The point estimates of these macro moments are trivial to compute. We then compute an estimate of the covariance matrix of these macro moments Ω_{Macro} using a stationary block bootstrap.

Note that for our later inference based on clustering at the firm level, we will rely upon an assumption that the macro sample length T^{macro} and the total number of observations in the microdata sample T behave proportionally with $T^{macro}/T \to \gamma$ for some constant γ as $N \to \infty$. This allows us to rely on asymptotics of the basic form

$$\sqrt{T^{macro}}(\hat{\mu} - \mu) \to_d N(0, \Omega), \tag{33}$$

where $\hat{\mu}$ is the estimated moment vector (with micro and macro moments) and Ω is the joint moment covariance matrix. Table 5 reports $\hat{\mu}$.

B.4.2 Point Estimate Calculation

We compute the point estimates $\hat{\beta}$ for the vector of estimated parameters β in Table 4 by solving the following standard SMM optimization problem

$$\min_{\beta} (\mu^S(\beta) - \hat{\mu})' \hat{\Omega}^{-1} (\mu^S(\beta) - \hat{\mu})$$

where $\mu^S(\beta)$ is the model value of the moments given β computed from simulated data, $\hat{\Omega}^{-1}$ is the asymptotically efficient weighting matrix given by the inverse of the estimated moment covariance matrix, and $\hat{\mu}$ is the empirical moment vector. We employ particle swarm optimization to solve this optimization problem, a stochastic global optimization routine that bears substantial similarity to simulated annealing and genetic algorithms.

B.4.3 Standard Error Calculation

Given the ratio between the number of observations T^{sim} in the model simulation used to compute $\mu^S(\beta)$ and the empirical number of observations T, the SMM estimator's asymptotic covariance matrix Σ follows

$$\sqrt{T}(\hat{\beta} - \beta) \to_d N(0, \Sigma) \tag{34}$$

where

$$\Sigma = \left(1 + \frac{T}{T^{sim}}\right) \left(\frac{\partial \mu^S(\beta)}{\partial \beta'} \Omega^{-1} \frac{\partial \mu^S(\beta)}{\partial \beta}\right)^{-1}. \tag{35}$$

Equation (35) yields a feasible formula for Σ after substitution of the estimated covariance matrix $\hat{\Omega}$ and numerical calculation of the moment Jacobian matrix $\frac{\partial \mu^S(\beta)}{\partial \beta'}$ within the model using forward differentiation from the point estimates $\hat{\beta}$. With these elements in hand, Tables 4 reports standard errors based on the approximating variance from (34).

B.5 GMM Regression Coefficients with Firm Fixed Effects

In this appendix section we provide a statistical framework for the analysis of forecast overreaction allowing for firm fixed effects through the use of within-firm differences and GMM estimators. We use these results directly to provide reduced-form regression coefficients with firm fixed effects in Appendix Table B.2 via GMM, and they also prove useful for generating intuition for the identification of the beliefs parameter θ in our quantitative SMM estimation of the neoclassical model in Section 5.1.

B.5.1 Data Generating Process

Let x_{it} be the observed value of the forecasted variable $\tilde{x}_{it}$ (e.g., profits) for firm i in period t, following an AR(1) with firm fixed effects μ_i^x and subject to measurement error ν_{it}^x with MA(1) dynamics:

$$\tilde{x}_{it} = \mu_i^x + \rho\tilde{x}_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim N(0, \sigma_\epsilon^2)$$

$$x_{it} = \tilde{x}_{it} + \nu_{it}^x + \gamma_x \nu_{it-1}^x, \quad \nu_{it}^x \sim N(0, \sigma_{x,\nu}^2)$$

Let y_{it} be the observed value of a variable $\tilde{y}_{it}$ (e.g., investment) linked linearly to the forecasted variable, subject to its own fixed effects μ_i^y and its own MA(1) measurement error ν_{it}^y:

$$\tilde{y}_{it} = \mu_i^y + \alpha\tilde{x}_{it}$$

$$y_{it} = \tilde{y}_{it} + \nu_{it}^y + \gamma_y \nu_{it-1}^y, \quad \nu_{it}^y \sim N(0, \sigma_{y,\nu}^2).$$

Allow forecasts $\tilde{x}_{it+1|t}^f$ to follow the DE formula, but also allow for forecast bias in the form of fixed effects μ_i^f and for MA(1) measurement noise ν_{it}^f in the observed value $x_{it+1|t}^f$

$$\tilde{x}_{it+1|t}^f = \mu_i^f + \rho\tilde{x}_{it} + \rho\theta\epsilon_{it}$$

$$x_{it+1|t}^f = \tilde{x}_{it+1|t}^f + \nu_{it}^f + \gamma_f \nu_{it-1}^f, \quad \nu_{it}^f \sim N(0, \sigma_{f,\nu}^2).$$

The implied observed forecast errors $fe_{it+1} = x_{it+1} - x_{it+1|t}^f$ are then given by

$$fe_{it+1} = \left(\mu_i^x + \rho\tilde{x}_{it} + \epsilon_{it+1} + \nu_{it+1}^x + \gamma_x\nu_{it}^x\right) - \left(\mu_i^f + \rho\tilde{x}_{it} + \rho\theta\epsilon_{it} + \nu_{it}^f + \gamma_f\nu_{it-1}^f\right)$$

$$fe_{it+1} = \left(\mu_i^x - \mu_i^f\right) + \epsilon_{it+1} + \nu_{it+1}^x + \gamma_x\nu_{it}^x - \rho\theta\epsilon_{it} - \nu_{it}^f - \gamma_f\nu_{it-1}^f$$

B.5.2 Nonoverlapping Difference Covariances as Useful Moments

Choose any difference horizons $\bar{s}, \underline{s} \geq 1$. Then the data generating process laid out above implies that the covariance of future forecast error growth with current profit growth in the firm is given by

$$cov(fe_{it+\bar{s}} - fe_{it}, x_{it-1} - x_{it-\underline{s}}) =$$

$$cov\left(\begin{array}{c} \epsilon_{it+\bar{s}} + \nu^x_{it+\bar{s}} + \gamma_x \nu^x_{it+\bar{s}-1} - \rho\theta\epsilon_{it+\bar{s}-1} - \nu^f_{it+\bar{s}-1} - \gamma_f \nu^f_{it+\bar{s}-2} \\ -\epsilon_{it} - \nu^x_{it} - \gamma_x \nu^x_{it-1} + \rho\theta\epsilon_{it-1} + \nu^f_{it-1} + \gamma_f \nu^f_{it-2} \end{array}, \begin{array}{c} \epsilon_{it-1} + \nu^x_{it-1} + \gamma_x \nu^x_{it-2} + \rho\tilde{x}_{t-2} \\ -\rho\tilde{x}_{t-2-\underline{s}} - \epsilon_{it-1-\underline{s}} - \nu^x_{it-1-\underline{s}} - \gamma_x \nu^x_{it-1-\underline{s}-1} \end{array} \right)$$

$$= \rho\theta cov(\epsilon_{it-1}, \epsilon_{it-1}) - \gamma_x cov(\nu^x_{it-1}, \nu^x_{it-1}) = \rho\theta\sigma^2_\epsilon - \gamma_x \sigma^2_{\nu,x}.$$

We immediately obtain our first result, that the covariance of future forecast growth with profit growth can offer evidence of diagnosticity in beliefs but may be inconclusive in the presence of MA(1) measurement error in profits.

Result 1: If $cov(fe_{it+\bar{s}} - fe_{it}, x_{it-1} - x_{it-\underline{s}}) = \rho\theta\sigma^2_\epsilon - \gamma_x\sigma^2_{\nu,x} > 0$ for some $\bar{s}, \underline{s} \geq 1$ and $\gamma_x > 0$, then $\theta > 0$.

To exploit similar intuition but avoid cross-contamination with profit measurement error, we can also compute the covariance of future forecast error growth with the growth of outcomes y_{it} in the firm such as investment or debt.

$$cov(fe_{it+\bar{s}} - fe_{it}, y_{it-1} - y_{it-\underline{s}}) = \alpha\rho\theta cov(\epsilon_{it-1}, \epsilon_{it-1}) = \alpha\rho\theta\sigma^2_\epsilon$$

We then immediately obtain our second result, which is that covariances of future forecast error growth with investment or debt growth in the firm should be zero unless expectations are diagnostic with $\theta = 0$.

Result 2: If $cov(fe_{it+\bar{s}} - fe_{it}, y_{it-1} - y_{it-\underline{s}}) = \alpha\rho\theta\sigma^2_\epsilon > 0$ for some $\bar{s}, \underline{s} \geq 1$ and $\alpha > 0$, then $\theta > 0$.

Result 2, which holds analytically in closed form and still holds approximately in our nonlinear neoclassical model, provides direct motivation for our use of the forecast error growth covariance moments in Section 5.1.

B.5.3 Obtaining Regression Coefficients

In this section, we derive formulas for the asymptotically normal and consistent GMM estimation of regression coefficients of future forecast errors on investment in the firm, allowing for both measurement error and fixed effects in all series. In particular, using the notation of our data generating process, our goal is to estimate regression coefficients linking the following "cleansed" variables to one another:

- Forecast errors without measurement error or fixed effects $fe^*_{it+1} = \epsilon_{it+1} - \rho\theta\epsilon_{it}$
- The linked variable y without measurement error or fixed effects $y^*_{it} = \tilde{y}_{it} - \mu^y_i$

Now, note a few simple results based on our data generating process above.

$$cov(fe^*_{it+1}, y^*_{it}) = cov\left(\epsilon_{it+1} - \rho\theta\epsilon_{it}, \alpha_y\epsilon_{it} + ...\right) = -\alpha_y\rho\theta\sigma^2_\epsilon$$

$$var(y^*_{it}) = \alpha^2\frac{\sigma^2_\epsilon}{1-\rho^2}.$$

Therefore, the probability limit of a univariate regression of forecast errors fe^*_{it+1} on the linked variable y^*_{it} is given by

$$\beta_y = \frac{cov(fe^*_{it+1}, y^*_{it})}{var(y^*_{it})} = \frac{-\alpha\rho\theta\sigma^2_\epsilon}{\alpha^2\frac{\sigma^2_\epsilon}{1-\rho^2}} = \frac{-\rho\theta\sigma^2_\epsilon}{\alpha\frac{\sigma^2_\epsilon}{1-\rho^2}}.$$

Now, this is true for each linked variable y, and in particular for $y = i$ (investment) and $y = b'$ (debt issuance). The statistical framework above implies the following moment relationships.

$$m_1 = cov(fe_{it+1} - fe_{it}, i_{it-1} - i_{it-1}) = \alpha_i\rho\theta\sigma^2_\epsilon$$

$$m_2 = cov(fe_{it+1} - fe_{it}, b'_{it-1} - b'_{it-1}) = \alpha_{b'}\rho\theta\sigma^2_\epsilon$$

$$m_3 = cov({b'}^*_{it-1}, i^*_{it}) = \alpha_i\alpha_{b'}\frac{\sigma^2_\epsilon}{1-\rho^2}$$

But all of these are estimable consistently. m_1 and m_2 are probability limits of covariances of differences. m_3 is the probability limit of the covariance of residualized investment and debt. Consider a general central limit theorem

$$\sqrt{N}(m(X) - m) \rightarrow_d N(0, \Sigma)$$

where $m(X) = (m_1, m_2, m_3)'$ is the moment estimate from the data X, N is the observation count and Σ is the asymptotic variance of $m(X)$ allowing for firm-level clustering. Note that Σ can be estimated via off-the-shelf closed-form form econometric formulas. Then consider the desired regression coefficient $\beta_i = -\frac{m_2}{m_3}$, which is a function of the underlying vector m. Via the Delta method we have

$$\sqrt{N}(\hat{\beta}_i - \beta_i) \rightarrow_d N(0, \Omega),$$

where the asymptotic variance of the regression coefficient vector is given by

$$\Omega = \frac{\partial\beta_i}{\partial m'}\Sigma\frac{\partial\beta_i}{\partial m}.$$

All of these objects have feasible estimators, since the Jacobian has a simple form given by

$$\frac{\partial \beta_i}{\partial m'} = \left[\begin{array}{ccc} 0 & \frac{-1}{m_3} & \frac{m_2}{m_3^2} \end{array} \right].$$

With this framework in hand, we note that Appendix Table B.2 in column (3) reports the regression coefficient point estimate $\hat{\beta}_i = \beta_i(\hat{m})$ as well as the standard error $\sqrt{\frac{diag\hat{\Omega}}{N}}$.

B.6 AR(1) with Misperceived Persistence

There are multiple departures from full information rational expectations which in principle can generate overreaction of beliefs, including both DE as well as models in which agents systematically inflate the perceived persistence of an underlying AR(1) model. To compare the two belief systems, consider data X_t which follows an underlying true AR(1)

$$\log X_{t+1} = \rho \log X_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0, \sigma^2).$$

A agent with diagnostic beliefs governed by parameter θ makes forecast

$$\mathbb{E}_t^\theta \log X_{t+1} = \rho \log X_t + \theta \rho \epsilon_t,$$

leading to forecast error

$$\begin{aligned} FE_{t+1}^\theta &= \log X_{t+1} - \mathbb{E}_t^\theta \log X_{t+1} \\ &= \rho \log X_{t+1} + \varepsilon_{t+1} - (\rho \log X_t + \theta \rho \epsilon_t) \\ &= \epsilon_{t+1} - \theta \rho \epsilon_t, \end{aligned}$$

implying a negative covariance, i.e., overreaction, given by

$$\begin{aligned} cov\left[FE_{t+1}^\theta, \log X_t\right] &= cov\left[\epsilon_{t+1} - \theta\rho\epsilon_t, \rho \log X_{t-1} + \epsilon_t\right] \\ &= -\theta \rho \sigma^2. \end{aligned}$$

Now, as an alternative, consider an agent who in the same context perceives an AR(1) process with persistence $\hat{\rho}$. The agent's forecast is

$$\mathbb{E}_t^{\hat{\rho}} \log X_{t+1} = \hat{\rho} \log X_t,$$

leading to forecast error

$$\begin{aligned} FE_{t+1}^{\hat{\rho}} &= \log X_{t+1} - \hat{\rho} \log X_t \\ &= \rho \log X_t + \epsilon_{t+1} - \hat{\rho} \log X_t \end{aligned}$$

$$= (\rho - \hat{\rho}) \log X_t + \epsilon_{t+1},$$

implying a covariance

$$cov\left[FE^{\hat{\rho}}_{t+1}, \log X_t\right] = cov\left[(\rho - \hat{\rho}) \log X_t + \epsilon_{t+1}, \log X_t\right]$$

$$= (\rho - \hat{\rho}) cov\left[\log X_t, \log X_t\right]$$

$$= (\rho - \hat{\rho}) \frac{\sigma^2}{1 - \rho^2}.$$

This covariance is negative, i.e., overreaction occurs on the part of the agent, whenever $\hat{\rho} > \rho$. So overestimated persistence maps to overreaction.

To compare the two belief systems, it is useful to consider the degree of perceived persistence $\hat{\rho}$ required to equalize the degree of overreaction across the diagnostic and misperceived AR(1) versions of beliefs. In this case $\hat{\rho}$ must satisfy

$$cov\left[FE^{\hat{\rho}}_{t+1}, \log X_t\right] = cov\left[FE^{\theta}_{t+1}, \log X_t\right]$$

$$(\rho - \hat{\rho}) \frac{\sigma^2}{1 - \rho^2} = -\theta \rho \sigma^2$$

$$\hat{\rho} = \rho\left(1 + \theta(1 - \rho^2)\right).$$

As expected, to match diagnostic expectations with overreaction and $\theta > 0$ requires $\hat{\rho} > \rho$.

Operationalizing this formula using our baseline micro and macro TFP process estimates, together with the estimated degree of diagnosticity, from Table 4 implies perceived persistence parameters given by

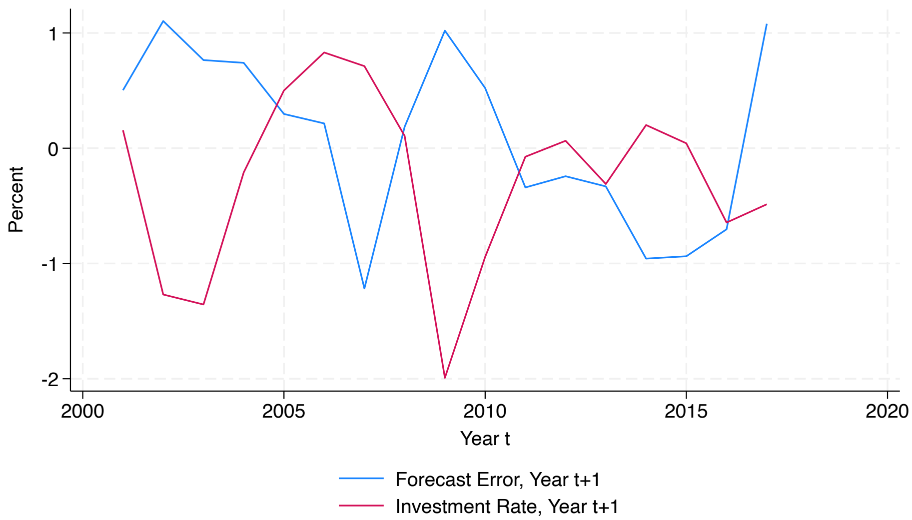
$$\text{Micro TFP: } \hat{\rho}_z = 1.0071$$

$$\text{Macro TFP: } \hat{\rho}_A = 1.0346.$$

Forecast overreaction to the degree we estimate in our baseline requires nonstationary AR(1) beliefs with $\hat{\rho} > 1$. Nonstationary beliefs are incompatible with the stationary Bellman Equations (15) and (16) characterizing firm decisionmaking. Intuitively, the degree of overreaction and forecast error volatility we see in the data are high, requiring a perceived persistence incompatible with stationarity because of the high fundamental persistence common in macroeconomic and firm-level analysis. An advantage of diagnostic beliefs in this context is therefore the maintained stationarity of beliefs which nevertheless match observed overreaction patterns.

B.7 Robustness Tables and Figures

Figure B.1: Aggregate Forecast Errors



Notes: Horizontal axis is year. Red line is mean investment rate. Blue line is mean forecast error in the following year. Variables standardized. Correlation in the two series is -0.61.

Table B.2: Predictable Forecast Errors, OLS vs GMM

	(1)	(2)	(3)
		Forecast Error_{t+1}	
Estimation Method:	OLS	OLS	GMM
Investment_t	-1.037^{***}	-0.457^{***}	-3.647^{*}
	(0.069)	(0.065)	(2.080)
Firm Effects		X	X
Year Effects	X	X	X
Years	1999-2018	1999-2018	1999-2018
Firm-Years	9666	9666	7095

Notes: The table reports panel estimates from the merged Compustat-IBES sample of the coefficients of a regression of forecast errors on the indicated variable. Columns 1 and 2 reflect OLS panel estimators, while column 3 reflects the GMM estimator outlined in Data Appendix B. The standard errors are clustered at the firm level. All variables are scaled by the firm's tangible capital stock and measured at the firm-fiscal year level. Forecast errors in $t+1$ are realized earnings in $t+1$ minus firm forecasts in t. Investment in t is capital expenditures. $* = 10\%$ level, $** = 5\%$ level, and $*** = 1\%$ level. The standard deviations of each variable in the baseline sample are 0.784 (forecast errors) and 0.233 (investment) where $0.01 = 1\%$ relative to the firm's capital stock.

Table B.3: Predictable Forecast Errors, Robustness

	(1)	(2)	(3)	(4)	(5)	(6)
			Forecast Error_{t+1}			
Sample:	Baseline	Inv. Grade	$\geq$ 5 Yrs.	Post-G.R.	High Inv.	Low Inv.
Investment_t	-0.457^{***}	-0.448^{***}	-0.365^{***}	-0.468^{***}	-0.372^{***}	-0.569^{***}
	(0.065)	(0.067)	(0.073)	(0.107)	(0.087)	(0.199)
Firm Effects	X	X	X	X	X	X
Year Effects	X	X	X	X	X	X
Years	1999-2018	1999-2018	1999-2018	2009-2018	1999-2018	1999-2018
Firm-Years	9666	8639	7973	4388	4625	4599

Notes: The table reports panel OLS estimates from the merged Compustat-IBES sample of the coefficients of a regression of forecast errors on the indicated variable. Each column, across rows, reports coefficients for a particular sample of interest. The standard errors are clustered at the firm level. All variables are scaled by the firm's tangible capital stock and measured at the firm-fiscal year level. Forecast errors in $t+1$ are realized earnings in $t+1$ minus firm forecasts in t. Investment in t is capital expenditures. $* = 10\%$ level, $** = 5\%$ level, and $*** = 1\%$ level. The standard deviations of each variable in the baseline sample are 0.784 (forecast errors) and 0.233 (investment) where $0.01 = 1\%$ relative to the firm's capital stock.

Table B.4: Predictable Forecast Errors by Horizon

	(1)	(2)	(3)
		Forecast Error$_{t+1}$	
Estimation Method:	OLS	OLS	OLS
Δ Investment$_t$	-0.179^{***}		
	(0.058)		
Δ Investment$_{t-1}$		-0.153^{***}	
		(0.055)	
Δ Investment$_{t-2}$			-0.029
			(0.049)
Year Effects	X	X	X
Years	1999-2018	2000-2018	2001-2018
Firm-Years	9747	9342	8937

Notes: The table reports panel OLS estimates from the merged Compustat-IBES sample of the coefficients of a regression of forecast errors on the indicated variable. The standard errors are clustered at the firm level. All variables are scaled by the firm's tangible capital stock and measured at the firm-fiscal year level. Forecast errors in $t+1$ are realized earnings in $t+1$ minus firm forecasts in t. Investment in t is capital expenditures. $* = 10\%$ level, $** = 5\%$ level, and $*** = 1\%$ level.

Table B.5: Linking Forecast Errors and Firm Reversals: Investment Grade Firms

	(1)	(2)	(3)	(4)
Panel A: Second Stage	Return_t	Return_t	Δ Investment_t	Δ Investment_t
Estimation Method:	OLS	IV	OLS	IV
Forecast Error_t	0.000	0.006^{*}	0.022^{**}	0.436^{***}
	(0.000)	(0.004)	(0.009)	(0.078)
Panel B: First Stage		Forecast Error_t		
Investment_{t-1}		-0.523^{***}		-0.523^{***}
		(0.118)		(0.118)
Year Effects	X	X	X	X
Years	2003-2018	2003-2018	2003-2018	2003-2018
Firm-Years	2000	2000	2000	2000
First Stage F		20		20

Notes: The table reports estimates of specifications on the merged Compustat - IBES - FISD/TRACE sample at the firm-fiscal year level, restricting to firms with Moody's rated investment grade debt. The top panel plots OLS and IV second-stage estimates. The bottom panel, where relevant, reports IV first-stage estimates. Columns (3)-(4) control for current profits in the second stage. Standard errors are clustered at the firm level. $*$ = 10% level, $**$ = 5% level, and $***$ = 1% level. The standard deviation of the bond return is 0.011, the standard deviation of investment growth is 0.072, the standard deviation of the forecast error is 0.381, and the standard deviation of lagged investment is 0.119. For all series, 0.01 = 1% relative to a firm's tangible capital stock.

Table B.6: Linking Forecast Errors and Firm Reversals: No Controls

	(1)	(2)	(3)	(4)
Panel A: Second Stage	Return_t	Return_t	Δ Investment_t	Δ Investment_t
Estimation Method:	OLS	IV	OLS	IV
Forecast Error_t	0.001	0.007^{*}	0.008	0.452^{***}
	(0.001)	(0.004)	(0.007)	(0.095)
Panel B: First Stage		Forecast Error_t		
Investment_{t-1}		-0.565^{***}		-0.565^{***}
		(0.104)		(0.104)
Year Effects	X	X	X	X
Years	2003-2018	2003-2018	2003-2018	2003-2018
Firm-Years	2852	2852	2852	2852
First Stage F		29		29

Notes: The table reports estimates of specifications on the merged Compustat - IBES - FISD/TRACE sample at the firm-fiscal year level. The top panel plots OLS and IV second-stage estimates. The bottom panel, where relevant, reports IV first-stage estimates. Standard errors are clustered at the firm level. $* = 10\%$ level, $** = 5\%$ level, and $*** = 1\%$ level. The standard deviation of the bond return is 0.014, the standard deviation of investment growth is 0.090, the standard deviation of the forecast error is 0.439, and the standard deviation of lagged investment is 0.133. For all series, $0.01 = 1\%$ relative to a firm's tangible capital stock.

Table B.7: Linking Forecast Errors and Industry Reversals in the Data: SIC2 Sectors

	(1)	(2)	(3)
	Forecast Error_t	Return_t	$\Delta\text{Investment}_t$
IV Stage:	First	Second	Second
Forecast Error_t		0.048^{***}	0.567^{***}
		(0.014)	(0.046)
Investment_{t-1}	-0.873^{***}		
	(0.246)		
Industry Effects	X	X	X
Industries	35	35	35
Industry-Years	453	453	453
First Stage F	13		

Notes: The table reports first- and second-stage IV estimates based on industry aggregated data from the Compustat- IBES-FISD/TRACE sample at the SIC2 $\times$ fiscal year level. Column (1) reports the first stage, and columns (2)-(3) report second-stage regressions. Standard errors are clustered at the industry level. $* = 10\%$ level, $** = 5\%$ level, and $*** = 1\%$ level. Forecast error is average realized minus expected profits normalized by capital stocks. Investment is the average investment rate, i.e., capital expenditures normalized by the capital stock. The return is the average realized bond return. Columns (2)-(3) control for the lagged spread and current profits relative to the capital stock. For all series, $0.01 = 1\%$.

Table B.8: Linking Forecast Errors and Industry Reversals in the Data: No Controls

	(1) Forecast Error_t	(2) Return_t	(3) $\Delta\text{Investment}_t$
IV Stage:	First	Second	Second
Forecast Error_t		0.047^{***}	1.266^{***}
		(0.016)	(0.408)
Investment_{t-1}	-0.476^{***}		
	(0.153)		
Industry Effects	X	X	X
Industries	111	111	111
Industry-Years	1291	1291	1291
First Stage F	10		

Notes: The table reports first- and second-stage IV estimates based on industry aggregated data from the Compustat- IBES-FISD/TRACE sample at the SIC3 $\times$ fiscal year level. Column (1) reports the first stage, and columns (2)-(3) report second-stage regressions. Standard errors are clustered at the industry level. $* = 10\%$ level, $** = 5\%$ level, and $*** = 1\%$ level. Forecast error is average realized minus expected profits normalized by capital stocks. Investment is the average investment rate, i.e., capital expenditures normalized by the capital stock. The return is the average realized bond return. For all series, $0.01 = 1\%$.

Table B.9: Linking Forecast Errors and Industry Reversals in the Data: No Industry Effects

	(1) Forecast Error_t	(2) Return_t	(3) $\Delta\text{Investment}_t$
IV Stage:	First	Second	Second
Forecast Error_t		0.009^{***}	0.242^{***}
		(0.003)	(0.025)
Investment_{t-1}	-0.956^{***}		
	(0.141)		
Industries	111	111	111
Industry-Years	1291	1291	1291
First Stage F	46		

Notes: The table reports first- and second-stage IV estimates based on industry aggregated data from the Compustat- IBES-FISD/TRACE sample at the SIC3 $\times$ fiscal year level. Column (1) reports the first stage, and columns (2)-(3) report second-stage regressions. Standard errors are clustered at the industry level. $* = 10\%$ level, $** = 5\%$ level, and $*** = 1\%$ level. Forecast error is average realized minus expected profits normalized by capital stocks. Investment is the average investment rate, i.e., capital expenditures normalized by the capital stock. The return is the average realized bond return. Columns (2)-(3) control for the lagged spread and current profits relative to the capital stock. For all series, $0.01 = 1\%$.