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Appendix A - Theoretical Details and Proofs

In this appendix we give more technical details and proofs of our main results. We begin with a full statement of the equilibrium concept for the closed economy, as well as a characterization of the closed-economy steady-state growth rate referred to in the main text. We then state the open economy equilibrium definition, characterize steady-state open economy growth rates, and define the trapped factors trade shock equilibrium.

Definition 1. *Closed-Economy Equilibrium*

Given initial conditions A_0, x_{j0} , an equilibrium is a path of wages, interest rates, stock prices, and intermediate goods prices w_t, r_t, q_{ft}, p_{jt} , together with stock portfolio decisions, debt levels, final goods firm input demands, intermediate goods firms input demands, intermediate goods firm innovation quantities, intermediate goods dividends, aggregate innovation quantities, firm variety portfolios, and aggregate variety quantities s_{ft}, b_t, H_t^D, x_{jt}^D, x_{jt+1}^S, M_{ft+1}, d_{ft}, A_t, A_{ft}, M_t, such that

Households Optimize: Taking wages w_t, interest rates r_t, and stock prices q_{ft} as given, the representative household maximizes the present discounted value of its consumption stream by choosing period consumption C_t, debt b_{t+1}, and share purchases s_{ft}, i.e. these decisions solve

$$\max_{C_t, b_{t+1}, s_{jt}} \sum_{t=0}^{\infty} \frac{\beta^t C_t^{1-\sigma}}{1-\sigma}$$

$$b_{t+1} + C_t + \sum_{f=1}^{N} q_{ft}(s_{ft} - s_{ft-1}) \leq (1 + r_{t+1})b_t + w_t H + \sum_{f=1}^{N} d_{ft} s_{ft}.$$

Final Goods Firm Optimizes: Taking wages w_t and intermediate goods prices p_{jt} as given, the competitive representative final goods firm statically optimizes profits by choosing labor demand H_t^D and intermediate goods input demands x_{jt}^D, i.e. these decisions solve

$$\max_{H_t, x_{kt}} (H_t)^{\alpha} \int_0^{A_t} (x_{jt})^{1-\alpha} dj - w_t H_t - \int_0^{A_t} p_{jt} x_{jt} dj.$$

Intermediate Goods Firms Optimize: Taking marginal utilities m_t , perfectly competitive off-patent intermediate goods prices p_{jt}, $j \leq A_{t-1}$, and aggregate variety and innovation levels A_t, M_{t+1} as given, intermediate goods firms maximize firm value, the discounted stream of dividends, by choosing the measure of newly innovated goods M_{ft+1} to add to the existing measure of varieties A_{ft} in their portfolios, the supply of all intermediate goods for use next period x_{jt+1}^S, and the price of on-patent intermediate goods $p_{jt}, j \in (A_{t-1}, A_t]$, i.e. these quantities solve

$$\max_{p_{jt}, M_{ft+1}, x_{jt+1}} \sum_{t=0}^{\infty} m_t d_{ft}$$

$$d_{ft} + \int_{A_{ft+1}} x_{jt+1} dj + Z_{ft} \leq \int_{A_{ft}} p_{jt} x_{jt} dj$$

$$Z_{ft} = \nu M_{ft+1}^{\gamma} A_t^{1-\gamma}, \quad \nu = \frac{N^{\gamma-1}}{\gamma}$$

Labor, Bond, Stock, and Intermediate Goods Markets Clear:

$$H_t^D = H,\ b_{t+1} = 0,\ s_{ft} = 1,\ x_{jt+1}^D = x_{jt+1}^S$$

Final Goods Market Clears:

$$Y_t = C_t + \int_0^{A_{t+1}} x_{jt+1} dj + \sum_{f=1}^{N} Z_{ft}$$

Innovation and Variety Consistency Conditions Hold:

$$A_{t+1} = A_t + M_{t+1},\ A_{ft+1} = A_{ft} + M_{ft+1},\ M_{t+1} = \sum_{f=1}^{N} M_{ft+1},\ A_t = \sum_{f=1}^{N} A_{ft}.$$

Proof of Proposition 1 To complete the proof of Proposition 1, we need to show that the rates of growth of output, consumption, and varieties are equal on a steady-state growth path. First, note that the first-order conditions of the intermediate goods firm monopoly pricing decision immediately yield

$$p_{Mt+1} = \frac{1 + r_{t+1}}{1 - \alpha},$$

i.e. they imply that the monopoly price in any future period $t+1$ is a fixed markup over firm marginal cost. Marginal cost is given here by the interest rate r_{t+1} from the current period t into the next period $t+1$. The household Euler equation immediately implies the interest rate $r_{t+1} = \frac{1}{\beta}(\frac{C_{t+1}}{C_t})^{\sigma}$. We then immediately obtain the optimal intermediate goods firm pricing rule $p_{Mt+1} = \frac{1}{\beta}(\frac{C_{t+1}}{C_t})^{\sigma}\frac{1}{1-\alpha}$. For later reference, note that the pricing of off-patent varieties, which we will label R goods, is given by $p_{R+1} = 1 + r_{t+1} = \frac{1}{\beta}(\frac{C_{t+1}}{C_t})^{\sigma}$ via perfect competition and the household Euler equation.

Now write the final goods market clearing condition

$$C_t = H^{\alpha}\left[M_t x_{Mt}^{1-\alpha} + R_t x_{Rt}^{1-a}\right] - M_{t+1} x_{Mt+1} - R_{t+1} x_{Rt+1} - \sum_{f=1}^{N} Z_{ft},$$

where we are using the notation that the measure of off-patent varieties is given by R_t and equal to $R_t = A_{t-1}$, and the measure of innovated varieties $M_t = gA_{t-1}$. Now, recall the assumption of steady-state growth. If we define the growth rate of consumption by g_C, and note that the by symmetry the individual firm patenting ratios $g^f = \frac{g}{n}$, we can use the intermediate goods firm pricing rules to rewrite the final goods market clearing condition as

$$\frac{C_t}{A_t} = \frac{1}{1+g} H \left[(1-\alpha)^{\frac{1-\alpha}{\alpha}} \left((1-\alpha)^{\frac{1-\alpha}{\alpha}} + 1 \right) \beta^{\frac{1-\alpha}{\alpha}} (1+g_C)^{-\frac{\sigma}{\alpha}} \right] - g(1-\alpha)^{\frac{2}{\alpha}} \beta^{\frac{1}{\alpha}} (1+g_C)^{-\frac{\sigma}{\alpha}} H$$
$$- (1-\alpha)^{\frac{1}{\alpha}} \beta^{\frac{1}{\alpha}} (1+g_C)^{-\frac{\sigma}{\alpha}} H - N\nu \left(\frac{g}{N} \right)^{\gamma}.$$

Since $\frac{C_t}{A_t}$ is constant, we conclude that $g = g_C$, so that the innovation optimality condition, i.e. the first-order condition of an intermediate goods firm with respect to R&D expenditures, reads

$$\frac{\nu\gamma}{N^{(\gamma-1)}} g^{\gamma-1} = \Omega \beta^{\frac{1}{\alpha}} (1+g)^{-\frac{\sigma}{\alpha}} H.$$

This expression motivates the choice of the scaling constant

$$\nu = \frac{N^{(\gamma-1)}}{\gamma},$$

so that the steady-state growth path growth rates are invariant to the number of firms or the degree of cost externalities across firms as well as the number of firms N . We obtain the steady-state growth path innovation optimality condition

$$g^{\gamma-1} = \Omega \beta^{\frac{1}{\alpha}} (1+g)^{-\frac{\sigma}{\alpha}} H.$$

The left-hand side, the marginal cost of innovation, is strictly increasing in g, is equal to 0 when $g = 0$, and limits to ∞ as $g \to \infty$. The right-hand side, the discounted monopoly profits from innovation, is strictly decreasing in g, is equal to $\Omega \beta^{\frac{1}{\alpha}} H > 0$ when $g = 0$, and limits to 0 as $g \to \infty$. We conclude that a steady-state growth path equilibrium exists and is uniquely determined by the value of g which satisfies the innovation optimality condition. This completes the proof.

Definition 2. *Open-Economy Equilibrium*

Given any initial conditions A_0, x_{j0}, x^*_{j0} , along with a sequence of trade restrictions ϕ_t , an equilibrium in the open economy is a set of terms of trade, interest rates, wages, stock prices, and intermediate goods prices q_t, r_t, r^*_t, w_t, w^*_t, q_{ft}, q^*_{ft}, p_{jt}, and p^*_{jt} , along with stock portfolio decisions, debt levels, final goods firm input demands, intermediate goods firms input demands, intermediate goods firm innovation quantities, intermediate goods firm portfolios, intermediate goods dividends, aggregate innovation quantities, imported variety measures, restricted variety measures, and aggregate variety quantities $s_{ft}, s^*_{ft}, b_{t+1}, b^*_{t+1}, H^D_t, H^{*D}_t, x^D_{jt}, x^{*D}_{jt}, x^S_{jt+1}, x^{*S}_{jt+1}, M_{ft+1}$, $A_{jt}, A^*_{ft}, d_{ft}, d^*_{ft}$, M_t, I_t, R_t, and A_t such that

Northern Household Optimizes: Taking wages w_t, interest rates r_t, and stock prices q_{ft} as given, the representative household in the North maximizes the present discounted value of its consumption stream by choosing period consumption C_t, debt b_{t+1}, and share purchases s_{ft}, i.e.

these decisions solve

$$\max_{C_t, b_{t+1}, s_{jt}} \sum_{t=0}^{\infty} \frac{\beta^t C_t^{1-\sigma}}{1-\sigma}$$

$$b_{t+1} + C_t + \sum_{f=1}^{N} q_{ft} \left(s_{ft} - s_{ft-1}\right) \leq (1 + r_{t+1}) b_t + w_t H + \sum_{f=1}^{N} d_{ft} s_{ft} .$$

Southern Household Optimizes: Taking wages w_t^*, interest rates r_t^*, and stock prices q_{ft}^* as given, the representative household in the South maximizes the present discounted value of its consumption stream by choosing period consumption C_t^*, debt b_{t+1}^*, and share purchases s_{ft}^*, i.e. these decisions solve

$$\max_{C_t^*, b_{t+1}^*, s_{ft}^*} \sum_{t=0}^{\infty} \frac{\beta^t \left(C_t^*\right)^{1-\sigma}}{1-\sigma}$$

$$b_{t+1}^* + C_t^* + \sum_{f=1}^{N} q_{ft}^* (s_{ft}^* - s_{ft-1}^*) \leq (1 + r_{t+1}^*) b_t^* + w_t^* H^* + \sum_{f=1}^{N} d_{ft}^* s_{ft}^*.$$

Northern Final Goods Firm Optimizes: Taking wages w_t and intermediate goods prices p_{jt} as given, the competitive representative final goods firm in the North statically optimizes profits by choosing labor demand H_t^D and intermediate goods input demands x_{jt}^D, i.e. these decisions solve

$$\max_{H_t, x_{jt}} (H_t)^{\alpha} \int_0^{A_t} (x_{jt})^{1-\alpha} dj - w_t H_t - \int_0^{A_t} p_{jt} x_{jt} dj.$$

Southern Final Goods Firm Optimizes: Taking wages w_t^* and intermediate goods prices p_{jt}^* as given, the competitive representative final goods firm in the South statically optimizes profits by choosing labor demand H_t^{*D} and intermediate goods input demands x_{jt}^{D*}, i.e. these decisions solve

$$\max_{H_t^*, x_{jt}^*} (H_t^*)^{\alpha} \int_0^{A_t} \left(x_{jt}^*\right)^{1-\alpha} dj - w_t^* H_t^* - \int_0^{A_t} p_{jt}^* x_{jt}^* dj .$$

Northern Intermediate Goods Firm Optimizes: Taking marginal utilities m_t, perfectly competitive off-patent intermediate goods prices $p_{jt}, j \leq A_{t-1}$, and aggregate variety, trade, and innovation levels A_t, R_t, and M_{t+1} as given, intermediate goods firms f in the North maximize firm value, the discounted stream of dividends, by choosing the measure of newly innovated goods M_{ft+1} to add to the existing measure of varieties A_{ft} in their portfolios, the supply of all intermediate goods in their portfolio for use next period x_{jt+1}^S, x_{jt+1}^{*S}, and the price of on-patent intermediate goods p_{jt}, $j \in (A_{t-1}, A_t]$, i.e. these quantities solve

$$\max_{p_{jt}, M_{ft+1}, x_{jt+1}, x_{jt+1}^*} \sum_{t=0}^{\infty} m_t d_{ft}$$

$$d_{ft} + \int\limits_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj + Z_{ft} \leq \int\limits_{A_{ft}} p_{jt}\, (x_{jt} + x^*_{jt})dj$$

$$Z_{ft} = \nu M^{\gamma}_{ft+1} A^{1-\gamma}_t.$$

Southern Intermediate Goods Firm Optimizes: Taking marginal utilities m^*_t and perfectly competitive off-patent intermediate goods prices p^*_{jt}, $j \leq A_{t-1}$ as given, intermediate goods firms f in the South maximize firm value, the discounted stream of dividends, by choosing the supply of all intermediate goods in their portfolios A^*_{ft} for use next period x^S_{jt+1}, x^{*S}_{jt+1}, i.e. these quantities solve

$$\max_{M_{ft+1}, x_{jt+1}, x^*_{jt+1}} \sum_{t=0}^{\infty} m^*_t\, d_{ft}$$

$$d^*_{ft} + \int\limits_{A^*_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj \leq \int\limits_{A^*_{ft}} p^*_{jt}(x_{jt} + x^*_{jt})dj.$$

Labor, Bond, Stock, and Intermediate Goods Markets Clear

$$H^D_t = H,\ H^{*D}_t = H^*,$$
$$b_{t+1} = 0,\ b^*_{t+1} = 0,$$
$$s_{ft} = 1,\ s^*_{ft} = 1,$$
$$x^D_{jt} = x^S_{jt},\ x^{*D}_{jt} = x^{*S}_{jt}.$$

Final Goods Markets Clear

$$Y_t = H^{\alpha} \int x^{1-\alpha}_{jt} dj = C_t + R_{t+1} x_{Rt+1} + M_{t+1}(x_{Mt+1} + x^*_{Mt+1}) + \sum_{f=1}^{N} Z_{ft}$$

$$Y^*_t = (H^*)^{\alpha} \int_0^{A_t} \left(x^*_{jt}\right)^{1-\alpha} dj = C^*_t + R_{t+1} x^*_{Rt+1} + I_{t+1}(x_{It+1} + x^*_{It+1})$$

No Arbitrage Pricing Condition Holds

$$p_{jt} = q_t p^*_{jt}$$

Trade is Balanced

$$I_t p_{It} x_{It} = M_t p_{Mt} x^*_{Mt}$$

Innovation and Variety Consistency Conditions Hold:

$$\phi_t\, (R_t + I_t) = I_t\, , I_t + R_t = A_{t-1}\, , I_t + R_t + M_t = A_t\, ,$$

$$A_{ft+1}= A_{ft}+M_{ft+1},\ M_t=\sum_{f=1}^{N} M_{ft},\ M_t+R_t=\sum_{f=1}^{N} A_{ft},\ I_t+R_t=\sum_{f=1}^{N} A^*_{ft}.$$

Southern Cost Advantage Condition Holds: Off-restriction goods are always produced in the Southern economy only.

Although the Fully Mobile economy with a trade shock has essentially the same equilibrium concept as laid out in the previous section initially discussing the open economy, we must be more explicit about the Trapped Factors environment. In the Trapped Factors equilibrium, Northern intermediate goods firms face an additional constraint due to the adjustment costs preventing them from immediately responding in their input usage to the new trade shock. Formally, they must solve the modified problem

$$\max_{p_{ft}, M_{ft+1}, x_{jt+1}, x^*_{jt+1}, X_{ft}} \sum_{t=0}^{\infty} m_t d_{ft}$$

$$d_{ft} + \int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj + Z_{ft} \leq \int_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt}) dj$$

$$\int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj + Z_{ft} \geq X_{ft} \left(\phi^E_{t,t+1}\right),$$

where $X_{ft}\left(\phi^E_{t,t+1}\right)$ is the optimal input demand for period t , given expectations of the trade restriction $\phi^E_{t,t+1}$ for the next period. X_{ft} is also indexed by f and depends both upon the number of M goods that the firm plans to produce for next period, as well as the number of R goods that the firm has in its portfolio and plans to produce for the next period. Therefore, although these portfolio shares are only allocative in a period in which a trade shock occurs, we must be explicit about the structure we assume for the pre-shock portfolios of R goods held by each firm f , as well as the actual allocation of the trade shock liberalization among existing firms' measures of R goods. We now define some additional notation. Let $\widetilde{s}_f$ be the share of off-patent R goods production firm f anticipates doing before the trade shock, where $\sum_{f=1}^{N} \widetilde{s}_f = 1$. Then, let the trade shock allocate destruction of R goods production opportunities across firms so that only the proportion χ_f of R goods varieties can still be produced in each firm. As long as we have the consistency condition

$$\sum_{f=1}^{N} \widetilde{s}_f \chi_f \left(1-\phi\right) A_t = (1-\phi') A_t,$$

an arbitrary choice of χ_f will be consistent with the trade shock $\phi \to \phi'$. We will henceforth make the assumption that $\widetilde{s}_f = \frac{1}{N}$ for all firms, i.e. that pre-shock allocations of R goods production is uniform across firms. This assumption grows naturally out of our structure in which we assume that firms continue to be the producers of goods which they invented, even after these goods fall off-patent and become perfectly competitive. We also will now assume that N is even, and that half of the firms in the economy are in the "No Shock" industry, industry 1. The other half of firms in the economy, those in the "Shocked" industry 2, experience a loss of R goods production opportunities during the trade shock with only a fixed proportion χ_2 of R goods remaining. This framework is a rough approximation of the heterogeneity in the direct effects on firms in developed countries during the trade liberalizations of the early 2000s. Seen in this light, industries such as textiles which experienced a substantial loss of protection against manufacturers in low-wage economies such as China, can be identified with industry 2, while other industries would be represented by firms in group 1 in our environment. We now define a trapped factors equilibrium formally.

Definition 3. *Trapped Factors Trade Shock Equilibrium*

Given any initial conditions A_0, x_{j0}, x^*_{j0} and a sequence of trade restrictions

$$\phi_s = \begin{cases} \phi, s \leq t, \\ \phi', s > t \end{cases},$$

where the trade shift from ϕ to $\phi' > \phi$ is unanticipated and affects only Shocked industry 2, leaving the proportion χ_2 of R goods in industry 2 restricted, a Trapped Factors equilibrium in the open economy is a set of terms of trade, interest rates, wages, stock prices, and intermediate goods prices q_t, r_t, r^*_t, w_t, w^*_t, q_{ft}, q^*_{ft}, p_{jt}, and p^*_{jt}, along with stock portfolio decisions, debt levels, final goods firm input demands, intermediate goods firms input demands, intermediate goods firm innovation quantities, intermediate goods firm portfolios, intermediate goods dividends, aggregate innovation quantities, imported variety measures, restricted variety measures, and aggregate variety quantities $s_{ft}, s^*_{ft}, b_{t+1}, b^*_{t+1}, H^D_t, H^{*D}_t, x^D_{jt}, x^{*D}_{jt}, x^S_{jt+1}, x^{*S}_{jt+1}$, $M_{ft+1}, A_{ft}, A^*_{ft}, d_{ft}, d^*_{ft}, M_t, I_t, R_t$, and A_t such that the following hold.

Northern Household Optimizes: Taking wages w_t, interest rates r_t, and stock prices q_{ft} as given, the representative household in the North maximizes the present discounted value of its consumption stream by choosing period consumption C_t, debt b_{t+1}, and share purchases s_{ft}, i.e. these decisions solve

$$\max_{C_t, b_{t+1}, s_{ft}} \sum_{t=0}^{\infty} \frac{\beta^t C_t^{1-\sigma}}{1-\sigma}$$

$$b_{t+1} + C_t + \sum_{f=1}^{N} q_{ft}(s_{ft} - s_{ft-1}) \leq (1 + r_{t+1})b_t + w_t H + \sum_{f=1}^{N} d_{ft} s_{ft}.$$

Southern Household Optimizes: Taking wages w^*_t, interest rates r^*_t, and stock prices q^*_{ft} as given, the representative household in the South maximizes the present discounted value of its consumption stream by choosing period consumption C^*_t, debt b^*_{t+1}, and share purchases s^*_{ft}, i.e. these decisions solve

$$\max_{C^*_t, b^*_{t+1}, s^*_{ft}} \sum_{t=0}^{\infty} \frac{\beta^t \left(C^*_t\right)^{1-\sigma}}{1-\sigma}$$

$$b^*_{t+1} + C^*_t + \sum_{f=1}^{N} q^*_{ft}(s^*_{ft} - s^*_{ft-1}) \leq (1 + r^*_{t+1})b^*_t + w^*_t H^* + \sum_{f=1}^{N} d^*_{ft} s^*_{ft}.$$

Northern Final Goods Firm Optimizes: Taking wages w_t and intermediate goods prices p_{jt} as given, the competitive representative final goods firm in the North statically optimizes profits by choosing labor demand H^D_t and intermediate goods input demands x^D_{jt}, i.e. these decisions solve

$$\max_{H_t, x_{jt}} (H_t)^{\alpha} \int_0^{A_t} (x_{jt})^{1-\alpha} dj - w_t H_t - \int_0^{A_t} p_{jt} x_{jt} dj.$$

Southern Final Goods Firm Optimizes: Taking wages w^*_t and intermediate goods prices p^*_{jt} as given, the competitive representative final goods firm in the South statically optimizes profits by choosing labor demand H_t^{*D} and intermediate goods input demands x_{jt}^{D*}, i.e. these decisions solve

$$\max_{H_t^*, x_{jt}^*} (H_t^*)^\alpha \int_0^{A_t} \left(x_{jt}^*\right)^{1-\alpha} dj - w_t^* H_t^* - \int_0^{A_t} p_{jt}^* x_{jt}^* dj.$$

Northern Intermediate Goods Firm Optimizes: Taking marginal utilities m_t, perfectly competitive off-patent intermediate goods prices p_{jt}, $j \leq A_{t-1}$, and aggregate variety, trade, and innovation levels A_t, R_t, M_{t+1} as given intermediate goods firms in the North maximize firm value, the discounted stream of dividends, by first choosing the quantity of inputs $X_{ft}\left(\phi_{t,t+1}^E\right)$ given their expectations of trade policy next period, then choosing the measure of newly innovated goods M_{ft+1} to add to the existing measure of varieties A_{ft} in their portfolios, the supply of all intermediate goods in their portfolio for use next period x_{jt+1}^S, x_{jt+1}^{*S}, and the price of on-patent intermediate goods p_{jt}, $j \in (A_{t-1}, A_t]$, i.e. these quantities solve

$$\max_{p_{jt}, M_{ft+1}, x_{jt+1}, x_{jt+1}^*, X_{ft}} \sum_{t=0}^{\infty} m_t d_{ft}$$

$$d_{ft} + \int_{A_{ft+1}} (x_{jt+1} + x_{jt+1}^*) dj + Z_{ft} \leq \int_{A_{ft}} p_{jt}(x_{jt} + x_{jt}^*) dj$$

$$\int_{A_{ft+1}} (x_{jt+1} + x_{jt+1}^*) dj + Z_{ft} \geq X_{ft}\left(\phi_{t,t+1}^E\right)$$

$$Z_{ft} = \nu M_{ft+1}^{\gamma} A_t^{1-\gamma}$$

where we have that

$$\phi_{s,s+1}^E = \begin{cases} \phi, & s \leq t \\ \phi', & s > t \end{cases}.$$

Southern Intermediate Goods Firm Optimizes: Taking marginal utilities m_t^* and perfectly competitive off-patent intermediate goods prices $p_{jt}^*, j \leq A_{t-1}$ as given, intermediate goods firms in the South maximize firm value, the discounted stream of dividends, by choosing the supply of all intermediate goods in their portfolios A_{ft}^* for use next period x_{jt+1}^S, x_{jt+1}^{*S}, i.e. these quantities solve

$$\max_{M_{ft+1}, x_{jt+1}, x_{jt+1}^*} \sum_{t=0}^{\infty} m_t^* d_{ft}$$

$$d_{ft}^* + \int_{A_{ft+1}} (x_{jt+1} + x_{jt+1}^*) dj \leq \int_{A_{ft}} p_{jt}^*(x_{jt} + x_{jt}^*) dj.$$

Labor, Bond, Stock, and Intermediate Goods Markets Clear

$$H_t^D = H,\ H_t^{*D} = H^*,$$
$$b_{t+1} = 0,\ b_{t+1}^* = 0,$$
$$s_{ft} = 1,\ s_{ft}^* = 1,$$
$$x_{jt}^D = x_{jt}^S,\ x_{jt}^{*D} = x_{jt}^{*S}.$$

Final Goods Markets Clear:

$$Y_t = H^\alpha \int x_{jt}^{1-\alpha} dj = C_t + \int_{R_{t+1}} x_{jt+1} dj + \int_{M_{t+1}} (x_{jt+1} + x_{jt+1}^*) dj + \sum_{f=1}^{N} Z_{ft}$$

$$Y_t^* = (H^*)^\alpha \int_0^{A_t} \left(x_{jt}^*\right)^{1-\alpha} dj = C_t^* + \int_{R_{t+1}} x_{jt+1}^* dj + \int_{I_{t+1}} (x_{jt+1} + x_{jt+1}^*) dj$$

No Arbitrage Pricing Condition Holds

$$p_{jt} = q_t p_{jt}^*$$

Trade is Balanced

$$I_t p_{It} x_{It} = M_t p_{Mt} x_{Mt}^*$$

Innovation and Variety Consistency Conditions Hold:

$$\phi_t (R_t + I_t) = I_t,\ I_t + R_t = A_{t-1},\ I_t + R_t + M_t = A_t,$$

$$A_{ft+1} = A_{ft} + M_{ft+1},\ M_t = \sum_{f=1}^{N} M_{ft},\ M_t + R_t = \sum_{f=1}^{N} A_{ft},\ I_t + R_t = \sum_{f=1}^{N} A_{ft}^*.$$

Southern Cost Advantage Condition Holds: Off-restriction goods are always produced in the Southern economy only.

Proof of Proposition 2: Open Economy Steady-State Growth Path The demand schedules for intermediate goods, based on the Northern and Southern final goods firms' technologies, are given by

$$x_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H p_{jt}^{-\frac{1}{\alpha}}$$
$$x_{jt}^* = (1-\alpha)^{\frac{1}{\alpha}} H^* \left(p_{jt}^*\right)^{-\frac{1}{\alpha}},$$

where p_{jt} and p_{jt}^* are the prices of intermediate good variety j in Northern and Southern output units, respectively, and $p_{jt} = q_t p_{jt}^*$. The optimality conditions for the Northern intermediate goods firm, combined with the Euler equations of the Northern representative household for debt and equity, are given by

$$
\begin{aligned}
p_{Rt+1} &= 1 + r_{t+1} \\
p_{Mt+1} &= \frac{1 + r_{t+1}}{1-\alpha} \\
\frac{\partial}{\partial M_{ft+1}} Z_{ft+1} &= \left(\frac{1}{1+r_{t+1}} p_{Mt+1} - 1 \right) (x_{Mt+1} + x^*_{Mt+1}).
\end{aligned}
$$

Differentiating the cost function and substituting in the optimal pricing rules we have that the third condition, the innovation optimality condition, is given by

$$\nu\gamma(g^f_{t+1})^{(\gamma-1)} = \Omega\beta^{\frac{1}{a}}(\frac{C_{t+1}}{C_t})^{-\frac{\sigma}{\alpha}}(H + q^{\frac{1}{\alpha}}_{t+1}H^*).$$

Now the balanced trade condition can be written

$$
\begin{aligned}
M_t p_{Mt} x^*_{Mt} &= I_t p_{It} x_{It} \\
g_t A_{t-1}\frac{(1+r_t)}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} H^* \left(\frac{(1+r_t)}{q_t(1-\alpha)} \right)^{-\frac{1}{\alpha}} &= \phi A_{t-1} q_t (1+r^*_t)(1-\alpha)^{\frac{1}{\alpha}} \left(q_t(1-\alpha)\right)^{-\frac{1}{\alpha}} H \\
q_t &= \left(\frac{\phi H}{g_t H^*} \right)^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_t}{1+r^*_t} \right)^{\frac{1-\alpha}{2-\alpha}},
\end{aligned}
$$

where $\Psi = (1-\alpha)^{\frac{\alpha-1}{2-\alpha}}$. Applying the assumption of steady-state growth, we immediately obtain from the Euler equations of both representative households that interest rates in the Northern and Southern economies, as well as the terms of trade, are constant. Also, exactly as in the proof of Proposition $A1$, the final goods market clearing conditions for each economy, together with the assumption of steady-state growth, imply that the ratios

$$\frac{C_t}{A_t}, \frac{C^*_t}{A_t}$$

are constant, so that we conclude that

$$(1+r) = (1+r^*) = \beta^{-1}(1+g)^{\sigma}, \quad q = \left(\frac{\phi H}{g H^*} \right)^{\frac{\alpha}{2-\alpha}} \Psi.$$

Now the innovation optimality condition can be rewritten as

$$g^{\gamma-1} = \Omega\beta^{\frac{1}{a}}(1+g)^{-\frac{\sigma}{\alpha}}(H + q^{\frac{1}{\alpha}}H^*).$$

Also, substituting the terms of trade formula/balanced trade condition into the innovation optimality condition yields

$$g^{\gamma-1}= \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}\left(H+\left(\frac{\phi H}{gH^{*}}\right)^{\frac{1}{2-\alpha}}\Psi^{\frac{1}{\alpha}}H^{*}\right).$$

As a function of g, the marginal cost of innovation on the left-hand side is strictly increasing in g, starting at 0 and growing exponentially to ∞ as $g\to\infty$. The right-hand side, the discounted monopoly profits from sale of newly patented goods in the North and the South, is strictly decreasing in g, asymptoting to ∞ as $g\to 0$ and to 0 as $g\to\infty$. We conclude both that there exists a steady-state growth path equilibrium for this economy, and that it is the unique steady-state growth path growth rate. For any given fixed value of ϕ, we denote this growth rate, and the associated terms of trade, by $g(\phi)$ and $q(\phi)$. This completes the proof.

Appendix B - Data and Model Robustness Checks

This Appendix first describes the firm-level data sources and variable construction used in for the production of Table 1 in Section 2. Then, we describe the aggregate data sources used to calibrate our model and for various figures throughout the paper. We conclude by listing the empirical strategy behind some alternative trade policy calibration exercises used in this paper and previous versions of the project.

Innovation & Trade Data

The empirical analysis in Table 1 in Section 2 of this paper draws from a dataset built from 4 distinct sources. The first source is Bureau van Dijk's Amadeus database, containing firm-fiscal year level accounting statements of public and private firms in European nations. Work by Bloom et al. (2015) matched this dataset to microdata on patenting from the European Patent Office, the second source of data. We use two measures from this matched dataset - described further in Bloom et al. (2015) - as dependent variables in Table 1. For Panel A, we define $\ln(\text{PATENTS})_{ijkt}=\ln(1+\text{PAT}_{ijkt})$, where PAT_{ijkt} is the number of successful patent applications per worker filed by firm i in industry j in country k in year t. The dependent variable is the five-year difference of this variable. For Panel B, the dependent variable is the five-year growth rate of sales.

The third source of information we use is the UN Comtrade database, from which we extracted HS-6 digit product by year trade flows from China into each of the nations in our Amadeus sample, plus the US. We concord the HS-6 digit product flows into 4-digit SIC industry codes, which is the uniform industrial classification matched by Bloom et al. (2015) to the European firm-level data. The fourth source of information we use is industry production tables from the US and Europe. We use the US NBER-CES manufacturing database, providing information on US production by SIC 4-digit manufacturing industry and year, as well as aggregates from the European Prodcom database in the Bloom et al. (2015) database.

To pull these together, we compute base-year production Y_{jk1996} in each 4-digit SIC manufacturing industry j in 1996 in the countries k including the US and the European nations in our sample. We define $IMP^{CH}_{jkt} = \frac{M^{CH}_{jkt}}{Y_{jk1996}}$ as the ratio of imports from China into industry j in country k in year t scaled by base-year production in that industry. Then, the change in Chinese imports ΔImp^{CH}_{jkt} for industry j in country k in year t is simply the 5-year difference of this variable. In Table 1, when k includes European nations this import growth measure is the endogenous trade outcome on the right hand side, and when $k = US$ we use the variable as the instrument for our IV specifications.

We combine these source of information to obtain a firm-fiscal year dataset that after censoring outliers in the trade flows results in a sample of patent growth, employment growth, and Chinese import flows for around 25,000 firm-fiscal years for around 7,000 firms in 235 manufacturing industries based on data from 1996-2005 spanning the eleven European nations Austria, Germany, Denmark, Spain, Finland, France, Great Britain, Ireland, Italy, Norway, and Sweden.

Calculating the ratio of H to H^* for model calibration

To calculate the ratio of H to H^*, we follow the human capital accounting approach in Hall (2009) and compute the human capital endowment in country c from the Barro and Lee (2013) data as $H_c = e^{\mu_c S_c} P_c$, where S_c is the average number of years of schooling completed in the adult population above age 25, and P_c is the size of the population of the country c in 2000. We take into account the differences in educational quality and the returns to schooling across countries by using the Mincerian returns to education of immigrants in the United States from country c , μ_c , from Table 4 in Schoellman (2012). If Mincerian returns for a country c are not available in Schoellman (2012), we take $\mu_c = 7\%$ for non-OECD countries and $\mu_c = 9\%$ for OECD countries. These are the averages of returns to schooling for the two categories in Schoellman's sample. We finally define $H_{non-OECD} = 2.1 \sum_{c \notin OECD} H_c$, where the ratio 2.1 corrects for the fact that not all non-OECD countries are represented in the Barro and Lee (2013) data. In particular 2.1 is equal to the ratio of the non-OECD to OECD population ratio in 2000 in the Wolfram Alpha database (with full global coverage) to the non-OECD to OECD population ratio in 2000 in the Barro and Lee (2013) data. Such a procedure relies on the implicit assumption that the schooling rates and returns to education in countries not represented in the Barro and Lee (2013) data are similar to those with data present. From the procedure above we obtain $\frac{H^*}{H} \approx 2.96$, which we round to 3.0 in the text discussion.

Calculating the Trade Shares for Figure 1

The real per-capita output growth rate is from the US NIPA tables, computed as the average annual real GDP per capita growth rate from 1960-2010. Trade data was downloaded from the OECD-STAN database, and OECD GDP data comes from the Penn World Tables, Version 9.0. The non-OECD country to OECD imports to OECD output ratios were computed over the years 1994-2014. All of the data and simple calculations performed in the calibration procedure are available on Nicholas Bloom's website: `http://www.stanford.edu/nbloom`. Figure 1 plots the non-OECD imports to OECD GDP ratio over this period, together with Chinese imports into the OECD.

Computing Patent Ratios for Figure B1

We downloaded United States Patent and Trademark Office (USPTO) microdata on patents granted from the mid-1970s onwards from the USPTO PatentsView website. Figure B1 plots the ratio of all patents with a foreign (non-US) assignee, non-OECD assignee, or Chinese assignee to the total number of patents granted from 1994-2014.

Trade Policy Substitution away from China

Total observed low-wage import growth into the OECD as a share of GDP from 1994-2014 is equal to 4.9%. Growth in Chinese import shares was equal to 2.5%, implying that non-China/non-OECD countries saw their import shares into the OECD increase by 2.4%. The no China counterfactual in the main text assumed that the growth in Chinese import shares was completely removed from liberalization over this period. If, however, policy-makers partially substituted towards other non-OECD imports in lieu of Chinese imports, we would still see import share growth in the counterfactual. To analyze the quantitative magnitude of this substitution effect, we consider a case where exactly one half of Chinese import growth is realized in the no China counterfactual, via substitution towards other non-OECD countries. Starting with a low-wage import share of 3.5%, this "half substitution" case exhibits import share growth of 0.5*2.5+2.4 = 3.65%, so that the resulting target import to output ratio post-liberalization in the counterfactual is 3.5+3.65 = 7.15%. Figure B2 plots the resulting two trapped factors transition paths, analogous to Figure 9, in the total observed import liberalization and "Half China" cases. As can be seen immediately, the transition paths differ by less than the case in which all Chinese import growth is removed, which works to reduce the marginal contribution of China to welfare to a total of 4.6% (North)

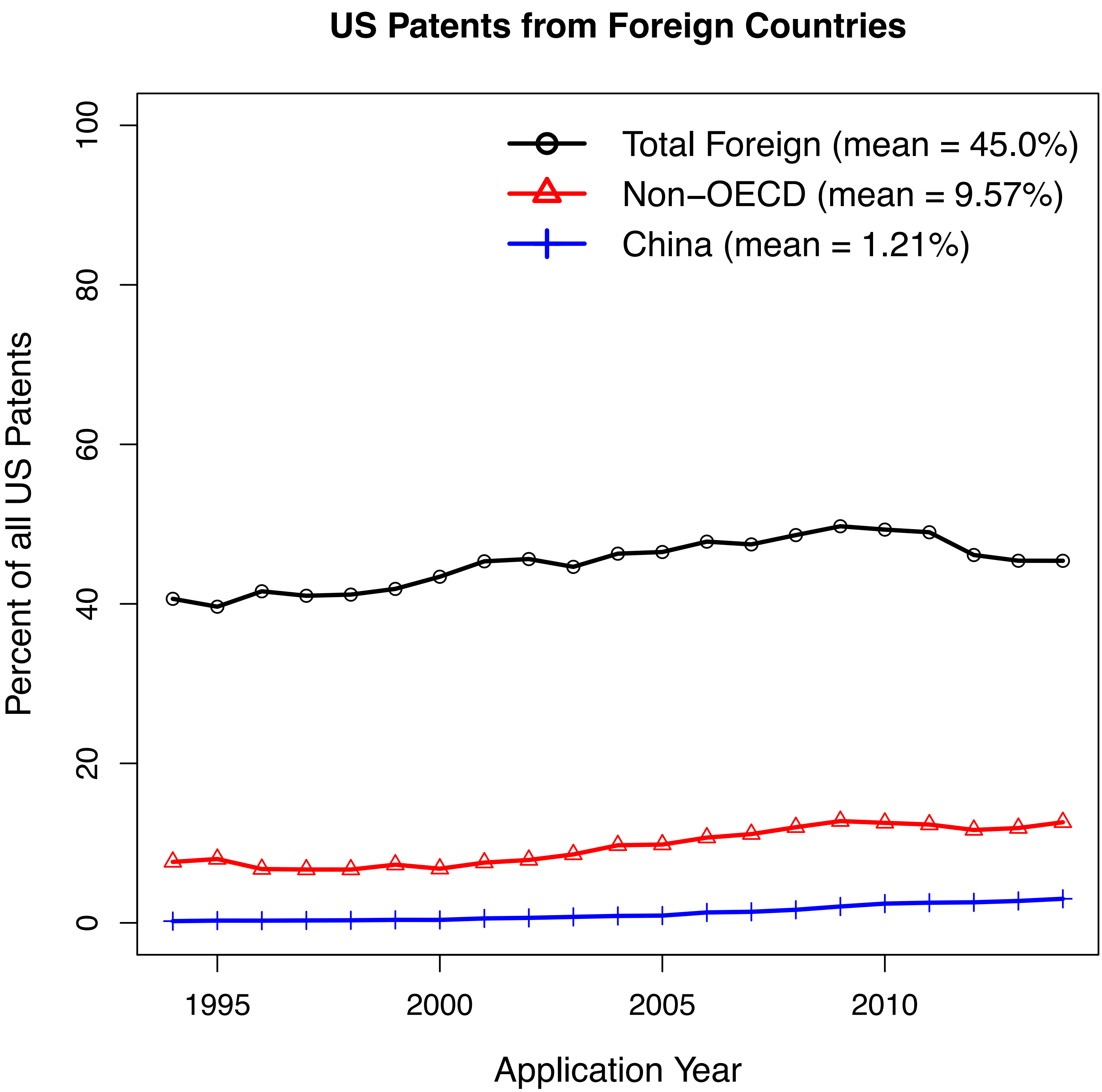

Figure B1: Non-OECD Patent Ratios are Small

Note: Patent fractions are computed from the USPTO PatentsView database. The classification of patents by assignee to the required OECD, non-OECD, and Chinese categories is done by the nationality of assignee, and a given country's OECD member status as of the application year. Each series is normalized by the total number of granted US Patent and Trademark Office applications in the same year. The reported mean ratios are computed over model calibration range 1994-2014.

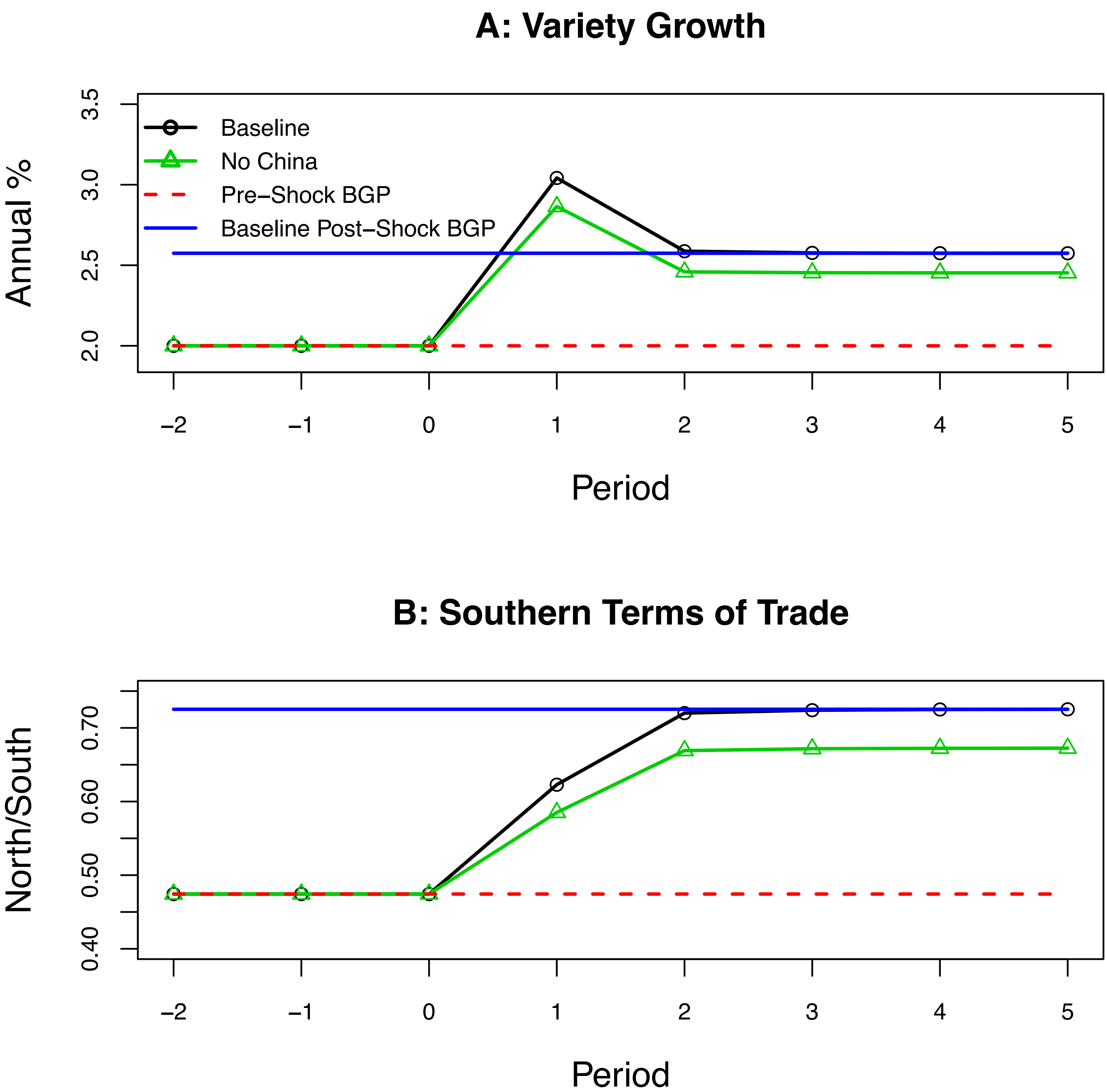

Figure B2: Trade Liberalization with Half of Chinese Import Growth

Note: The figure displays the transition path in response to trade liberalization in two scenarios. The first transition path, in solid black, "Baseline," replicates the Trapped Factors transition path displayed in Figure 6 above. A permanent and unanticipated trade liberalization from ϕ to $\phi' > \phi$ is announced in period 0 to become effective in period 1. The second transition path in green with triangle symbols, "Half China," plots the Trapped Factors transition path, starting with the same initial conditions as "Baseline" but instead considering a counterfactual increase of ϕ to a level between ϕ and ϕ' which matches post-liberalization imports to GDP ratios assuming that half the growth in Chinese imports into the OECD occurs through policy substitution to non-China, non-OECD countries. The upper horizontal solid blue line is the post-shock steady-state growth path, and the lower horizontal dashed red line is the pre-shock steady-state growth path.

and 4.5% (South). In this alternative counterfactual, the impact of China is equal to 18% (North) and 19% (South) of the overall welfare gains from trade observed in the data.

Alternative Calibration from 2013 Paper

Note that a previous version of our calibration strategy, with results published in "A Trapped Factors Model of Innovation," (*American Economic Review: Papers and Proceedings,* 2013) yielded smaller dynamic impacts of trade liberalization. Our improved calibration strategy here differs from that earlier work in four respects. First, we consider a model period of ten years rather than one year to match a more plausible effective monopoly length. Second, we base the calibration on imports to value added ratios rather than imports to gross output ratios, since data availability for China is better for value added. Third, instead of calibrating the post-liberalization trade openness via a "limiting" highest ϕ' which still maintained product-cycle trade (i.e. $q(\phi') < 1$), the first two calibration changes allow us to now directly match observed pre- and post-liberalization trade ratios, which results in larger growth impacts more aligned with observed trade liberalization. Fourth, we now have access to data on import liberalization spanning a larger number of years 1994-2014 instead of 1997-2006. In the larger time span, liberalization expanded before stabilizing, increasing the implied dynamics gains from trade.

Appendix C - Solution Technique and Equilibrium Conditions for the Calibration

Please find code for the quantitative results in the paper on Nicholas Bloom's website at `http://www.stanford.edu/nbloom/`. We solve each of the systems of nonlinear equations laid out below using particle swarm optimization as implemented in R . This is a robust global nonlinear optimization technique.

Steady-State growth Path

As documented in the proof of Proposition 2 , the steady-state growth path growth rate $g(\phi)$ of the open economy given trade restriction ϕ is fully characterized by the equilibrium innovation optimality condition

$$g(\phi)^{\gamma-1} = \Omega\beta^{\frac{1}{a}}(1+g(\phi))^{-\frac{\sigma}{\alpha}}\left(H+\left(\frac{\phi H}{g(\phi)H^*}\right)^{\frac{1}{2-\alpha}}\Psi^{\frac{1}{\alpha}}H^*\right).$$

All other long-run quantities, in particular the interest rates and exchange rate, are direct functions of this steady-state growth path growth rate through the Euler equations and balanced trade condition

$$(1+r(\phi)) = (1+r^*(\phi)) = \beta^{-1}(1+g(\phi))^{\sigma}$$

$$q(\phi) = \left(\frac{\phi H}{g(\phi)H^*}\right)^{\frac{\alpha}{2-\alpha}}\Psi.$$

Fully Mobile Transition Dynamics

To compute the transition dynamics of the fully mobile model in response to a trade shock in period 0 , starting from the steady-state growth path associated with trade restriction ϕ , we first pick a horizon T . We also normalize $A_0 = 1$. Then, we assume that the model has converged to the steady-state growth path associated with ϕ' by period T . This structure requires that we solve for $3(T-1)$ prices, $\{q_t, r_t, r_t^*\}_{t=2}^{T}$. These $3(T-1)$ prices are pinned down by $3(T-1)$ equations: the balanced trade condition, the Northern Euler equation, and the Southern Euler equation, in periods $t = 1, ..., T-1$. These equations are given by

$$q_t = \left(\frac{\phi H}{g_t H^*}\right)^{\frac{\alpha}{2-\alpha}}\Psi\left(\frac{1+r_t}{1+r_t^*}\right)^{\frac{1-\alpha}{2-\alpha}},$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta(1+r_{t+1}),$$

$$\left(\frac{C^*_{t+1}}{C^*_t}\right)^{\sigma} = \beta(1 + r^*_{t+1}).$$

We note that all allocations in the transition path are a function of these three prices. Intermediate goods prices follow the monopoly markup or competitive pricing conditions

$$p_{Mt}=\frac{1+r_t}{1-\alpha}, p_{Rt}= (1 + r_t), p_{It}= q_t(1 + r^*_t)$$

$$p^*_{Mt}= q_t^{-1}\frac{1+r_t}{1-\alpha}, p^*_{Rt}= (1 + r^*_t), p^*_{It}= (1 + r^*_t).$$

The final goods firms demand schedules then yield

$$x_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H p_{jt}^{-\frac{1}{\alpha}},$$

$$x^*_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H^* (p^*_{jt})^{-\frac{1}{\alpha}},$$

The first-order condition for innovation at Northern intermediate goods firms, together with symmetry across firms and the equilibrium price and quantity decisions laid out above, yields the innovation optimality conditions

$$g^{\gamma-1}_{t+1}= \Omega(1 + r_{t+1})^{-\frac{1}{\alpha}} \left(H + q^{\frac{1}{\alpha}}_{t+1} H^*\right),$$

which uniquely pin down the variety growth rate g_{t+1} as a function of terms of trade and interest rates. Given our characterization of g_t as a function of prices, it only remains to pin down C_t and C^*_t as a function of prices. But this is easily accomplished by noting that

$$C_t+M_{t+1}(x_{Mt+1}+x^*_{Mt+1}) + R_{t+1}x_{Rt+1}+Z_t= Y_t$$

$$Y_t= H^{\alpha}\left[M_t x^{1-\alpha}_{Mt} + R_t x^{1-\alpha}_{Rt} + I_t x^{1-\alpha}_{It}\right]$$

$$Z_t=\sum_{f=1}^{N} Z_{ft}=\frac{g^{\gamma}_{t+1}}{\gamma}A_t$$

$$C^*_t+I_{t+1}(x_{It+1}+x^*_{It+1}) + R_{t+1}x^*_{Rt+1}= Y^*_t$$

$$Y^*_t= (H^*)^{\alpha}\left[M_t(x^*_{Mt})^{1-\alpha} + R_t(x^*_{Rt})^{1-\alpha} + I_t(x^*_{It})^{1-\alpha}\right]$$

$$A_{t+1}= (1 + g_{t+1})A_t$$

$$M_{t+1}= g_t A_t$$

$$R_{t+1}= (1 - \phi_{t+1})A_t$$

$$I_{t+1}= \phi_{t+1}A_t.$$

Since all allocations in this economy are therefore a function of the $3(T-1)$ prices, we can construct the errors in $3(T-1)$ equations above given any input sequence of prices. The percentage squared errors of this system of equation are minimized using particle swarm optimization. After solving for the transition path price paths, we check to see if the cost advantage for I goods production is maintained by the South, justifying our M, R, I goods partitioning. This is equivalent to checking that, for each period

$$(1+r_t^*)q_t \leq (1+r_t).$$

In the baseline results shown in Section 5 , we choose $T=7$.

Trapped Factors Transition Dynamics

The equilibrium conditions which we must solve to compute the transition dynamics for the trapped factors model are identical to those in the fully mobile economy, for period $2,...,T-1$. There are, however, differences in the equilibrium conditions in the period of the shock. In particular, there is heterogeneity in the response of the affected and unaffected industries to the shock, and instead of solving for simply the $3(T-1)$ prices $\{q_t, r_t, r_t^*\}_{t=2}^{T}$ as in the fully mobile case, we must solve for these prices and the four additional variables $\{g_2^1, g_2^2, \mu^1, \mu^2\}$. These variables are patenting rates and shadow values of inputs within Northern firms in the unaffected industry (1) and the affected industry (2). Therefore, we must pin down $3(T-1)+4$ quantities, which we do with $3(T-1)+4$ equations:

$$q_1 = \left[\frac{\phi' H}{H\left[\left(\frac{n}{2}\right)(\mu^1)^{\frac{\alpha-1}{\alpha}} g_1^1 + \left(\frac{n}{2}\right)(\mu^2)^{\frac{\alpha-1}{\alpha}} g_1^2\right]}\right]^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_1}{1+r_1^*}\right)^{\frac{1-\alpha}{2-\alpha}}$$

$$q_t = \left(\frac{\phi' H}{g_t H^*}\right)^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_t}{1+r_t^*}\right)^{\frac{1-\alpha}{2-\alpha}}, 2, ..., T-1$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta(1+r_{t+1}), t=1,...,T-1$$

$$\left(\frac{C_{t+1}^*}{C_t^*}\right)^{\sigma} = \beta(1+r_{t+1}^*), t=1,...,T-1$$

$$(Ng_1^1)^{\gamma-1} = \Omega(1+r_1)^{-\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(H + q_1^{\frac{1}{\alpha}} H^*)$$

$$(Ng_1^2)^{\gamma-1} = \Omega(1+r_1)^{-\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(H + q_1^{\frac{1}{\alpha}} H^*)$$

$$\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}} H + \frac{1}{N}\frac{g(\phi)^{\gamma}}{\gamma} + \frac{g(\phi)}{N}(1-\alpha)^{\frac{2}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}(H + q(\phi)^{\frac{1}{\alpha}} H^*)$$

$$=\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(1+r_1)^{-\frac{1}{\alpha}}H+\frac{N^{\gamma-1}}{\gamma}\left(g_1^1\right)^{\gamma}$$

$$+g_1^1(1-\alpha)^{\frac{2}{\alpha}}(1+r_1)^{-\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(H+q_1^{\frac{1}{\alpha}}H^*)$$

$$\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}H+\frac{1}{N}\frac{g(\phi)^{\gamma}}{\gamma}+\frac{g(\phi)}{N}(1-\alpha)^{\frac{2}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}(H+q(\phi)^{\frac{1}{\alpha}}H^*)$$

$$=\frac{1}{N}\chi_2(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(1+r_1)^{-\frac{1}{\alpha}}H+\frac{N^{\gamma-1}}{\gamma}\left(g_1^2\right)^{\gamma}$$

$$+g_1^2(1-\alpha)^{\frac{2}{\alpha}}(1+r_1)^{-\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(H+q_1^{\frac{1}{\alpha}}H^*).$$

The first $3(T-1)$ equations are simply the balanced trade and Euler equations for the Northern and Southern households in periods $1,...,T-1$. The balanced trade condition must be modified in period 1 to reflect the fact that flows of M goods from North to South come from both industry 1 and industry 2, with different prices and quantities for each. The final four equations represent the innovation optimality conditions for firms in industry 1 and industry 2, as well as the trapped factors constraints for firms in each industry. The innovation optimality conditions are simply the first-order conditions of firms with respect to the mass of new varieties to be innovated in period 0 for use in period 1. Note that we are defining $\mu^1 = 1-\lambda^1$ and $\mu^2 = 1-\lambda^2$, where $m_1\lambda^1$ and $m_1\lambda^2$ are the multipliers on the trapped factors input constraints in the optimization problem for Northern intermediate goods firms in period 1. A fall in μ below 1 represents a fall in the shadow value of inputs for an intermediate goods firm. Also, if M_{f1} is the number of new patents innovated by a firm in industry f in period 0 for use in period 1, we are following the conventions $g_1^f = \frac{M_{f1}}{A_0}$, and imposing the consistency condition

$$g_1=\left(\frac{N}{2}\right)(g_1^1+g_1^2).$$

The trapped factors constraints are simply the input demands for R goods production and M goods innovation and production expenditure pre-shock (left-hand side) and post-shock (right-hand side). The input constraints differ across industries because the R goods available in the post-shock period in industry 2 for production are reduced by the factor χ_2, where χ_2 satisfies

$$\frac{1+\chi_2}{2}=\frac{1-\phi'}{1-\phi},$$

which is the consistency condition discussed in the equilibrium definition. Also, the right-hand side on the trapped factors constraints take into account the following optimal pricing rules in the period of the shock:

$$p_{M1}^1=\mu^1\frac{1+r_1}{1-\alpha}, p_{R1}^1=(1+r_1),$$

$$p_{M1}^2=\mu^2\frac{1+r_1}{1-\alpha}, p_{R1}^2=(1+r_1).$$

The demand conditions are identical to those laid out in the fully mobile section. Intermediate goods firm innovation costs on the right hand side of the trapped factors constraint are given by

$$Z_1^1 = \frac{N^{\gamma-1}}{\gamma}\left(g_1^1\right)^{\gamma}$$

$$Z_1^2 = \frac{N^{\gamma-1}}{\gamma}\left(g_1^2\right)^{\gamma},$$

which is a direct application of the definition of the innovation cost function. All of the other quantities needed for construction of the Euler equation errors and balanced trade conditions are identical to those in the fully mobile economy, with the exception of the resource constraints in the North and South in periods 0 and 1 which must be modified to read

$$Y_0 = C_0 + \left(\frac{N}{2}\right) g_1^1 A_0 (x_{M1}^1 + x_{M1}^{*1}) + \left(\frac{N}{2}\right) g_1^2 A_0 (x_{M1}^2 + x_{M1}^{*2}) + \left(\frac{N}{2}\right)\frac{1-\phi}{2} A_0 x_{R1}^1 + \left(\frac{N}{2}\right)\frac{(1-\phi)\chi_2}{2} A_0 x_{R1}^2$$

$$+ Z_1^1 + Z_1^2$$

$$Y_0^* = C_0^* + (1-\phi')A_0 x_{R1}^* + \phi' A_0 (x_{I1}^* + x_{I1})$$

$$Y_1 = H^{\alpha}\left[\left(\frac{N}{2}\right) g_1^1 A_0 (x_{M1}^1)^{1-\alpha} + \left(\frac{N}{2}\right) g_1^2 A_0 (x_{M1}^2)^{1-\alpha} + \left(\frac{N}{2}\right)\frac{1-\phi}{2} A_0 (x_{R1}^1)^{1-\alpha} + \right.$$

$$\left.\left(\frac{N}{2}\right)\frac{(1-\phi)\chi_2}{2} A_0 (x_{R1}^2)^{1-\alpha} + \phi' A_0 x_{I1}^{1-\alpha}\right]$$

$$Y_1^* = (H^*)^{\alpha}\left[\left(\frac{N}{2}\right) g_1^1 A_0 (x_{M1}^{*1})^{1-\alpha} + \left(\frac{N}{2}\right) g_1^2 A_0 (x_{M1}^{*2})^{1-\alpha} + (1-\phi')A_0 (x_{R1}^*)^{1-\alpha} + \phi' A_0 (x_{I1}^*)^{1-\alpha}\right].$$

After computing the transition path in the above manner, we must verify that $\mu^1, \mu^2 < 1$, justifying our imposition of the trapped factors inequality constraint as an equality constraint. We must also check the Southern cost dominance condition for I goods in each period, i.e.

$$\min(\mu^1, \mu^2)(1+r_1) \geq q_1(1+r_1^*),$$

$$(1+r_t) \geq q_t(1+r_t^*), t = 2, .., T-1,$$

$$q_0, q_T \leq 1.$$

Welfare Calculations

We illustrate our method of computing the consumption equivalent variation by explicitly laying out the formulas used to compute the welfare gains to trade from the fully mobile trade shock.

All other welfare calculations are similar.

$$W^{NS}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{NS}\right)^{1-\sigma}}{1-\sigma},W^{*NS}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{*NS}\right)^{1-\sigma}}{1-\sigma}$$

$$W^{FM}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{FM}\right)^{1-\sigma}}{1-\sigma},W^{*FM}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{*FM}\right)^{1-\sigma}}{1-\sigma},$$

where the consumption allocations on the fully mobile "FM" computed transition path from $0,...,T-1$ are directly computed and consumption is assumed to grow at the rate $g(\phi')$ for all economies from period T onwards. The no shock "NS" case is consumption assuming that allocations are those of the pre-shock steady-state growth path with constant growth at rate $g(\phi)$. Then, we solve for x and x^* ,

$$\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{NS}(1+x)\right)^{1-\sigma}}{1-\sigma}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{FM}\right)^{1-\sigma}}{1-\sigma},$$

$$\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{*NS}(1+x^*)\right)^{1-\sigma}}{1-\sigma}=\sum_{t=0}^{\infty}\beta^t\frac{\left(C_t^{*FM}\right)^{1-\sigma}}{1-\sigma}.$$

The welfare numbers reported in the text are $100x$ and $100x^*$.

Decomposing Output Growth

Figure 10 and the discussion in Section 4.6 in the main text introduce a decomposition of output growth into components due to various price and variety factors. For any two periods $t-1$ and t between which we want to decompose output growth, note that Northern output in each period is given by

$$Y_{t-1}=H^{\alpha}\left[M_{t-1}x_{Mt-1}^{1-\alpha}+R_{t-1}x_{Rt-1}^{1-\alpha}+I_{t-1}x_{It-1}^{1-\alpha}\right]$$

$$Y_t=H^{\alpha}\left[M_tx_{Mt}^{1-\alpha}+R_tx_{Rt}^{1-\alpha}+I_tx_{It}^{1-\alpha}\right].$$

Clearly, total output growth between the two periods is dependent on the ratio $\frac{Y_t}{Y_{t-1}}$. However, to perform the decomposition displayed in Figure 10, we simply define two intermediate values of output. The first value

$$Y_t^{RtoI}=H^{\alpha}\left[M_{t-1}x_{Mt-1}^{1-\alpha}+(R_{t-1}-I_t+I_{t-1})x_{Rt-1}^{1-\alpha}+I_tx_{It-1}^{1-\alpha}\right]$$

simply converts the mass of goods which are restricted R goods in $t-1$ into imported I goods varieties at the extensive margin which will prevail in period t. The change from Y_{t-1} to Y_t^{RtoI} is driven solely by the reduced prices and higher intensive margins on I goods in the North, although the total mass of varieties remains fixed. The second value

$$Y_t^{MtoR} = H^\alpha \left[R_t x_{Rt-1}^{1-\alpha} + I_t x_{It-1}^{1-\alpha} \right]$$

further converts the existing monopoly M varieties into off-patent R goods varieties. The change from Y_t^{RtoI} to Y_t^{MtoR} is driven solely by the lower price on pre-existing M goods in $t-1$ that are converted into lower-cost R goods and used more intensively. Clearly, the remaining difference from Y_t^{MtoR} to the observed Y_t is driven mostly by the introduction of new M goods varieties, i.e., new extensive margin effects, although the switch from intensive margins prevailing in $t-1$ to t also has a minor effect.

We now turn to a more detailed explanation of the figure itself. The bottom black area labelled "Cheaper R to I Goods" within each bar in Figure 10 reflects the ratio $\frac{Y_t^{RtoI}}{Y_{t-1}}$. The middle blue area within each bar reflects the ratio $\frac{Y_t^{MtoR}}{Y_{t-1}}$. And the bar height itself reflects the total ratio $\frac{Y_t}{Y_{t-1}}$. Each of these ratios is expressed in annualized percentage changes, i.e., we compute

$$g_{Yt}^{RtoI} = 100 \left[\left(\frac{Y_t^{RtoI}}{Y_{t-1}} \right)^{\frac{1}{10}} - 1 \right]$$

$$g_{Yt}^{MtoR} = 100 \left[\left(\frac{Y_t^{MtoR}}{Y_{t-1}} \right)^{\frac{1}{10}} - 1 \right]$$

$$g_{Yt} = 100 \left[\left(\frac{Y_t}{Y_{t-1}} \right)^{\frac{1}{10}} - 1 \right],$$

and Figure 10 plots these growth rates cumulatively in the order g_{Yt}^{RtoI}, then g_{Yt}^{MtoR}, then g_{Yt}. For the Southern economy, the growth rates are defined analogously, although the lack of a price difference for Southern final goods firms between R and I goods implies that the southern equivalent of $g_{Yt}^{RtoI} = 0$ for all periods.

Appendix D - Semi-endogenous Growth Model

In this Appendix we consider the semi-endogenous growth model approach to show that it delivers quantitatively similar results to our fully endogenous growth model. As documented in Jones (1995a,b) the implication of a model like that considered in the main text, with "strong scale effects" implying that the long-term growth rate is dependent upon the level of human capital, is rejected by the time series evidence which documents the concurrence of rising populations and researcher numbers with constant growth rates. Jones proposes a small modification to the production function for new varieties, or alternatively, to the cost function for innovation, which implies smaller returns from the existing stock of varieties in the production of new patents. This change to the model converts the structure into a semi-endogenous growth model with "weak scale effects," since the long-term growth rate is now proportional to the growth rate of human capital rather than the level of human capital. Analogously, in our context with product-cycle trade, such a modification of the model leads to long-term growth rates proportional to human capital growth rates and, crucially, independent of the trade liberalization policy ϕ. As we will see, however, a reasonable calibration of a semi-endogenous growth model consistent with the data on both per-capita growth rates and population growth displays extremely long transition dynamics and considerable temporary effects on variety growth rates from trade liberalization. Therefore, the temporary growth effects of liberalization (and the permanent level effects), imply similar results for welfare regardless of whether one considers a strong or weak scale effects model. Given that the model with strong scale effects delivers closed-form expressions for the steady-state growth path growth rates dependent upon the trade policy parameter ϕ, and given that the transition dynamics for the strong scale effects model are of a more reasonable length, we prefer to work with the strong scale effects model as our baseline version.

Model

We now lay out the model structure and equilibrium concept in the semi-endogenous growth framework, for the fully mobile environment only.

Population and Human Capital: We assume that in the North and in the South there is a continuum of identical households of measure 1 , each with an expanding set of members $[0, L_t]$ and $[0, L_t^*]$, respectively. We further assume that there is an constant level of human capital per member of the population, i.e. $H_t = hL_t$ and $H_t^* = hL_t^*$, respectively. This assumption implies that preferences of the CRRA form defined over per-capita consumption or over consumption expressed relative to human capital differ only by a constant, and for convenience we express preferences as per unit of human capital.[11]

[11] Note that we omit below a term multiplying per capita preferences by the size of the population, which would be proportional to H_t^* given our assumptions. Such an assumption, as will be seen below, results in a level shift in interest rates. However, and importantly, our assumption prevents the mechanical inflation of the welfare gains from trade liberalization (relative to our baseline strong scale effects model with no population growth) simply

Northern Households: Given a sequence of wages w_t , firm stock prices q_{ft} , firm dividends D_{ft} , and interest rates r_t , a Northern household supplies labor inelastically and chooses consumption C_t , portfolio positions S_{ft} , and bond purchases B_{t+1} to solve the problem

$$\max_{C_t,B_{t+1},S_{ft}} \sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{C_t}{H_t}\right)^{1-\sigma}}{1-\sigma}$$

$$C_t + B_{t+1} + \sum_{f=1}^{N} q_{ft}(S_{ft} - S_{ft-1}) \leq w_t H_t + (1 + r_t)B_t + \sum_{f=1}^{N} S_{ft} D_{ft}$$

Southern Households: Given a sequence of wages w_t^* , firm stock prices q_{ft}^* , firm dividends D_{ft}^* , and interest rates r_t^* , a Southern household supplies labor inelastically and chooses consumption C_t^* , portfolio positions S_{ft}^* , and bond purchases B_{t+1}^* to solve the problem

$$\max_{C_t^*,B_{t+1}^*,S_{ft}^*} \sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{C_t^*}{H_t^*}\right)^{1-\sigma}}{1-\sigma}$$

$$C_t^* + B_{t+1}^* + \sum_{f=1}^{N} q_{ft}^*(S_{ft}^* - S_{ft-1}^*) \leq w_t^* H_t^* + (1 + r_t^*)B_t^* + \sum_{f=1}^{N} S_{ft}^* D_{ft}^*$$

Northern Final Goods Firms: Taking as given a sequence of wages w_t and intermediate goods prices p_{jt} for each variety $j \in [0, A_t]$ as given, perfectly competitive Northern final goods firms choose input demands H_t and x_{jt} to solve the static problem

$$\max_{H_t,x_{jt}} Y_t - \int_0^{A_t} p_{jt} x_{jt} dj - w_t H_t$$

$$\max_{H_t,x_{jt}} H_t^{\alpha} \int_0^{A_t} x_{jt}^{1-\alpha} dj - \int_0^{A_t} p_{jt} x_{jt} dj - w_t H_t$$

Southern Final Goods Firms: Taking as given a sequence of wages w_t^* and intermediate goods prices p_{jt}^* for each variety $j \in [0, A_t]$ as given, perfectly competitive Southern final goods firms choose input demands H_t^* and x_{jt}^* to solve the static problem

$$\max_{H_t^*,x_{jt}^*} Y_t^* - \int_0^{A_t} p_{jt}^* x_{jt}^* dj - w_t^* H_t^*$$

$$\max_{H_t^*,x_{jt}^*} (H_t^*)^{\alpha} \int_0^{A_t} (x_{jt}^*)^{1-\alpha} dj - \int_0^{A_t} p_{jt}^* x_{jt}^* dj - w_t^* H_t^*$$

because liberalization gains occur in the future with a larger population. In unreported results, however, we also solved an alternative model with per-capita preferences weighted by population size. Predictably, this resulted in larger welfare gains from trade liberalization.

Northern Intermediate Goods Firms: Taking as given a sequence of interest rates r_t , along with aggregate variety stocks A_t , as well as Northern and Southern final goods firms' intermediate demand schedules, each of N Northern intermediate goods firms f makes monopoly production x_{Mjt+1} and x^*_{Mjt+1} , perfectly competitive production x_{Rjt+1} , and innovation decisions M_{ft+1} to solve the following problem

$$\max_{x_{Rjt+1}, x_{Mjt+1}, x^*_{Mjt+1}, M_{ft+1}} \sum_{t=0}^{\infty} m_t D_{ft},$$

$$D_{ft} + Z_{ft} + \int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj \leq \int_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt}) dj,$$

where $\frac{m_{t+1}}{m_t} = \frac{1}{1+r_{t+1}}$ or $m_t = \Pi_{\tau=1}^{t} \frac{1}{1+r_\tau}$. This is equivalent to stock price or value maximization as can be seen from iteration on the Northern Household's first order condition for S_{ft} and insertion of the Northern household first order condition for B_{t+1}. At all times, the innovation cost function is given by

$$Z_{ft} = \nu M_{ft+1}^{\gamma} A_t^{1-\frac{\delta}{\rho}},$$

where $\gamma = \frac{1}{\rho}$ and $\delta \in (0,1)$, and $\nu = \frac{N^{\gamma-1}}{\gamma}$ is again a scaling constant discussed in more detail below. This innovation cost function is identical to the strong scale effects innovation cost function, with the exception that $\delta < 1$ here and $\delta = 1$ in that case.

Southern Intermediate Goods Firms: Taking as given a sequence of interest rates r^*_t , as well as Northern and Southern final goods firms' intermediate demand schedules, each Southern intermediate goods firm makes perfectly competitive production x_{Ijt} , x^*_{Ijt} , and x^*_{Rjt} decisions to solve the following problem

$$\max_{x_{Ijt}, x^*_{Ijt}, x^*_{Rjt}} \sum_{t=0}^{\infty} m^*_t D^*_{ft},$$

$$D^*_{ft} + \int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj \leq \int_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt}) dj$$

where $\frac{m^*_{t+1}}{m^*_t} = \frac{1}{1+r^*_{t+1}}$ or $m^*_t = \Pi_{\tau=1}^{t} \frac{1}{1+r^*_\tau}$. This is equivalent to stock price or value maximization as can be seen from iteration on the Southern Household's first order condition for S_{ft} and insertion of the Southern Household's first order condition for B^*_{t+1} .

Terms of Trade Notation/No Arbitrage Condition:

$$p_{jt} = q_t p^*_{jt}$$

Trade Restrictions and Monopoly Structure: There is one-period monopoly protection for any newly innovated M goods, trade restriction for an exogenously set proportion $1 - \phi_t$ of off-patent goods labeled R goods, and imports from South to North of the exogenously set proportion ϕ_t of off-patent goods labeled I goods.

Equilibrium Summary

- Some sequence of ϕ_t is exogenously set by the Northern government
- Northern households optimize consumption, savings, and equity purchase decisions
- Southern households optimize consumption, savings, and equity purchase decisions
- Perfectly competitive Northern final goods sector optimizes human capital and intermediate goods demand
- Perfectly competitive Southern final goods sector optimizes human capital and intermediate goods demand
- Northern intermediate goods firms optimize M goods innovation, M goods monopoly production, and perfectly competitive R goods production decisions
- Southern intermediate goods firms optimize perfectly competitive R and I goods production decisions
- Trade is balanced: $I_t p_{It} x_{It} = M_t p_{Mt} x^*_{Mt}$
- Bond markets clear: $B_t = B^*_t = 0$
- Equity markets clear: $S_{ft} + S^*_{ft} = 1$
- Human capital market clear $H^D_t = H_t$, $(H^*)^D_t = H^*_t$
- Final goods market clears/resource constraint is satisfied in the North

$$Y_t = H^\alpha_t \int_0^{A_t} x^{1-\alpha}_{jt} dj = C_t + \int_{A_{t+1}} (x_{jt+1} + x^*_{jt+1}) dj + \sum_{f=1}^{N} Z_{ft}$$

- Final goods market clears/resource constraint is satisfied in the South

$$Y_t = H^\alpha_t \int_0^{A_t} x^{1-\alpha}_{jt} dj = C^*_t + \int_{A_{t+1}} (x_{jt+1} + x^*_{jt+1}) dj$$

- Consistency conditions hold

$$\sum_{f=1}^{N} M_{ft+1} = M_{t+1} = A_{t+1} - A_t$$

$$\phi A_t = I_t, (1-\phi) A_t = R_t$$

$$\frac{H_t^*}{H_t} = \frac{H_0^*}{H_0} = \frac{\bar{H}}{H^*}$$

- Southern cost dominance for I goods

$$q_t(1 + r_t^*) < (1 + r_t)$$

Equilibrium Conditions for Reference

For later reference in the proof of Proposition $D1$, we now list the equilibrium conditions in this environment. Northern Households' (HH) First Order Conditions (FOC)

$$\beta^t H_t^{\sigma-1} C_t^{-\sigma} = \lambda_t$$

$$\lambda_t = (1 + r_{t+1})\lambda_{t+1}$$

$$\lambda_t (D_{ft} - q_{ft}) + \lambda_{t+1} q_{ft+1} = 0$$

$$\rightarrow (1 + r_{t+1}) = \frac{1}{\beta} \frac{H_{t+1}}{H_t} \left(\frac{C_{t+1}}{H_{t+1}} \frac{H_t}{C_t} \right)^{\sigma} = \frac{1}{\beta}(1 + g_H) \left(\frac{c_{t+1}}{c_t} \right)^{\sigma}, \quad c_t \equiv \frac{C_t}{H_t}$$

$$\rightarrow q_{ft} = \sum_{t=0}^{\infty} m_t D_{ft}, \quad m_t \equiv \frac{\lambda_t}{\lambda_0} = \prod_{\tau=1}^{t} \frac{1}{1 + r_\tau}$$

Southern Households' FOC's

$$\rightarrow (1 + r_{t+1}^*) = \frac{1}{\beta} \frac{H_{t+1}^*}{H_t^*} \left(\frac{C_{t+1}^*}{H_{t+1}^*} \frac{H_t^*}{C_t^*} \right)^{\sigma} = \frac{1}{\beta}(1 + g_H) \left(\frac{c_{t+1}^*}{c_t^*} \right)^{\sigma}, \quad c_t^* \equiv \frac{C_t^*}{H_t^*}$$

$$\rightarrow q_{ft}^* = \sum_{t=0}^{\infty} m_t^* D_{ft}^*, \quad m_t^* \equiv \frac{\lambda_t^*}{\lambda_0^*} = \prod_{\tau=1}^{t} \frac{1}{1 + r_\tau^*}$$

Northern Final Goods Firm FOC's

$$(1-\alpha) H_t^{\alpha} x_{jt}^{-\alpha} - p_{jt} = 0 \rightarrow x_{jt} = (1-\alpha)^{\frac{1}{\alpha}} p_{jt}^{-\frac{1}{\alpha}} H_t$$

$$\alpha H_t^{\alpha-1} x_{jt}^{1-\alpha} - w_t = 0$$

Southern Final Goods Firm FOC's

$$(1-\alpha)(H_t^*)^{\alpha}(x_{jt}^*)^{-\alpha} - p_{jt}^* = 0 \rightarrow x_{jt}^* = (1-\alpha)^{\frac{1}{\alpha}} (p_{jt}^*)^{-\frac{1}{\alpha}} H_t^*$$

$$\alpha (H_t^*)^{\alpha-1} (x_{jt}^*)^{1-\alpha} - w_t^* = 0$$

Northern Intermediate Goods Firm FOC's

$$\max_{x_{Mt+1},M_{ft+1},x_{Rt+1}} \sum_{t=0}^{\infty} m_t D_{ft}$$

$$D_{ft} = \int_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt})dj - Z_{ft} - \int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj$$

$$-m_t \left[\frac{\partial}{\partial M_{ft+1}} Z_{ft} + x_{Mt+1} + x^*_{Mt+1} \right] + m_{t+1} p_{Mt+1}(x_{Mt+1} + x^*_{Mt+1}) = 0$$

$$p_{Mt+1} = \arg\max_{p} -m_t(1-\alpha)^{\frac{1}{\alpha}} p^{-\frac{1}{\alpha}} (H_{t+1} + q_{t+1}^{\frac{1}{\alpha}} H^*_{t+1}) + m_{t+1}(1-\alpha)^{\frac{1}{\alpha}} p^{1-\frac{1}{\alpha}} (H_{t+1} + q_{t+1}^{\frac{1}{\alpha}} H^*_{t+1})$$

$$p_{Mt+1} = \frac{m_t}{m_{t+1}} \frac{1}{1-\alpha}$$

$$-m_t + m_{t+1} p_{Rt+1} = 0$$

$$\to p_{Mt+1} = \frac{1+r_{t+1}}{1-\alpha}, \quad x_{Mt+1} = (1-\alpha)^{\frac{2}{\alpha}} (1+r_{t+1})^{-\frac{1}{\alpha}} H_{t+1}, \quad x^*_{Mt+1} = (1-\alpha)^{\frac{2}{\alpha}} (1+r_{t+1})^{-\frac{1}{\alpha}} q_{t+1}^{\frac{1}{\alpha}} H^*_{t+1}$$

$$\to p_{Rt+1} = 1 + r_{t+1}, \quad x_{Rt+1} = (1-\alpha)^{\frac{1}{\alpha}} (1+r_{t+1})^{-\frac{1}{\alpha}} H_{t+1}$$

$$\to \frac{\partial}{\partial M_{ft+1}} Z_{ft+1} = g_{At+1}^{\gamma-1} A_t^{\frac{1-\delta}{\rho}}, \text{ imposes symmetry } g_{Aft+1} = (1/N) g_{At+1}$$

$$\to Z_t = \sum_{f=1}^{N} Z_{ft} = \frac{g_{At+1}^{\gamma} A_t^{1+\frac{1-\delta}{\rho}}}{\gamma}, \text{ imposes symmetry } g_{Aft+1} = (1/N) g_{At+1}$$

$$\to g_{At+1}^{\gamma-1} A_t^{\frac{1-\delta}{\rho}} = \Omega (1+r_{t+1})^{-\frac{1}{\alpha}} \left(H_{t+1} + q_{t+1}^{\frac{1}{\alpha}} H^*_{t+1} \right)$$

Southern Intermediate Goods Firm FOC's

$$\max \sum_{t=0}^{\infty} m^*_t D^*_{ft},$$

$$D^*_{ft} = \int_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt})dj - \int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1})dj$$

$$-m^*_t + m^*_{t+1} p^*_{Rt+1} = 0$$

$$-m^*_t + m^*_{t+1} p^*_{It+1} = 0$$

$$\to p^*_{Rt+1} = (1 + r^*_{t+1}), \quad x^*_{Rt+1} = (1-\alpha)^{\frac{1}{\alpha}} (1+r^*_{t+1})^{-\frac{1}{\alpha}} H^*_{t+1}$$

$$\to p^*_{It+1} = (1 + r^*_{t+1}), p_{It+1} = q_{t+1} p^*_{It+1}, \quad x^*_{It+1} = (1-\alpha)^{\frac{1}{\alpha}} (1+r^*_{t+1})^{-\frac{1}{\alpha}} H^*_{t+1},$$

$$x_{It+1} = (1-\alpha)^{\frac{1}{\alpha}} (1+r^*_{t+1})^{-\frac{1}{\alpha}} q_{t+1}^{-\frac{1}{\alpha}} H_{t+1}$$

Balanced Trade Condition

$$I_t p_{It} x_{It} = M_t p_{Mt} x^*_{Mt}$$

$$\phi_t A_{t-1} q_t (1+r_t^*)(1-\alpha)^{\frac{1}{\alpha}}(1+r_t^*)^{-\frac{1}{\alpha}} q_t^{-\frac{1}{\alpha}} H_t = g_{At} A_{t-1} \frac{1+r_t}{1-\alpha}(1-\alpha)^{\frac{2}{\alpha}}(1+r_t)^{-\frac{1}{\alpha}} q_t^{\frac{1}{\alpha}} H_t^*$$

$$q_t = \left(\frac{\phi_t H_t}{g_{At} H_t^*}\right)^{\frac{\alpha}{2-\alpha}} \left(\frac{1+r_t}{1+r_t^*}\right)^{\frac{1-\alpha}{2-\alpha}} \Psi, \quad \Psi = (1-\alpha)^{\frac{\alpha-1}{2-\alpha}}$$

Northern Resource Constraint

$$Y_t = H_t^\alpha \left[M_t x_{Mt}^{1-\alpha} + R_t x_{Rt}^{1-\alpha} + I_t x_{It}^{1-\alpha}\right]$$

$$= C_t + M_{t+1}\left(x_{Mt+1} + x^*_{Mt+1}\right) + R_{t+1} x_{Rt+1} + Z_t$$

Southern Resource Constraint

$$Y_t^* = (H_t^*)^\alpha \left[M_t (x^*_{Mt})^{1-\alpha} + R_t (x^*_{Rt})^{1-\alpha} + I_t (x^*_{It})^{1-\alpha}\right]$$

$$= C_t^* + R_{t+1} x^*_{Rt+1} + I_{t+1}(x_{It+1} + x^*_{It+1})$$

Consistency Conditions and Terms of Trade Notation Convention

$$M_{t+1} = A_{t+1} - A_t, \quad R_{t+1} = (1-\phi_{t+1})A_t, \quad I_{t+1} = \phi_{t+1} A_t$$

$$M_{t+1} = \sum_{f=1}^{N} M_{ft+1}, \quad p_{jt} = q_t p^*_{jt}$$

Southern Cost Dominance for I Goods

$$q_t(1+r_t^*) \le (1+r_t)$$

Proposition D1 *A steady-state growth path with constant ϕ exists and is unique. On this steady-state growth path the growth rate g_A of varieties satisfies*

$$(1+g_A)^{\frac{1-\delta}{\rho}} = (1+g_H),$$

interest rates satisfy

$$1 + r = 1 + r^* = \frac{1}{\beta}(1+g_H)(1+g_A)^\sigma,$$

and the terms of trade satisfies

$$q = \left(\frac{\phi}{g_A}\frac{\bar{H}}{H^*}\right)^{\frac{\alpha}{2-\alpha}} \Psi, \Psi = (1-\alpha)^{\frac{\alpha-1}{2-\alpha}}.$$

On this unique steady-state growth path, output and consumption grow as the factor $(1+g_H)(1+g_A)$ *and per capita consumption has growth rate equal to the number of varieties* g_A.

Proof of Proposition D1: Semi-endogenous Steady-state Growth Path Assume constant growth rates of quantities and a constant ϕ. Then the HH Euler equations yield

$$1 + r = \frac{1}{\beta}(1 + g_H)(1 + g_c)^{\sigma}$$

$$1 + r^* = \frac{1}{\beta}(1 + g_H)(1 + g_{c^*})^{\sigma},$$

which implies that interest rates are constant. But the BT condition is then

$$q = \left(\frac{\phi}{g_A}\frac{\bar{H}}{H^*}\right)^{\frac{\alpha}{2-\alpha}} \left(\frac{1+r}{1+r^*}\right)^{\frac{1-\alpha}{2-\alpha}} \Psi,$$

which implies that the terms of trade are constant. But the innovation FOC is

$$g_A^{\gamma-1} A_t^{\frac{1-\delta}{\rho}} = \Omega(1+r)^{-\frac{1}{\alpha}} \left(H_{t+1} + q^{\frac{1}{\alpha}} H^*_{t+1}\right).$$

$$LHS \propto \left((1+g_A)^{\left(\frac{1-\delta}{\rho}\right)}\right)^t, \quad RHS \propto (1+g_H)^t$$

$$\rightarrow (1+g_A)^{\frac{1-\delta}{\rho}} = (1+g_H) \text{ on any BGP.}$$

Now note that prices of all goods are constant because they are functions of interest and terms of trade, so the intensive demand margins are also constant multiples of human capital. In particular,

$$x_{Mt} = (1-\alpha)^{\frac{2}{\alpha}}(1+r)^{-\frac{1}{\alpha}} H_t, \quad x^*_{Mt} = (1-\alpha)^{\frac{2}{\alpha}}(1+r)^{-\frac{1}{\alpha}} q^{\frac{1}{\alpha}} H^*_t$$

$$x_{Rt} = (1-\alpha)^{\frac{1}{\alpha}}(1+r)^{-\frac{1}{\alpha}} H_t, \quad x^*_{Rt} = (1-\alpha)^{\frac{1}{\alpha}}(1+r^*)^{-\frac{1}{\alpha}} H^*_t$$

$$x_{It} = (1-\alpha)^{\frac{1}{\alpha}}(1+r^*)^{-\frac{1}{\alpha}} q^{-\frac{1}{\alpha}} H_t$$

$$x^*_{It} = (1-\alpha)^{\frac{1}{\alpha}}(1+r^*)^{-\frac{1}{\alpha}} H^*_t$$

Note also that by the consistency conditions $M_t = g_A A_{t-1}, R_t = (1-\phi)A_{t-1}, I_t = \phi A_{t-1}$ are all constant multiples of A_t (given the fact that $A_{t-1} = \frac{1}{1+g_A} A_t$).

$$Y_t = H_t^{\alpha}\left[M_t x_{Mt}^{1-\alpha} + R_t x_{Rt}^{1-\alpha} + I_t x_{It}^{1-\alpha}\right]$$

$$Y_t \propto H_t A_t \propto ((1+g_H)(1+g_A))^t$$

Now from the uses identity we also have

$$Y_t = C_t + M_{t+1}\left(x_{Mt+1} + x^*_{Mt+1}\right) + R_{t+1}x_{Rt+1} + Z_t$$

But from above

$$M_{t+1}\left(x_{Mt+1} + x^*_{Mt+1}\right) \propto H_t A_t$$

$$R_{t+1}x_{Rt+1} \propto H_t A_t$$

$$Z_t = \frac{g_A^\gamma}{\gamma} A_t^{1+\frac{1-\delta}{\rho}} \propto A_t^{1+\frac{1-\delta}{\rho}} \propto \left((1+g_A)^{1+\frac{1-\delta}{\rho}}\right)^t$$

But since $1 + g_H = (1+g_A)^{\frac{1-\delta}{\rho}}$ on any BGP by the innovation FOC, we have

$$Z_t \propto ((1+g_H)(1+g_A))^t ,$$

Therefore, we have

$$C_t \propto ((1+g_H)(1+g_A))^t , \quad c_t \propto (1+g_A)^t ,$$

implying that $g_c = g_A$, so that

$$1 + r = \frac{1}{\beta}(1+g_H)(1+g_A)^\sigma .$$

Now similar reasoning shows that

$$Y_t^* \propto H_t^* A_t, \quad C_t^* \propto H_t^* A_t, \quad c_t^* \propto A_t,$$

so that

$$1 + r^* = 1 + r$$

$$q = \left(\frac{\phi}{g_A}\frac{\bar{H}}{H^*}\right)^{\frac{\alpha}{2-\alpha}} \left(\frac{1+r}{1+r^*}\right)^{\frac{1-\alpha}{2-\alpha}} \Psi = \left(\frac{\phi}{g_A}\frac{\bar{H}}{H^*}\right)^{\frac{\alpha}{2-\alpha}} \Psi.$$

Note that this final expression implies that for sufficiently small ϕ, $q < 1$, which is equivalent along the BGP to Southern cost dominance in I goods. Finally, uniqueness follows from the innovation FOC

$$g_A^{\gamma-1} A_t^{\frac{1-\delta}{\rho}} = \Omega(1+r)^{-\frac{1}{\alpha}} \left(H_{t+1} + q^{\frac{1}{\alpha}} H^*_{t+1}\right).$$

After dividing both sides by $(1+g_H)^t$, we have that

$$g_A^{\gamma-1} \propto \Omega(1+r)^{-\frac{1}{\alpha}} \left(H_1 + q^{\frac{1}{\alpha}} H^*_1\right).$$

Since $\gamma > 1$, the LHS is increasing in g_A . Since r is increasing in g_A and q is decreasing in g_A , there is at most one solution for g_A . Since all other prices are functions of g_A , they are unique as well. Existence is shown by noting that the increasing LHS asymptotes to ∞ as $g_A \to \infty$ and to 0 as $g_A \to 0$. The decreasing RHS asymptotes to ∞ as $g_A \to 0$ (see the formula for q) and to 0 as $g_A \to \infty$ (see the formulas for r and q). By the continuity and monotonicity of everything involved, as well as the intermediate value theorem, g_A exists uniquely. This completes the proof.

Calibration Strategy

We would like to consider, as in the Fully Mobile environment described above, the transition path associated with a shock from the balanced growth path associated with trade policy parameter ϕ to the balanced growth path associated with trade policy parameter ϕ' . As before, we will consider the impact of a permanent and unanticipated shock moving the policy parameter from ϕ to ϕ' . The timing conventions are identical to those discussed in the Fully Mobile trade shock timing section in the main text. According to the OECD National Accounts Main Aggregates dataset and Population dataset, as current in early May 2013, the average total OECD real GDP per-capita growth rate from $1984 - 2000$ is equal to approximately 2.37% per year. The average OECD population growth rates over this same period is approximately equal to 0.78% per year. Now note that the steady-state growth path relationship above between g_H and g_A is a logarithmic equation whose solution yields

$$\delta = 1 - \rho \frac{\log(1+g_H)}{\log(1+g_A)}.$$

Above, note that g_A and g_H are 10-year versions of the annual growth rates taken from OECD data. Now, with the calibration $\rho = 0.5$ from above, we have that $\delta = 0.83$. The remaining parameters to calibrate in the model are β , σ , α , $\frac{\bar{H}^*}{H}$, H_{-1} , ϕ , and ϕ' . The values for $\alpha = 2/3$, $\sigma = 1$, $\beta = 1/1.02$, and $\frac{H_t^*}{H_t} = 2.96$ are unchanged from before. The final three parameters which must be calibrated are ϕ , ϕ' , and H_1 . We jointly pick these three parameters so that the following three conditions hold: $\frac{I}{Y}_{\phi,BGP} = 3.5\%$, $\frac{I}{Y}_{\phi,BGP} = 8.4\%$, and the innovation first order condition for the pre-shock ϕ steady-state growth path is satisfied. The first two conditions require that the model match the non-OECD to OECD trade shares which the strong scale effects model is calibrated to match. The final condition requires that the scaling of varieties to human capital at the initial condition of the transition path is consistent with the equilibrium conditions. Given the calibration, the transition path in response to a fully mobile shock moving the economy from ϕ to ϕ' can be written as a minimization problem in r_t, r_t^*, and q_t , as in the strong scale effects case. The endpoints of each series are known, because they reflect steady-state growth path values.

Table D1: Semi-endogenous Transition Path Summary

Quantity	Value
max g_{At}	3.07%
(max g_{At})$-g_A$	0.70%
Half Life	15 periods
r	5.23%
$q(\phi)$	0.43
$q(\phi')$	0.76
$\frac{I}{Y}_\phi$	3.5%
$\frac{I}{Y}_{\phi'}$	8.4%
ΔW	26.19%
ΔW^*	24.48%

Note: The table above displays a summary of the quantitative exercise performed for the semi-endogenous model given a calibrated trade liberalization. The long-run annualized value of the interest rate is given as r , and all other quantities are computed from a transition path in response to an unanticipated, permanent movement of trade policy ϕ to $\phi' > \phi$, where ϕ and ϕ' are chosen to match the movement in low-cost imports to OECD GDP observed in the data from 1994-2014 and also displayed in the table. The pre- and post-shock Southern terms of trade $q(\phi)$ and $q(\phi')$ vary permanently with the trade policy parameter and reflect the steady-state growth path for the indicated policy. The maximum level of variety growth $\max g_{At}$ and the maximum difference in variety growth from its long-run level over the transition path are displayed in the first two rows, while the half life of the shock to variety growth induced by trade liberalization is indicated in the third row. The model calibration of a period is one decade. ΔW and ΔW^* refer to the permanent consumption equivalent of trade liberalization for a Northern and Southern household, respectively. In particular, this percentage is the permanent fraction by which consumption for a household must increase in each period without the trade shock to make the household indifferent to the allocation with trade liberalization.

Results

Figure D1 plots the transition path for the semi-endogenous economy in response to the trade liberalization, for variety growth, the Southern terms of trade, and Northern and Southern per-capita output growth. In fact, the transition is not complete 25 periods. Recall that a period in this calibration is one decade, so this represents a transition path which is not complete 250 years after the initial shock. However, the broad pattern of the transition path is similar to that observed in the strong scale effects model. In particular, we have that in response to trade liberalization, the appreciation of the Southern terms of trade due to the increased flow of I goods from South to North causes an increase in the variety growth rate, as well as Northern and Southern output growth rates. Variety growth rates immediately begin to fall, however, as the gains from increased variety levels fade in the semi-endogenous innovation cost function. This process is incredibly persistent, however, because the level of δ implied by OECD evidence on per capita GDP and population growth rates is quite close to 1 , yielding something quantitatively similar to the strong

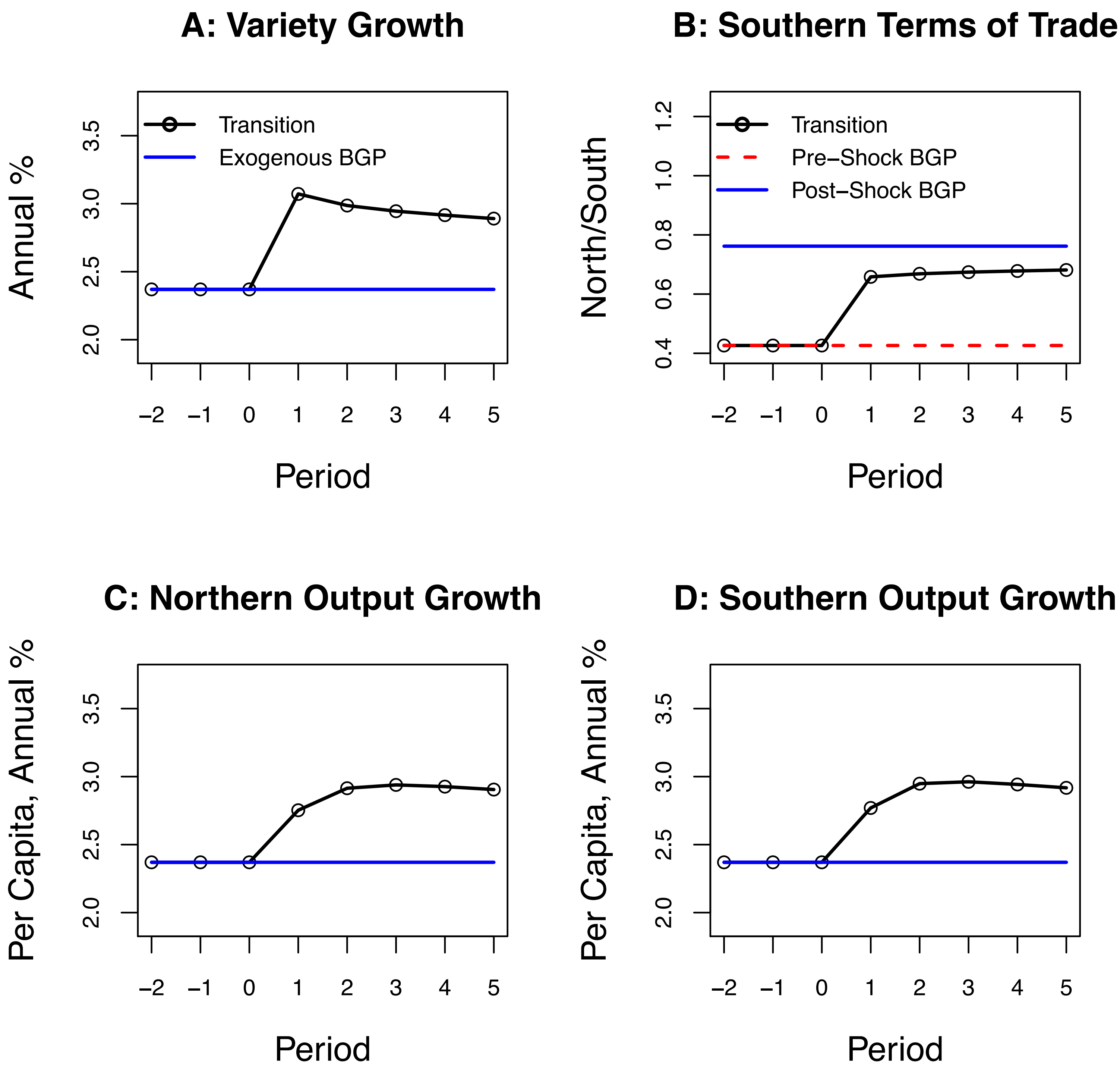

Figure D1: Semi-endogenous Growth Model Trade Liberalization

Note: The figure displays the Fully Mobile transition path in the semi-endogenous growth model in response to a permanent, unanticipated trade liberalization from policy parameter ϕ to $\phi' > \phi$, which is announced in period 0 to become effective in period 1. Intermediate goods firms may respond to the information about trade liberalization without short-term adjustment costs. The solid black line is the transition path, the upper horizontal solid blue line is the post-shock steady-state growth path, and the lower horizontal dashed red line is the pre-shock steady-state growth path. Note that since the semi-endogenous growth model's value for variety growth and output growth in the long run does not vary with trade policy, there is only one steady-state growth marker for these series.

scale effects model. Because of consumption smoothing and the implied movements in interest rates, Northern and Southern output growth rates are smoother than variety growth, yet just as persistent. Finally, as the variety growth rate and interest rates begin to return to their normal long-run levels, the Southern terms of trade q slowly converges to its new long-run value associated with ϕ'.

More precisely, in Table $D1$ we present the detailed statistics associated with trade liberalization in the semi-endogenous model. In particular, note that the half-life of the shock to the variety growth rate is 15 periods, or 150 years. Also, note that the welfare gains to the North and to the South from liberalization, 26.2% and 24.5%, which are permanent consumption equivalent welfare gains defined analogously to before, are qualitatively similar to those obtained from the strong scale effects model.

Appendix E - R&D Cost Externalities

As noted in the main text, to allow for the problem that firms face in coordinating search and innovation in larger teams, we allow for a form of diminishing marginal productivity for the inputs to innovation in any given period. This diminishing marginal productivity can be internal in the sense that it depends only on the inputs devoted to innovation within the firm, or it could be external in the sense that it depends on total inputs devoted to innovation in the economy. We start first with the fully internal case, which is our benchmark structure considered in the main paper. In this case, the number of new designs at firm f is a function of innovation expenditures Z_{ft} within firm f :

$$M_{ft+1} = (Z_{ft})^{\rho} A_t^{1-\rho},$$

where $0 < \rho < 1$. This yields an internal R&D cost function given by

$$Z_{ft} = IC(M_{ft+1}^{\gamma}, A_t) = M_{ft+1}^{\gamma} A_t^{1-\gamma},$$

where $\gamma = \frac{1}{\rho} > 1$ and the function name IC is a mnemonic for Internal Costs. The other extreme, which is the extension we consider in this section, would be to assume that the costs of innovation for any one firm depend on the total amount of innovation that is taking place in the economy because independent firms could develop redundant designs. In this case, with fully external increasing costs, the aggregate production function for innovation is given by

$$M_{t+1} = (Z_t)^{\rho} A_t^{1-\rho},$$

where Z_t is the aggregate quantity of final good devoted to innovation. The corresponding aggregate cost function is

$$Z = M_{t+1}^{\gamma} A_t^{1-\gamma}.$$

In this case, the cost per new patent to an individual firm would be the average economy-wide cost of innovation

$$Z_{ft} = EC(M_{ft+1}, M_{t+1}, A_t) = \frac{M_{ft+1}}{M_{t+1}} M_{t+1}^{\gamma} A_t^{1-\gamma}.$$

where EC is a mnemonic for external costs. To allow for intermediate degrees of internal and external costs of innovation, we nest these two versions in a cost function for firm f of the form

$$Z_{ft} = \nu \left(IC(\bullet)\right)^{\eta} \left(EC(\bullet)\right)^{1-\eta},$$

where $0 \leq \eta \leq 1$ and the inputs for the functions $IC(\bullet)$ and $EC(\bullet)$ are as given above. As η increases, the cost function exhibits a steeper marginal cost curve within each firm, with less redundancy across firms and hence weaker innovation externalities. The fully internal and fully external innovation cost benchmarks are the cases of $\eta = 1$ and $\eta = 0$, respectively. The introduction of η requires a slight change in the scaling constant ν to deliver invariance of steady-state growth path growth rates to N, η, ρ . However, the equilibrium definition and structure is identical to that considered above, except for the obvious modifications to the innovation first-order conditions and resource constraints. For the Fully Mobile environment, the symmetry across firms causes invariance of the aggregate allocation to the level of η . Only the Trapped Factors transition dynamics are modified. For completeness, we reproduce below the modified system of equations solved numerically to compute the transition path in the Trapped Factors case with an arbitrary level of η . These equations are the direct analogues of those in Appendix C above.

$$q_2 = \left[\frac{\phi' H}{H\left[\left(\frac{n}{2}\right)\left(\mu^1\right)^{\frac{\alpha-1}{\alpha}} g_2^1 + \left(\frac{n}{2}\right)\left(\mu^2\right)^{\frac{\alpha-1}{\alpha}} g_2^2\right]}\right]^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_2}{1+r_2^*}\right)^{\frac{1-\alpha}{2-\alpha}}$$

$$q_t = \left(\frac{\phi' H}{g_t H^*}\right)^{\frac{\alpha}{2-\alpha}} \Psi \left(\frac{1+r_t}{1+r_t^*}\right)^{\frac{1-\alpha}{2-\alpha}}, 3, ..., T$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta(1+r_{t+1}), t = 2, ..., T$$

$$\left(\frac{C_{t+1}^*}{C_t^*}\right)^{\sigma} = \beta(1+r_{t+1}^*), t = 2, ..., T$$

$$(Ng_2^1)^{\eta(\gamma-1)}(g_2)^{(\gamma-1)(1-\eta)} = \Omega(1+r_2)^{-\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(H + q_2^{\frac{1}{\alpha}} H^*)$$

$$(Ng_2^2)^{\eta(\gamma-1)}(g_2)^{(\gamma-1)(1-\eta)} = \Omega(1+r_2)^{-\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(H + q_2^{\frac{1}{\alpha}} H^*)$$

$$\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}} H + \frac{1}{N}\frac{g(\phi)^{\gamma}}{\eta(\gamma-1)+1} + \frac{g(\phi)}{N}(1-\alpha)^{\frac{2}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}(H + q(\phi)^{\frac{1}{\alpha}} H^*)$$

$$= \frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}} H + \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1}\left(g_2^1\right)^{\eta(\gamma-1)+1}(g_2)^{(\gamma-1)(1-\eta)}$$

$$+ g_2^1(1-\alpha)^{\frac{2}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}}(\mu^1)^{-\frac{1}{\alpha}}(H + q_2^{\frac{1}{\alpha}} H^*)$$

$$\frac{1}{N}(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}} H + \frac{1}{N}\frac{g(\phi)^{\gamma}}{\eta(\gamma-1)+1} + \frac{g(\phi)}{N}(1-\alpha)^{\frac{2}{\alpha}}(1+r(\phi))^{-\frac{1}{\alpha}}(H + q(\phi)^{\frac{1}{\alpha}} H^*)$$

$$= \frac{1}{N}\chi_2(1-\phi)(1-\alpha)^{\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}} H + \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1}\left(g_2^2\right)^{\eta(\gamma-1)+1}(g_2)^{(\gamma-1)(1-\eta)}$$

$$+ g_2^2(1-\alpha)^{\frac{2}{\alpha}}(1+r_2)^{-\frac{1}{\alpha}}(\mu^2)^{-\frac{1}{\alpha}}(H + q_2^{\frac{1}{\alpha}} H^*).$$

$$g_2 = \left(\frac{N}{2}\right)(g_2^1 + g_2^2).$$

$$\frac{1+\chi_2}{2} = \frac{1-\phi'}{1-\phi},$$

$$p^1_{M2} = \mu^1 \frac{1+r_2}{1-\alpha}, p^1_{R2} = (1+r_2),$$

$$p^2_{M2} = \mu^2 \frac{1+r_2}{1-\alpha}, p^2_{R2} = (1+r_2).$$

$$Z^1_2 = \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1} \left(g^1_2\right)^{\eta(\gamma-1)+1} (g_2)^{(\gamma-1)(1-\eta)}$$

$$Z^2_2 = \frac{N^{\eta(\gamma-1)}}{\eta(\gamma-1)+1} \left(g^2_2\right)^{\eta(\gamma-1)+1} (g_2)^{(\gamma-1)(1-\eta)},$$

$$Y_1 = C_1 + \left(\frac{N}{2}\right) g^1_2 A_1 (x^1_{M2} + x^{*1}_{M2}) + \left(\frac{N}{2}\right) g^2_2 A_1 (x^2_{M2} + x^{*2}_{M2}) + \left(\frac{N}{2}\right) \frac{1-\phi}{2} A_1 x^1_{R2} + \left(\frac{N}{2}\right) \frac{(1-\phi)\chi_2}{2} A_1 x^2_{R2}$$

$$+ Z^1_2 + Z^2_2$$

$$Y^*_1 = C^*_1 + (1-\phi') A_1 x^*_{R2} + \phi' A_1 (x^*_{I2} + x_{I2})$$

$$Y_2 = H^\alpha \left[\left(\frac{N}{2}\right) g^1_2 A_1 (x^1_{M2})^{1-\alpha} + \left(\frac{N}{2}\right) g^2_2 A_1 (x^2_{M2})^{1-\alpha} + \left(\frac{N}{2}\right) \frac{1-\phi}{2} A_1 (x^1_{R2})^{1-\alpha} \right.$$

$$\left. + \left(\frac{N}{2}\right) \frac{(1-\phi)\chi_2}{2} A_1 (x^2_{R2})^{1-\alpha} + \phi' A_1 x^{1-\alpha}_{I2} \right]$$

$$Y^*_2 = (H^*)^\alpha \left[\left(\frac{N}{2}\right) g^1_2 A_1 (x^{*1}_{M2})^{1-\alpha} + \left(\frac{N}{2}\right) g^2_2 A_1 (x^{*2}_{M2})^{1-\alpha} + (1-\phi') A_1 (x^*_{R2})^{1-\alpha} + \phi' A_1 (x^*_{I2})^{1-\alpha} \right].$$

$$\min(\mu^1, \mu^2)(1+r_2) \geq q_2(1+r^*_2),$$

$$(1+r_t) \geq q_t(1+r^*_t), t = 3, .., T,$$

$$q_1, q_{T+1} \leq 1.$$

Appendix F - Southern Innovation

In the baseline model we assume that the Southern economy cannot innovate. In this appendix we analyze an economy with Southern innovation, allowed under the assumption that Southern firms produce patents or ideas with a different productivity than Northern firms. The remainder of the structure of the economy is identical to the baseline environment. After laying out the optimality conditions characterizing this equilibrium, we first calibrate the relative productivities of Northern and Southern innovation to match observed patent rates. Then, we show that the quantitative impact of a trade liberalization in a global economy with Southern innovation is similar to the baseline case.

Model

First, we'll overview the structure of the economy, outlining each agent and their optimization problem. In particular, the North and South are populated by a set of households which provide labor and make consumption and savings choices. Northern and Southern final good sectors operates a constant returns to scale competitive technology, while intermediate goods firms in both economies innovate new varieties and supply existing intermediate goods varieties to the final goods sectors. Balanced trade in intermediate goods takes place between each economy, subject to various exogenous trade restrictions.

Northern Household

Taking wages w_t, interest rates r_t, intermediate goods firm stock prices q_{ft}, and intermediate goods firm dividends D_{ft} as given, a unit measure of identical Northern households supplies labor with in effective units H inelastically and chooses consumption C_t, portfolio positions S_{ft}, and bond purchases B_{t+1} to maximize their discounted utility as follows:

$$\max_{C_t, B_{t+1}, S_{ft}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

$$C_t + B_{t+1} + \sum_{f=1}^{N} q_{ft}(S_{ft} - S_{ft-1}) \leq w_t H + (1+r_t)B_t + \sum_{f=1}^{N} D_{ft}.$$

Southern Household

Taking wages w_t^*, interest rates r_t^*, intermediate goods firm stock prices q_{ft}^*, and intermediate goods firm dividends D_{ft}^* as given, a unit measure of identical Southern households supplies labor with in effective units H^* inelastically as chooses consumption C_t^*, portfolio positions S_{ft}^*, and bond purchases B^*_{t+1} to maximize their discounted utility as follows:

$$\max_{C^*_t, B^*_{t+1}, S^*_{ft}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{*1-\sigma}}{1-\sigma}$$

$$C^*_t + B^*_{t+1} + \sum_{f=1}^{N} q^*_{ft}(S^*_{ft} - S^*_{ft-1}) \leq w^*_t H + (1+r^*_t)B^*_t + \sum_{f=1}^{N} D^*_{ft}.$$

Northern Final Goods Sector

The Northern final good serves as a numeraire in this economy. Taking wages w_t and intermediate goods prices in Northern units p_{jt} as given, the Northern final goods sector chooses labor input H_t^D and intermediate goods inputs x_{jt}^D optimally in order to maximize their profits as follows:

$$\max_{H_t^D, \{x_{jt}^D\}} Y_t - \int_0^{A_t} p_{jt} x_{jt}^D dj - w_t H_t^D$$

$$Y_t = H_t^{D\alpha} \int_0^{A_t} x_{jt}^{D1-\alpha} dj.$$

Southern Final Goods Sector

Taking wages w^*_t and intermediate goods prices in Southern units p^*_{jt} as given, the Southern final goods sector chooses labor input H_t^{*D} and intermediate goods inputs x_{jt}^{*D} optimally in order to maximize their profits as follows:

$$\max_{H_t^{*D}, \{x_{jt}^{*D}\}} Y^*_t - \int_0^{A_t} p^*_{jt} x_{jt}^{*D} dj - w^*_t H_t^{*D}$$

$$Y^*_t = H_t^{*D\alpha} \int_0^{A_t} x_{jt}^{*D1-\alpha} dj.$$

Northern Intermediate Goods Firms

Taking as given a sequence of interest rates r_t, along with aggregate variety stocks A_t, as well as Northern and Southern final goods firms' intermediate demand schedules, each of N Northern intermediate goods firms f makes monopoly production x_{Mjt+1} and x^*_{Mjt+1}, perfectly competitive production x_{Rjt+1}, and innovation decisions M_{ft+1} to solve the following problem

$$\max_{x_{Rjt+1}, x_{Mjt+1}, x^*_{Mjt+1}, M_{ft+1}} \sum_{t=0}^{\infty} m_t D_{ft},$$

$$D_{ft} + Z_{ft} + \int_{A_{ft+1}} (x_{jt+1} + x^*_{jt+1}) dj \leq \int_{A_{ft}} p_{jt}(x_{jt} + x^*_{jt}) dj,$$

where $\frac{m_{t+1}}{m_t} = \frac{1}{1+r_{t+1}}$ or $m_t = \Pi_{\tau=1}^{t} \frac{1}{1+r_\tau}$. This is equivalent to stock price or value maximization as can be seen from iteration on the Northern Household's first order condition for S_{ft} and insertion of the Northern household first order condition for B_{t+1}. At all times, the innovation cost function is given by

$$Z_{ft} = \nu M_{ft+1}^{\gamma} A_t^{1-\gamma}, \quad \gamma = \frac{1}{\rho}, \quad \nu = \frac{N^{\gamma-1}}{\gamma}.$$

Southern Intermediate Goods Firms

Taking as given a sequence of interest rates r_t^*, along with aggregate variety stocks A_t, as well as Northern and Southern final goods firms' intermediate demand schedules, each of N Southern intermediate goods firms f makes monopoly production $x_{M^*jt+1}^*$ and $x_{M^*jt+1}^*$, perfectly competitive production x_{Rjt+1}^*, and innovation decisions M_{ft+1}^* to solve the following problem

$$\max_{x_{Rjt+1}, x_{M^*jt+1}, x_{M^*jt+1}^*, M_{ft+1}^*} \sum_{t=0}^{\infty} m_t D_{ft},$$

$$D_{ft}^* + Z_{ft}^* + \int_{A_{ft+1}^*} (x_{jt+1} + x_{jt+1}^*) dj \leq \int_{A_{ft}} p_{jt}^* (x_{jt} + x_{jt}^*) dj,$$

where $\frac{m_{t+1}^*}{m_t^*} = \frac{1}{1+r_{t+1}^*}$ or $m_t^* = \Pi_{\tau=1}^{t} \frac{1}{1+r_\tau^*}$. This is equivalent to stock price or value maximization as can be seen from iteration on the Southern Household's first order condition for S_{ft}^* and insertion of the Southern household first order condition for B_{t+1}^*. At all times, the innovation cost function is given by $Z_{ft}^* = \Delta^{-\gamma} \nu {M_{ft+1}^*}^{\gamma} A_t^{1-\gamma}, \quad \gamma = \frac{1}{\rho}, \quad \nu = \frac{N^{\gamma-1}}{\gamma}$. Note that $\Delta \in [0,1]$ is a parameter equal to the relative productivity of Southern firms to Northern firms in the innovation of new intermediate varieties.

Trade Restrictions and Market Structure

The total mass of varieties A_t in existence in any period is made up of newly innovated Northern varieties M_t, newly innovated Southern varieties M_t^*, as well as previously innovated varieties. For one period after innovation M and M^* goods are sold under patent or effective monopoly protection. Previously innovated varieties are produced in a competitive environment but split into two groups. A sequence of trade policy is given by fractions $\{\phi_t\}$ of off-patent goods is allowed to flow from South to North in mass I_t, while the remaining fraction $1 - \phi_t$ and mass R_t of off-patent goods is exogenously restricted to not flow from South to North. The masses of varieties satisfy the following equations:

$$A_t = M_t + M_t^* + A_{t-1}, \quad A_{t-1} = R_t + I_t, \quad I_t = \phi_t A_t.$$

Terms of Trade/No Arbitrage Condition

Northern and Southern intermediate goods trade at a relative price or terms of trade q_t in each period which translates pricing of each intermediate goods variety to the units relevant for final goods sector optimization in each economy. This can be expressed as $p_{jt} = q_t p^*_{jt}$.

Equilibrium Conditions

Given some sequence ϕ_t of trade restrictions, we now discuss conditions which characterize the equilibrium of the economy above. First, the demand curve for each intermediate variety is implied by profit maximization in the final goods sector, i.e. in equilibrium

$$x_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H p_{jt}^{-\frac{1}{\alpha}}, \quad x^*_{jt} = (1-\alpha)^{\frac{1}{\alpha}} H^* {p^*_{jt}}^{-\frac{1}{\alpha}}.$$

Competitive pricing of off-patent varieties, monopoly pricing of newly innovated varieties, and the trade structure of the economy imply that prices for each good are given by

$$p_{Mt} = \frac{1+r_t}{1-\alpha}, \quad p^*_{Mt} = \frac{1}{q_t} p_{Mt} \text{ (Northern innovated } M \text{ goods)}$$
$$p_{M^*t} = q_t p^*_{M^*t}, \quad p^*_{M^*t} = \frac{1+r^*_t}{1-\alpha} \text{ (Southern innovated } M^* \text{ goods}$$
$$p_{Rt} = 1 + r_t, \quad p_{R^*t} = 1 + r^*_t \text{ (Off-patent trade-restricted } R \text{ goods)}$$
$$p_{It} = q_t(1+r_t), \quad p_{I^*t} = 1 + r^*_t \text{ (Off-patent non-restricted } I \text{ goods)}$$

Let $\tilde{g}_{t+1} = \frac{M_{t+1}}{A_t}$ and $\tilde{g}^*_{t+1} = \frac{M^*_{t+1}}{A_t}$ be pseudo-growth rates representing the ratio of patents or new varieties created in the Northern and Southern economies in period t for first use in period $t+1$ relative to the total mass of varieties available in period t. It follows that the overall rate of growth of varieties in the global economy is given by $g_{t+1} = \tilde{g}_{t+1} + \tilde{g}^*_{t+1}$. Furthermore, simplified version of the first order conditions for innovation within the Northern and Southern intermediate goods firms can be written

$$\tilde{g}_{t+1}^{\gamma-1} = \Omega(1+r_{t+1})^{-\frac{1}{\alpha}}(H + q_{t+1}^{\frac{1}{\alpha}} H^*)$$
$$\{\tilde{g}^*_{t+1}\}^{\gamma-1}\Delta^{-\gamma} = \Omega(1+r^*_{t+1})^{-\frac{1}{\alpha}}(q_{t+1}^{-\frac{1}{\alpha}} H + H^*)$$

where $\Omega = \alpha(1-\alpha)^{\frac{2-\alpha}{\alpha}}$. Above, the interest rates in the Northern and Southern economies are pinned down by the household first-order conditions with respect to the one-period bond, i.e.

$$1 + r_{t+1} = \frac{1}{\beta}\left(\frac{C_{t+1}}{C_t}\right)^{\sigma}$$

$$1 + r^*_{t+1} = \frac{1}{\beta}\left(\frac{C^*_{t+1}}{C^*_t}\right)^{\sigma}$$

The balanced trade condition in the economy can be written and simplified after substitution of intermediate goods demand and pricing as

$$M_t p_{Mt} x^*_{Mt} = I_t p_{It} x_{it} + M^*_t p_{M^*t} x_{M^*t}$$

$$q_t = \left[\frac{H^*}{H}\frac{\tilde{g}_t(1-\alpha)^{\frac{1-\alpha}{\alpha}}(1+r_t)^{\frac{\alpha-1}{\alpha}}}{\phi_t(1+r^*_t)^{\frac{\alpha-1}{\alpha}} + \tilde{g}^*_t(1-\alpha)^{\frac{1-\alpha}{\alpha}}(1+r^*_t)^{\frac{\alpha-1}{\alpha}}}\right]^{\frac{\alpha}{\alpha-2}}.$$

Consumption in each economy must satisfy a resource constraint, and below we list the resource constraint for each economy as well as various simplifications of the output and R&D terms which follow directly from the definitions of each technology.

$$Y_t = C_t + M_{t+1}(x_{Mt+1} + x^*_{Mt+1}) + R_{t+1}x_{Rt+1} + Z_t$$

$$Z_t = \frac{1}{\gamma}\tilde{g}^{\gamma}_{t+1}A_t, \quad Y_t = H^{\alpha}\left[M_t x^{1-\alpha}_{Mt} + M^*_t x^{1-\alpha}_{M^*t} + R_t x^{1-\alpha}_{Rt} + I_t x^{1-\alpha}_{It}\right]$$

$$Y^*_t = C^*_t + M^*_{t+1}(x_{M^*t+1} + x^*_{M^*t+1}) + R_{t+1}x^*_{Rt+1} + I_{t+1}(x_{It+1} + x^*_{It+1}) + Z^*_t$$

$$Z^*_t = \frac{\Delta^{-\gamma}}{\gamma}\{\tilde{g}\}^{*}_{t+1}{}^{\gamma}A_t, \quad Y^*_t = H^{*\alpha}\left[M_t x^*_{Mt}{}^{1-\alpha} + M^*_t x^*_{M^*t}{}^{1-\alpha} + R_t x^*_{Rt}{}^{1-\alpha} + I_t x^*_{It}{}^{1-\alpha}\right]$$

These conditions jointly characterize the equilibrium of the economy with Southern innovation, conditional upon the trade decomposition assumed throughout the paper which requires Southern production of imported I varieties. For this to be consistent with cost minimization by the Northern final goods producer, it must be the case that $(1 + r^*_t)q_t \leq (1 + r_t)$ at all times. Also, note that the assumptions on the timing or mobility of inputs with respect to announcements of trade restrictions here follow the conventions of the Fully Mobile economies discussed in the main text.

Steady-State Growth Path Conditions

By arguments identical to those contained within the proofs of Propositions 1 and 2 in Appendix A above, we can immediately see that in any steady-state growth path associated with a constant trade restriction ϕ as well as a stable overall rate of global growth $g = \tilde{g} + \tilde{g}^*$ that each aggregate quantity in the model other than consumption must grow at the rate g. This implies via the resource constraint of each economy that consumption itself grows at rate g. This implies that interest rates along a steady-state growth path must be constant and satisfy

$$1 + r = 1 + r^* = \frac{1}{\beta}(1+g)^{\sigma}.$$

Table F1: Quantitative Exercise with Southern Innovation	
Quantity	Value
g_ϕ	2.0%
$g_{\phi'}$	3.05%
$\tilde{g}_\phi$	1.86%
$\tilde{g}_{\phi'}$	2.99%
$q(\phi)$	0.44
$q(\phi')$	0.97
ΔW	47.76%
ΔW^*	53.80%

Note: The variety growth rates and economy-specific pseudo-growth rates g and $\tilde{g}$ reported above are translated to annual percentage rates. The Southern terms of trade q is expressed in proportions. Quantities with subscript ϕ (ϕ') are calculated from the steady-state growth path associated with trade policy ϕ (ϕ'). The welfare gains from trade liberalization ΔW and ΔW^* reflect the percentage consumption equivalent gains from a trade liberalization $\phi \to \phi'$ relative to remaining on the pre-liberalization steady-state growth path with trade policy ϕ, taking into account the full transition path.

At that point, we can write the innovation first-order conditions and balanced trade conditions characterizing a steady-state growth path as

$$\tilde{g}^{\gamma-1} = \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}(H + q^{\frac{1}{\alpha}}H^*)$$

$$\{\tilde{g}\}^{*\gamma-1}\Delta^{-\gamma} = \Omega\beta^{\frac{1}{\alpha}}(1+g)^{-\frac{\sigma}{\alpha}}(q^{-\frac{1}{\alpha}}H + H^*)$$

$$q = \left[\frac{H^*}{H}\frac{\tilde{g}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\phi + \tilde{g}^*(1-\alpha)^{\frac{1-\alpha}{\alpha}}}\right]^{\frac{\alpha}{2-\alpha}}.$$

Calibration and Quantitative Results

As in the quantitative analysis in the main text, we now wish to consider the response of this economy to a trade liberalization shock. We follow the conventions of the Fully Mobile case from the main text. We assume that the economy is moving along the steady-state growth path associated with $\phi_s = \phi$ for all $s \leq t$. Then, in period t, we consider an announcement of an unanticipated and permanent change in the trade restriction parameter from ϕ to $\phi_s = \phi'$ for all $s > t$, where $\phi' > \phi$. The objects of interest in this exercise include not only the growth rates and terms of trade in the pre-shock and post-shock steady-state growth paths $(g_\phi, \tilde{g}_\phi, \tilde{g}^*_\phi, q_\phi)$ and $(g_{\phi'}, \tilde{g}_{\phi'}, \tilde{g}^*_{\phi'}, q_{\phi'})$ but also the transitional dynamics of the economy.

Before analyzing the transitional dynamics of the economy, we must first fix the calibration of the underlying parameters, which include β, σ, α, $\frac{H^*}{H}$, H, ϕ, ϕ', and Δ. Following the logic laid out in the main text, we externally calibrate a ten-year per period economy with the values of $\beta = 0.98^{10}$, $\frac{H^*}{H} = 2.96$, $\alpha = \frac{2}{3}$, and $\sigma = 1$. This approach leaves four parameters left to determine:

H, ϕ, ϕ', and Δ. We jointly calibrate the values of each of the four parameters by targeting four moments drawn from OECD trade and production data spanning the years of Chinese WTO accession as well as NBER data on patents filed in the US. The data sources and calculations are described in detail in Appendix B above. Implicit throughout this calibration exercise is a matching of the Northern model economy to the OECD countries.

- The Northern pre-shock imports to GDP ratio in the model along the steady-state growth path with parameter ϕ is equal to the non-OECD imports to OECD GDP ratio in 1994, i.e. $\frac{I}{Y}_{\phi} = 3.5\%$.
- The Northern post-shock imports to GDP ratio in the model along the steady-state growth path with parameter ϕ' is equal to the non-OECD imports to OECD GDP ratio in 2014, i.e. $\frac{I}{Y}_{\phi'} = 8.4\%$.
- The pre-shock global growth rate along the steady-state growth path with parameter ϕ is equal to the rate of growth of real GDP per capita in the US from 1960-2010, i.e. $g_{\phi} = 2.0\%$.
- The pre-shock ratio of Northern to Southern patents along the steady-state growth path with parameter ϕ is equal to the ratio of non-OECD patents filed in the US in 1994 to the total number of patents filed in the US in 1994, i.e. $\frac{M}{M+M^*}_{\phi} = 7.6\%$.

This calibration procedure is joint with no exact one-to-one correspondence between moments and parameters. Intuitively, however, the trade ratios are particularly influential in pinning down the values of ϕ and ϕ', while pre-shock growth rates determine the scale of the global economy as given by H. Finally, the patenting ratios are informative for the relative productivities of Northern and Southern innovation technologies. The calibration procedure results in parameter values of $H \approx 2.8$, $\phi \approx 5\%$, $\phi' \approx 48\%$, and $\Delta \approx 16\%$. Although the human capital level H is in model units difficult to interpret, the other parameters indicate a liberalization from a regime allowing 5% of off-patent goods into Northern markets to a regime allowing 48% of those goods into Northern markets. To match low Southern patenting rates, the productivity of Southern innovation must be only 16% of Northern innovation productivity.

Note that by contrast, calibration of the model without Southern innovation in the main text to match the same import ratios required a much smaller trade shock from $\phi \approx 8\%$ to $\phi' \approx 26\%$. The size of the trade shock is larger with Southern innovation because, as a function of ϕ, the curve of imports to GDP ratios shifts up and flattens. The curve shifts up because of the additional M^* goods flowing from North to South. The curve flattens or responds less to increases in ϕ because the induced Southern terms of trade appreciation results in lower Northern demand for Southern innovated goods, slowing import growth.

Once the calibration is complete, we compute the transition dynamics of the economy following an approach entirely analogous to the one presented in Appendix C for the baseline Fully Mobile

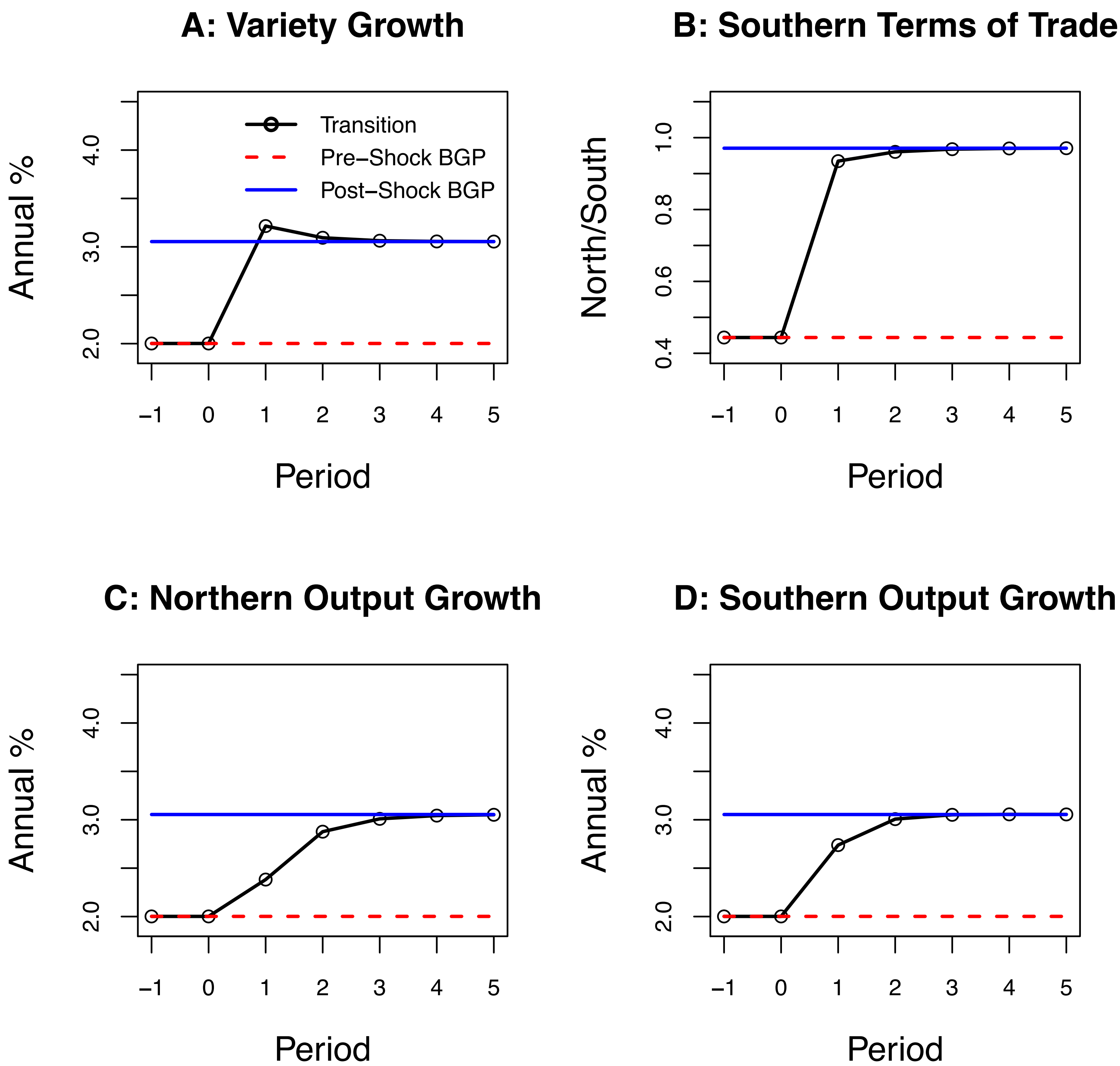

Figure F1: Southern Innovation Model Trade Liberalization

Note: The figure displays the Fully Mobile transition path in the growth model with Southern innovation in response to a permanent, unanticipated trade liberalization from policy parameter ϕ to $\phi' > \phi$, which is announced in period 0 to become effective in period 1. Intermediate goods firms may respond to the information about trade liberalization without short-term adjustment costs. The solid black line is the transition path, the upper horizontal solid blue line is the post-shock steady-state growth path, and the lower horizontal dashed red line is the pre-shock steady-state growth path.

economy. The main results of this quantitative exercise are given in Table F1. Qualitatively, the addition of a Southern innovation capacity to the baseline framework with fully mobile inputs changes little. Global growth increases from a pre-shock rate of 2.0% annually to 3.05% with Southern innovation. In the economy without Southern innovation, global growth increased from 2% to 2.57% in the long run. The substantially larger change in growth rates in response to trade liberalization is entirely driven by the larger calibrated trade shock $\phi' - \phi$ required in the economy with Southern innovation.

Over the full transition path plotted in Figure F1, which normalizes the period of the trade shock to 0, we see similar dynamics as in the baseline Figure 5 without Southern innovation. Trade liberalization leads to a rapid increase in the global variety growth rate and the Southern terms of trade, while output growth rates converge more slowly to their new and higher long-run levels. The gradual behavior of output growth rates relative to variety growth rates is due to underlying and gradual movements in interest rates due to consumption smoothing. Since interest rates determine pricing of intermediate goods, the intensive margins of intermediate goods use and hence overall output growth are slower to respond than the extensive margin or variety growth alone.

As in the baseline case without Southern innovation, the welfare gains from liberalization are large. The consumption equivalent gain from liberalization, computed exactly as laid out in Appendix C and taking into account the full transition path, are 47.8% for the North and 53.8% for the South.