A bunch of practice problems

Derivatives

1. $\dfrac{d}{dx}\sec(e^{\sin x})$

Solution: Use the chain rule a bunch of times: $\sec(e^{\sin x})\tan\left(e^{\sin x}\right)e^{\sin x}\cos x$

2. $\dfrac{d}{dx}(\sin x)^{\cos x}$

Solution: Start by taking natural logs: let $y = (\sin x)^{\cos x}$. Then $\ln y = \cos x \ln \sin x$. THis means $\dfrac{1}{y}\dfrac{dy}{dx} = -\sin x \ln \sin x + \cos x \dfrac{1}{\sin x}\cos^x$. Multiply both sides by $y = (\sin x)^{\cos x}$ to get the answer.

3. $\dfrac{d}{dx}\log_3(x^{2x})$

Solution: Start by simplifying: $\log_3(x^{2x}) = 2x\log_3(x) = 2x\dfrac{\ln x}{\ln 3}$.

4. $\dfrac{d}{dx}\dfrac{(x+2)^{3/2}e^{x^2}}{\sqrt{x^3+2}}$

Solution: Use logarithmic differentiation, similar to problem 2.

Integrals

5. $\displaystyle\int \frac{1}{x^2+6x+10}\,dx$

Solution: Start by completing the square

6. $\displaystyle\int \frac{2x+6}{x^2+6x+10}\,dx$

Solution: Use a the substitution $u = x^2 + 6x + 10$.

7. $\displaystyle\int x^2 e^x\,dx$

Solution: Use integration by parts with $u = x^2$, $dv = e^x\,dx$.

8. $\displaystyle\int \frac{\ln x}{x^3}\,dx$

Solution: Use integration by parts with $u = \ln x,\ dv = \dfrac{1}{x^3}\,dx$.

9. $\displaystyle\int \frac{1}{\sqrt{x^2 + 4x + 5}}\,dx$

Solution: Start by completing the square. Then use a trigonometric substitution.

10. $\displaystyle\int \sec^2 x\,dx$

Solution: By the formula sheet, $\dfrac{d}{dx}\tan x = \sec^2 x$. So the answer is $\tan x$.

11. $\displaystyle\int \tan^2 x \sec^2 x\,dx$

Solution: Use the substitution $u = \tan x,\ du = \sec^2 x\,dx$.