Math 1220-003, Summer 2018

Exam 2

Please write your name on the front and back of the exam. Remember to turn off your phone before starting this exam. Show all of your work for full credit. You may not use any notes or calculators during this exam.

Name: Solutions (Noethn / Pink)

UID: ______________________

1. (15 points) Determine whether the each of following statements is true or false. If true, write "True." If false, write "False." In what follows, $a_i \geq 0$ and $b_i \geq 0$ for all i.

(a) If the series $\sum_{i=1}^{\infty} c_i$ converges, then $\sum_{i=1}^{\infty} |c_i|$ converges.

False

(b) If $\sum_{i=1}^{\infty} b_i$ converges and $\lim_{i\to\infty} \frac{a_i}{b_i} = 3$, then $\sum_{i=1}^{\infty} a_i$ converges.

True

(c) $\int_{-1}^{1} \frac{1}{x^5}\, dx = 0.$

False $\left(\int_{-1}^{0} \frac{1}{x^5}\, dx = \infty \right)$

(d) If $\lim_{x\to 2} f(x) = 1$ and $\lim_{x\to 2} g(x) = \infty$, then $\lim_{x\to 2} f(x)^{g(x)} = 1$.

False (need L'Hopital)

(e) If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 3$, then $\sum_{n=1}^{\infty} a_n$ diverges.

True (ratio test)

2. (15 points) Find the integral:

$$\int \frac{x^2 - 2x + 6}{(x+1)(x-2)^2}\, dx$$

$$\frac{x^2-2x+6}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\Rightarrow x^2-2x+6 = A(x-2)^2 + B(x+1)(x-2) + C(x+1)$$

Plug in $x=2$: $4-4+6 = C\cdot(2+1) \Rightarrow C=2$

Plug in $x=-1$: $1+2+6 = A(-3)^2 \Rightarrow A=1$

Plug in $x=0$: $6 = 1\cdot(-2)^2 + B(1)(-2) + 2\cdot(1)$

$= 4 + -2B + 2 \quad \Rightarrow B=0$

Thus, $\displaystyle\int \frac{x^2-2x+6}{(x+1)(x-2)^2}\, dx = \int \frac{1}{x+1}\, dx + \int \frac{2}{(x-2)^2}\, dx$

$= \ln|x+1| - 2(x-2)^{-1} + C$

3. Find the following limits. If the limit is infinite, write ∞ or $-\infty$ accordingly. If the limit does not exist, write "does not exist".

(a) (10 points) $\lim_{x\to\infty} xe^{-x}$

$$\lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = 0.$$

(b) (10 points) $\lim_{x\to 0}(\cos x)^{\csc x}$

$$\lim_{x\to 0} \csc x \cdot \ln(\cos x) = \lim_{x\to 0} \frac{\ln(\cos x)}{\sin x}$$

$$= \lim_{x\to 0} \frac{\frac{1}{\cos x} \cdot \sin x}{\cos x} = 0.$$

4. (15 points) Does the following series converge?

$$\sum_{n=1}^{\infty} \frac{n+1}{2n^2+n+1}$$

Limit comparison test : compare to $\frac{1}{n}$

$$\lim_{n\to\infty} \frac{\frac{n+1}{2n^2+n+1}}{1/n} = \lim_{n\to\infty} \frac{n^2+n}{2n^2+n+1} = \frac{1}{2}$$

Thus, the two series both converge or both diverge.

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (Harmonic series)

So $\sum_{n=1}^{\infty} \frac{n+1}{2n^2+n+1}$ $\boxed{\text{diverges}}$.

5. (15 points) Does the following series converge?

$$\sum_{i=1}^{\infty} \frac{i^2+1}{3^i}$$

Ratio test:

$$\lim_{i\to\infty} \frac{(i+1)^2+1/3^{i+1}}{i^2+1/3^i} = \lim_{i\to\infty} \frac{[(i+1)^2+1]\cdot 3^i}{(i^2+1)\cdot 3^{i+1}}$$

$$= \lim_{i\to\infty} \frac{i^2+2i+1}{(i^2+1)\cdot 3} = \frac{1}{3}$$

$\boxed{\text{Converges}}$

6. For each of the following series, say whether it diverges, converges conditionally, or converges absolutely.

(a) (10 points) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4^n}$

$\sum \left| \frac{(-1)^{n+1}}{4^n} \right| = \sum \frac{1}{4^n}$ converges (geometric series)

$\boxed{\text{Converges absolutely}}$

(b) (10 points) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n^2+1}}$

$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}} = 0$, $\frac{d}{dx} \frac{1}{\sqrt{x^2+1}} = -\frac{1}{2}(x^2+1)^{-3/2} \cdot 2x < 0$

$\Rightarrow$ converges, by alternating series test.

Does $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$ converge? compare to $\sum_{n=1}^{\infty} \frac{1}{n}$.

$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{1/n} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty} \sqrt{\frac{n^2}{n^2+1}} = 1$

$\Rightarrow \sum \frac{1}{\sqrt{n^2+1}}$ diverges, since $\sum \frac{1}{n}$ diverges

$\Rightarrow \boxed{\text{Converges conditionally}}$

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Name: ______________________________

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2	15	
3	15	
4	20	
5	15	
6	15	
7	20	
Total:	100	