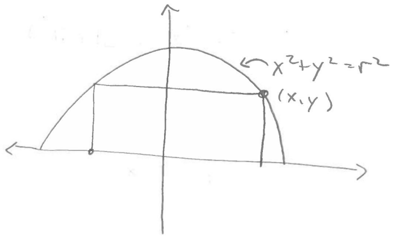
1.



Student A
Homework 8
Math 1310

$A = (2x)\,y$

$y^2 = r^2 - x^2$

$y = \sqrt{r^2 - x^2}$

$A = 2x\sqrt{r^2 - x^2}$

$2\sqrt{r^2-x^2} + \cancel{2}x\left(\cancel{\tfrac{1}{2}}\right)(r^2-x^2)^{-1/2}(-2x)$

$2\dfrac{r^2-x^2}{\sqrt{r^2-x^2}} - 2\dfrac{x^2}{\sqrt{r^2-x^2}}$

$2\dfrac{r^2-2x^2}{\sqrt{r^2-x^2}} = 0$

$r^2 - 2x^2 = 0$

$r^2 = 2x^2$

$x^2 = r^2/2$

$\boxed{x = \pm \dfrac{r}{\sqrt{2}}}$ ← Max.

$y = \sqrt{r^2 - \left(\pm \frac{r}{\sqrt{2}}\right)^2} = \sqrt{\frac{r^2}{2}} = r/\sqrt{2}$ $A = 2\left(\frac{r}{\sqrt{2}}\right)\left(\frac{r}{\sqrt{2}}\right) = \boxed{r^2}$

Ask yourself:

- Can I tell what is the question being answered?
- Can I tell what is being assumed and what is being shown?
- Did the author skip any steps?
- Is it easy to see why each step followed from the last?
- While reading this solution, can I see where the author is going with their work?

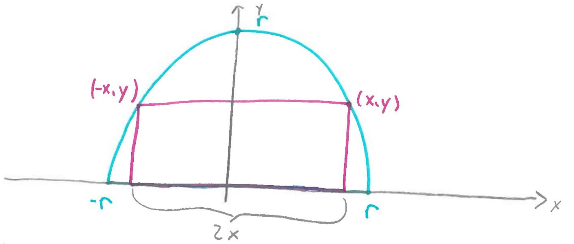
Pro tips:

- Pretend you're teaching someone who's never seen this problem how to solve it.
- Look at the textbook for examples of good mathematical writing

Student B
Homework 8
Math 1310

1. We wish to find the area of the largest rectangle that can be inscribed in a semicircle of radius r.

Let's consider the semi-circle oriented in the plane as below:



which has equations $x^2+y^2=r^2$, $y\geq 0$.

Let (x,y) be the corner of an inscribed rectangle. Then the Area of that rectangle is $A=2xy$. This is the function we want to maximize, but we need to use the constraint, $x^2+y^2=r^2$ to eliminate a variable:

$$x^2+y^2=r^2$$

$$x^2=r^2-y^2$$

$$x=\sqrt{r^2-y^2}$$

Now,

$$A=2xy=2\left(\sqrt{r^2-y^2}\right)y$$

To find the max we need critical points:

$$A=2y\sqrt{r^2-y^2}$$

$$\frac{dA}{dy}=2\sqrt{r^2-y^2}+2y\left(\tfrac{1}{2}\right)\left(r^2-y^2\right)^{-1/2}(-2y)$$

$$=2\,\frac{r^2-2y^2}{\sqrt{r^2-y^2}}$$

$$2\,\frac{r^2-2y^2}{\sqrt{r^2-y^2}}=0 \Rightarrow r^2-2y^2=0$$

$$r^2=2y^2$$

$$y=\pm\frac{r}{\sqrt{2}}$$

We're interested in $y\geq 0$, so max area is when $y=\frac{r}{\sqrt{2}}$, and is

$$A=2\sqrt{r^2-\left(\frac{r}{\sqrt{2}}\right)^2}\left(\frac{r}{\sqrt{2}}\right)$$

$$=2\sqrt{\frac{r^2}{2}}\left(\frac{r}{\sqrt{2}}\right)=\boxed{r^2}.$$