

Name: Solutions

Math 1220-003 Quiz 2

June 19, 2018

You have until the next class to complete this quiz. Make sure to write your name at the top of the quiz. This quiz is two questions, worth 20 points.

1. (10 points) If a radioactive substance loses 20% of its radioactivity in 3 years, what is its half-life?

$$P = P_0 e^{kt} \quad \text{After 3 years, lose 20\%}$$
$$\Rightarrow 80\% \text{ remains.}$$

$$0.8P_0 = P_0 e^{k \cdot 3} \quad \leadsto \ln(0.8) = k \cdot 3$$

$$\Rightarrow k = \ln(0.8)/3.$$

If  $t = \text{half-life}$ , then

$$\frac{1}{2}P_0 = P_0 e^{kt} \quad \Rightarrow \frac{1}{2} = e^{\frac{\ln(0.8)}{3} \cdot t}$$

$$\Rightarrow \ln\left(\frac{1}{2}\right) = \frac{\ln(0.8)}{3} \cdot t \quad \Rightarrow t = 3 \cdot \frac{\ln(1/2)}{\ln(0.8)}$$

$$\approx 9.3 \text{ years.}$$

2. (10 points) Solve the differential equation for  $y$ :  $\frac{dy}{dx} = e^{2x} - 3y$ , where  $y = 1$  when  $x = 0$ .

$$\frac{dy}{dx} + 3y = e^{2x}$$

$$P(x) = 3.$$

$$\text{Integrating factor} = e^{\int 3 dx} = e^{3x}$$

$$\Rightarrow e^{3x} \left( \frac{dy}{dx} + 3y \right) = e^{5x}$$
$$\underbrace{\hspace{10em}}_{\frac{d}{dx} [y \cdot e^{3x}]}$$

$$\Rightarrow y \cdot e^{3x} = \int e^{5x} dx = \frac{1}{5} \int e^u du = \frac{1}{5} e^u + C = \frac{1}{5} e^{5x} + C$$

$\uparrow$   
 $u = 5x$   
 $du = 5 dx$

$$\Rightarrow y = \frac{\frac{1}{5} e^{5x} + C}{e^{3x}} = \frac{1}{5} e^{2x} + C \cdot e^{-3x}$$

$$y(0) = 1 \Rightarrow 1 = \frac{1}{5} e^0 + C \cdot e^0 = \frac{1}{5} + C$$

$$\Rightarrow C = \frac{4}{5}$$

$$\Rightarrow \boxed{y = \frac{1}{5} e^{2x} + \frac{4}{5} e^{-3x}}$$