

Solutions

Name: _____

Math 1220-003 Quiz 5

June 28, 2018

You have until the next class to complete this quiz. Make sure to write your name at the top of the quiz. This quiz is two questions, worth 20 points.

1. (10 points) Find the integral: $\int \sqrt{x^2 + 8x + 15} dx$.

$$x^2 + 8x + 15 = x^2 + 8x + 16 - 16 + 15$$

$$= (x+4)^2 - 1$$

$$\int \sqrt{(x+4)^2 - 1} dx$$

trig sub: $x+4 = \sec t \Rightarrow$

$$dx = \sec t \cdot \tan t dt$$

$$t = \sec^{-1}(x+4)$$

$$\int \sqrt{\sec^2 t - 1} \sec t \cdot \tan t dt = \int |\tan t| \cdot \sec t \cdot \tan t dt$$

$$= \pm \int \tan^2 t \sec t dt$$

$$\int \tan^2 t \sec t dt = \tan(t) \cdot \sec(t) - \int \sec^3(t) dt = \tan(t) \sec(t) - \int (1 + \tan^2 t) \sec t dt$$

$$\begin{array}{l} u = \tan t \quad du = \sec^2 t \cdot \tan t dt \\ du = \sec^2 t \quad v = \sec t \end{array}$$

$$= \tan(t) \sec(t) - \int \sec(t) dt - \int \tan^2 t \sec t dt$$

$$\Rightarrow \int \tan^2 t \sec t dt = \frac{1}{2} (\tan(t) \sec(t) - \int \sec(t) dt) = \frac{1}{2} (\tan(t) \cdot \sec(t) + \ln |\sec(t) + \tan(t)|)$$

$$\Rightarrow \pm \frac{1}{2} (\tan(\sec^{-1}(x+4)) \cdot (x+4) + \ln |x+4 + \tan(\sec^{-1}(x+4))|)$$

2. (10 points) Find the integral: $\int \tan^3 x \sec x \, dx$.

$$\int \tan^2 x \cdot \tan(x) \sec(x) \, dx$$

$$= \int (\sec^2 x - 1) \cdot \tan(x) \sec(x) \, dx = \int u^2 - 1 \, du$$

$u = \sec x$
 $du = \sec x \tan x \, dx$

$$= \frac{u^3}{3} - u + C = \boxed{\frac{\sec^3 x}{3} - \sec x + C}$$