

Name:

Solutions

Math 1220-003 Quiz 6

July 5, 2018

You have until the next class to complete this quiz. Make sure to write your name at the top of the quiz. This quiz is two questions, worth 20 points.

1. (10 points) Find the integral: $\int \frac{1}{x^4 - 16} dx$.

$$x^4 - 16 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x - 2)(x + 2)$$

$$\frac{1}{(x^2 + 4)(x + 2)(x - 2)} = \frac{Ax + B}{x^2 + 4} + \frac{C}{x + 2} + \frac{D}{x - 2}$$

$$1 = (Ax + B)(x + 2)(x - 2) + C(x^2 + 4)(x - 2) + D(x^2 + 4)(x + 2)$$

Plug in $x = 2$: $1 = D(4 + 4)(2 + 2) = D \cdot 32$
 $\Rightarrow D = 1/32$

Plug in $x = -2$: $1 = C(4 + 4)(-4) \Rightarrow C = -1/32$

$$\Rightarrow 1 = (Ax + B)(x + 2)(x - 2) + \frac{-1}{32}(x^2 + 4)(x - 2) + \frac{1}{32}(x^2 + 4)(x + 2)$$

Plug in $x = 0$: $1 = B(2)(-2) - \frac{1}{32}(4)(-2) + \frac{1}{32}(4)(2)$

$$\Rightarrow 1 = -4B + \frac{8}{32} + \frac{8}{32} = -4B + \frac{1}{2} \Rightarrow B = -1/8$$

Plug in $x = 1$: $1 = (A - 1/8)(3)(-1) - \frac{1}{32}(5)(-1) + \frac{1}{32}(5)(3)$

$$\Rightarrow 1 = -3A + \frac{3}{8} + \frac{5}{32} + \frac{15}{32} = -3A + 1 \Rightarrow A = 0$$

$$\int \frac{1}{x^4 - 16} dx = \int \frac{-1/8}{x^2 + 4} dx + \int \frac{-1/32}{x + 2} dx + \int \frac{1/32}{x - 2} dx = \frac{1}{8} \int \frac{-1}{x^2 + 4} dx + \frac{1}{32} \int \frac{-1}{x + 2} dx + \frac{1}{32} \int \frac{1}{x - 2} dx$$

$$= \left[-\frac{1}{16} \tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{32} \ln|x + 2| + \frac{1}{32} \ln|x - 2| \right] + C$$

2. (10 points) Find the integral: $\int \sin^2 x \cos^2 x dx$.

Power-reduction Formula:

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \quad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\int \frac{1 - \cos(2x)}{2} \cdot \frac{1 + \cos(2x)}{2} dx = \frac{1}{4} \int (1 - \cos(2x))(1 + \cos(2x)) dx$$

$$= \frac{1}{4} \int 1 - \cos^2(2x) dx = \frac{1}{4} \int 1 - \frac{1 + \cos(4x)}{2} dx$$

$$= \frac{1}{4} \int \frac{1}{2} - \frac{1}{2} \cos(4x) dx = \frac{1}{4} \left(\frac{x}{2} - \frac{1}{8} \sin(4x) \right) + C$$

$$= \boxed{\frac{x}{8} - \frac{1}{32} \sin(4x) + C}$$