Name: Solutions

Math 1220-003 Quiz 8

July 12, 2018

You have until the next class to complete this quiz. Make sure to write your name at the top of the quiz. This quiz is two questions, worth 20 points.

1. (10 points) For which numbers n does the integral $\int_0^1 \frac{1}{x^n}\,dx$ converge? You may assume that $n > 0$.

$$\int \frac{1}{x^n}\,dx = \begin{cases} \frac{x^{-n+1}}{-n+1}, & \text{if } n \neq 1 \\ \ln(x), & \text{if } n = 1. \end{cases}$$

if $n=1$, $\int_0^1 \frac{1}{x^n}\,dx = \lim_{t\to 0^+} \left[\ln(x)\right]_t^1 = \lim_{t\to 0^+} \ln(1) - \ln(t)$,

$= \infty$ (diverges)

otherwise, $\int_0^1 \frac{1}{x^n}\,dx = \lim_{t\to 0^+} \left.\frac{x^{-n+1}}{-n+1}\right|_t^1 = \lim_{t\to 0^+}\left(\frac{1}{-n+1} - \frac{t^{-n+1}}{-n+1}\right)$

$$\lim_{t\to 0^+} t^{-n+1} = \begin{cases} 0, & -n+1 > 0 \Rightarrow \text{integral converges} \\ \infty, & -n+1 < 0 \Rightarrow \text{integral diverges.} \end{cases}$$

$-n+1 > 0$ means $1 > n$, $-n+1 < 0$ means $1 < n$.

So $\int_0^1 \frac{1}{x^n}\,dx$ converges if $n < 1$ and

diverges if $n \geq 1$.

2. (10 points) Write whether the given series converges or diverges. If it converges, find its sum.

(a) $\displaystyle\sum_{i=1}^{\infty} \frac{3}{i}$

$$\sum_{i=1}^{\infty} 3/i = 3 \underbrace{\sum_{i=1}^{\infty} \frac{1}{i}}_{\text{Harmonic series}} = 3 \cdot \infty$$

Diverges

(b) $\displaystyle\sum_{i=1}^{\infty} \left(\frac{2}{i+1} - \frac{2}{i}\right)$ (Hint: start by finding the partial sum $\displaystyle\sum_{i=1}^{n} \left(\frac{2}{i+1} - \frac{2}{i}\right)$)

$$\sum_{i=1}^{\infty} \frac{2}{i+1} - \frac{2}{i} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{i+1} - \frac{2}{i}$$

$$= \lim_{n\to\infty} \left(\cancel{\frac{2}{2}} - \frac{2}{1} + \cancel{\frac{2}{3}} - \cancel{\frac{2}{2}} + \dots + \frac{2}{n+1} - \cancel{2/n}\right)$$

$$= \lim_{n\to\infty} -2 + \frac{2}{n+1} = \boxed{-2}$$

(converges)