Name: Solutions

Math 1220-003 Quiz 9

July 17, 2018

You have until the next class to complete this quiz. Make sure to write your name at the top of the quiz. This quiz is two questions, worth 20 points.

1. Determine whether the following series converge or diverge. If it converges, find the sum.

(a) (5 points) $\sum_{i=2}^{\infty} \frac{5}{3^i}$

$$= \frac{5}{3^2} \cdot \sum_{i=0}^{\infty} \frac{1}{3^i} = \frac{5}{3^2}\left(\frac{1}{1-1/3}\right)$$

$$= \frac{5}{9} \cdot \left(\frac{1}{2/3}\right)$$

$$= \frac{5}{9} \cdot \frac{3}{2} = 5/6$$

Converges: geometric series with ratio < 1.

(b) (5 points) $\sum_{i=10}^{\infty} \frac{1}{i^{1/2}}$

Diverges: p-series with $p = 1/2$.

2. Does the series $\sum_{i=1}^{\infty} \sin^2\left(\frac{1}{i}\right)$ converge or diverge?

Limit comparison test: compare to $\sum 1/i^2$

$$\lim_{i\to\infty} \frac{\sin^2(1/i)}{1/i^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{i\to\infty} \frac{2\sin(1/i)\cos(1/i)\cdot -1/i^2}{-2/i^3}$$

$$= \lim_{i\to\infty} \frac{\sin(1/i)\cos(1/i)}{1/i}$$

product rule + chain rule.

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{i\to\infty} \frac{\cos(1/i)\cdot \cancel{1/i^2}\cdot\cos(1/i) + \sin(1/i)\cdot(-\sin 1/i)\cdot\cancel{-1/i^2}}{\cancel{-1/i^2}}$$

$$= \lim_{i\to\infty} \cos^2(1/i) - \sin^2(1/i) = \cos^2(0) - \sin^2(0)$$

$$= 1.$$

$\Rightarrow \sum \sin^2\left(\frac{1}{i}\right)$ and $\sum \frac{1}{i^2}$ either both converge or both diverge.

$\Rightarrow \sum \sin^2(1/i)$ converges.

Alternatively, $\lim_{i\to\infty} \frac{\sin^2(1/i)}{1/i^2} = \lim_{i\to 0^+} \frac{\sin^2(i)}{i^2}$

$$= \lim_{i\to 0^+} \frac{2\sin(i)\cos(i)}{2i} = \lim_{i\to 0^+} \frac{-\sin^2(i)+\cos^2(i)}{1}$$

$$= 1.$$