I. Matrices / vocab

II. Linear systems

III. Using matrices to solve lin. sys.

Vocab $\mathbb{R}$ = set of real #s

e.g. 2, $\sqrt{5}$, π,

NOT $2+i$

- A matrix is a rectangular array of #s.

e.g. $M = \begin{bmatrix} 1 & 6 & 4 \\ 3 & 7 & 2 \end{bmatrix}$ is a 2×3 matrix

row column

convention: use lower-case letters with two subscripts to denote the entries of a matrix

e.g. $m_{21} = 3$, $m_{23} = 2$, $m_{13} = 4$

- A vector is a matrix with only one column

e.g. $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}$ is a vector.

in fact, it is a 4-dimensional vector.

- Given a whole # n, we write $\mathbb{R}^n$ to denote the set of n-dimensional vectors with entries in $\mathbb{R}$ 1.2

(Aside: $\mathbb{C}$ = set of complex #s.

$\mathbb{C}^n$ = set of n-dimensional vectors with entries in $\mathbb{C}$.

e.g. $\begin{bmatrix}1\\2\\3\end{bmatrix}$ is in $\mathbb{R}^3$. It is not in $\mathbb{R}^4$

$\begin{bmatrix}1\\2\\3\end{bmatrix}$ is also in $\mathbb{C}^3$.

$\begin{bmatrix}1+i\\4\\6\end{bmatrix}$ is in $\mathbb{C}^3$ but <u>not</u> in <u>$\mathbb{R}^3$</u>

$\mathbb{R}^3 \subseteq \mathbb{C}^3$

"is a subset of"

"is contained in"

Q Solve the following linear systems:

① $x+y=1$
$2x+3y=5$

② $7x+7y=7$
$2x+3y=5$

③ $x+y=1$
$3x+4y=6$

④ $x+y=1$
$y=3$

⑤ $2x+3y=5$
$x+y=1$

What do you notice? Why?

A They all have same solutions!

~~$x=3, y=2$~~ $x=-2, y=3$

Why?

① and ②: if you divide $7x+7y=7$ by 7, you get $x+y=1$

i.e., you get ② from ① by multiplying the top row by 7.

① and ③: you get ③ from ① by ~~add~~ replacing the bottom row with bottom row + top row

$$\begin{aligned} x+y&=1 \\ 2x+3y&=5 \end{aligned} \implies \begin{aligned} x+y&=1 \\ 3x+4y&=6 \end{aligned}$$

$$\begin{array}{r} x+y=1 \\ +\quad 2x+3y=5 \\ \hline 3x+4y=6 \end{array}$$

④ = get this by subtracting ~~3~~ 2 · (top row of ①) from the bottom row

In summary, you can perform the following operations on a sys. of lin. eqs without changing the solutions:

1. Multiply any equation by a constant
2. Add a multiple of one equation to another
3. Swap the order of the equations

Solving lin. sys. with matrices

eg. Solve
$$\begin{aligned} x + 4y + 2z &= 1 \\ y + z &= -1 \\ -6y - 9z &= -3 \end{aligned}$$

First: rewrite this system using a augmented matrix

$$\begin{array}{cccc} & & & \text{RHS} \\ \left[\begin{array}{ccc|c} 1 & \boxed{4} & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & -6 & -9 & -3 \end{array}\right] \\ \quad x \quad y \quad z \quad "=" \end{array}$$

add $-4 \cdot$ row 2 to row 1

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -1 \\ 0 & \boxed{-6} & -9 & -3 \end{array}\right]$$

add $6 \cdot$ row 2 to row 3

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -3 & -9 \end{array}\right]$$

divide row 3 by -3

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

add $-1 \cdot$ row3 to row 2

Q how can we add multiples of row 3 to other rows to clear circled entries?

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

add $2 \cdot$ row 3 to row 1

$$\begin{array}{c} \boxed{\left[\begin{array}{ccc|c} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ x & y & z \end{array} \end{array}$$

In summary, we rewrote our system as a matrix, then we performed these 3 operations, to get this matrix

$$\begin{aligned} 1x + 0y + 0z &= 11 \\ 0x + 1y + 0z &= -4 \\ 0x + 0y + 1z &= 3 \end{aligned}$$

$x = 11$, $y = -4$, $z = 3$

page 12

Vocab: these three operations are called "elementary row operations"

Notation $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -4 \\ 3 \end{bmatrix}$