8/31/22

Agenda: Finish 1.2 (solving lin sys, Row Reduced Echelon Form)

Assignment: Read 1.3, watch week 1 vids.

Recall Solving lin sys w/ row ops.

$$\begin{aligned} x+2y&=1 \\ x+4y&=1 \end{aligned} \rightsquigarrow \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & 4 & 1 \end{array}\right] \xrightarrow{R2-R1} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{R1-R2} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 2 & 0 \end{array}\right] \xrightarrow{R2\cdot\frac{1}{2}} \underbrace{\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \end{array}\right]}_{\text{Goal}} \qquad \begin{aligned} 1x+0y&=1 \\ 0x+1y&=0 \end{aligned}$$

i.e. $x=1$, $y=0$

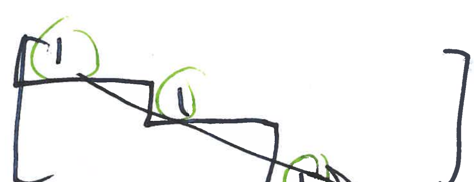
This is an example of "Reduced Row Echelon Form"

$\underbrace{\text{Echelon}}_{=\text{staircase}}$

Def (RREF):

1. Leftmost nonzero entry of each row (if it exists) is 1. This entry is called a "pivot"
2. All entries above and below a pivot are 0.
3. Pivots form a "staircase." I.e., if row A is above row B, then the pivot in Row A is to the left of the pivot in row B.

3* Any row of all zeros is always at the bottom.



E.g.

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1 ✓
2 ✓
3 ✓

IS RREF

$$\begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Fails 3

$$\begin{bmatrix} 1 & 2 & \cancel{3} & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Fails 2

Exc. 1.2 # 18–20

18a: Fails rule 2

b: Good ✓

c: Row of zeros must be at the bottom

d: ✓

19. $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

FACT each matrix has a unique RREF which can be obtained through elementary row ops.

There is an algorithm for finding it (p.15)

- for each row, divide by a const to make its leftmost nonzero entry a 1
- ~~Use row op~~ Add / subtract this row from others to clear the entries above / below that 1
- Swap the order of rows so that the pivots form a staircase

E.g. Put the matrix $\begin{bmatrix} 2 & 4 & 6 & 8 \\ 0 & 1 & 2 & 1 \\ 1 & 3 & 5 & 5 \end{bmatrix}$ in RREF

$R1 \cdot \frac{1}{2}$: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & 3 & 5 & 5 \end{bmatrix}$ $R3-R1$: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

$R1-2\cdot R2$: $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ $R3-R2$: $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Q Find the solutions to the system of eqs,

$$\begin{aligned} 2x+4y+6z &= 8 \\ y+2z &= 1 \\ x+3y+5z &= 5 \end{aligned} \rightsquigarrow \left[\begin{array}{ccc|c} 2 & 4 & 6 & 8 \\ 0 & 1 & 2 & 1 \\ 1 & 3 & 5 & 5 \end{array}\right]$$

$$\downarrow \text{RREF}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Solutions of the original sys are the solutions of

$x - z = 2, \quad y + 2z = 1$

ONLY 2 eqs! 3 vars.

$\Rightarrow$ There are inf. many solutions.

The third equation above was actually redundant!

Solutions: $x = 2 + z, \quad y = 1 - 2z$, z is any real #.

Exit ticket: NAME, Ask me a question.