Math 214-007 Singular Value Decomposition worksheet

1. Let A have the singular value decomposition

$$A = \begin{bmatrix} | & | & | & | \\ \vec{u}_1 & \vec{u}_2 & \vec{u}_3 & \vec{u}_4 \\ | & | & | & | \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} — & \vec{v}_1^\top & — \\ — & \vec{v}_2^\top & — \\ — & \vec{v}_3^\top & — \end{bmatrix}$$

   Which vectors $\vec{v}_i$ are in the kernel of A? Which vectors $\vec{u}_i$ are in the image of A?

2. Let B have the singular value decomposition

$$A = \begin{bmatrix} | & | & | \\ \vec{u}_1 & \vec{u}_2 & \vec{u}_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} — & \vec{v}_1^\top & — \\ — & \vec{v}_2^\top & — \\ — & \vec{v}_3^\top & — \\ — & \vec{v}_4^\top & — \end{bmatrix}$$

   Which vectors $\vec{v}_i$ are in the kernel of B? Which vectors $\vec{u}_i$ are in the image of A?

3. Explain how to use the SVD of a matrix to quickly see: its rank, its nullity, an orthonormal basis of its image, and an orthonormal basis of its kernel

4. True or false! (Taken from the textbook)

   (a) If A is a 2×2 matrix with singular values 3 and 5, then there is some $\vec{w} \in \mathbb{R}^2$ with $||\vec{w}|| = 1$ and $||A\vec{w}|| = 2$

   (b) If A is a 2×2 matrix with singular values 3 and 5, then there is some $\vec{w} \in \mathbb{R}^2$ with $||\vec{w}|| = 1$ and $||A\vec{w}|| = 4$

   (c) The product of the n singular values of an $n \times n$ matrix must be $|\det A|$.