Agenda:
- warm-up problem
- more on change of coords for vectors
- change of coords for transforms

§2.4

104. The color of light can be represented in a vector

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix},$$

where R = amount of red, G = amount of green, and B = amount of blue. The human eye and the brain transform the incoming signal into the signal

$$\begin{bmatrix} I \\ L \\ S \end{bmatrix},$$

where

$$\begin{aligned} \text{intensity} \quad I &= \frac{R+G+B}{3} = \tfrac{1}{3}R + \tfrac{1}{3}G + \tfrac{1}{3}B \\ \text{long-wave signal} \quad L &= R - G = 1R - 1G + 0B \\ \text{short-wave signal} \quad S &= B - \frac{R+G}{2}. = -\tfrac{1}{2}R - \tfrac{1}{2}G + B \end{aligned}$$

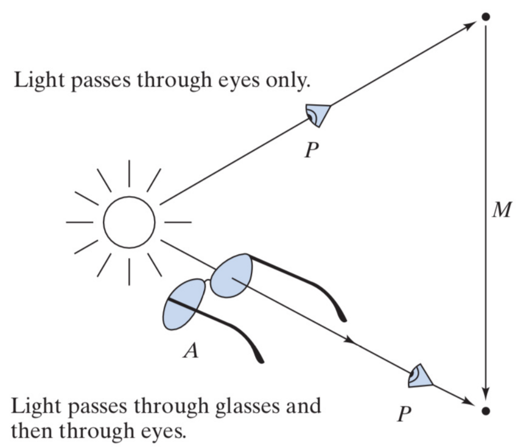
a. Find the matrix P representing the transformation from

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} \text{ to } \begin{bmatrix} I \\ L \\ S \end{bmatrix}.$$

b. Consider a pair of yellow sunglasses for water sports that cuts out all blue light and passes all red and green light. Find the 3×3 matrix A that represents the transformation incoming light undergoes as it passes through the sunglasses. All the entries of your matrix A will be 0's and 1's.

c. Find the matrix for the composite transformation that light undergoes as it first passes through the sunglasses and then the eye.

d. As you put on the sunglasses, the signal you receive (intensity, long- and short-wave signals) undergoes a transformation. Find the matrix M of this transformation. Feel free to use technology.



a) Find P s.t. $P\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} I \\ L \\ S \end{bmatrix}$ $\qquad P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1 & -1 & 0 \\ -1/2 & -1/2 & 1 \end{bmatrix}$

b) Want A s.t. $A \cdot \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} R \\ G \\ 0 \end{bmatrix}$

$A\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad A\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad A \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} R \\ G \\ B \end{bmatrix} \xrightarrow[\substack{A \\ \text{first}}]{} \begin{bmatrix} R \\ G \\ 0 \end{bmatrix} \xrightarrow[\substack{P \\ \text{second}}]{\text{translate to ILS}} \begin{bmatrix} I \\ L \\ S \end{bmatrix}$

$P \cdot A = \begin{bmatrix} 1/3 & 1/3 & 0 \\ 1 & -1 & 0 \\ -1/2 & -1/2 & 0 \end{bmatrix}$

d) $\begin{bmatrix} R \\ g \\ B \end{bmatrix} \xrightarrow{\text{sunglasses}} \begin{bmatrix} R \\ g \\ B \end{bmatrix}$

$$\begin{tikzcd}
\begin{bmatrix} I \\ L \\ S \end{bmatrix} \arrow[r, "\text{sunglasses}"] \arrow[d, "P^{-1}"', "\text{translate to RGB}"] & ??? \\
\begin{bmatrix} R \\ g \\ B \end{bmatrix} \arrow[r, "A"] & \begin{bmatrix} R \\ g \\ B \end{bmatrix} \arrow[u, "P"']
\end{tikzcd}$$

a): $\begin{bmatrix} R \\ g \\ B \end{bmatrix} \underset{P^{-1}}{\overset{P}{\rightleftarrows}} \begin{bmatrix} I \\ L \\ S \end{bmatrix}$

PAP^{-1}

Similar to

Sep 14

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§3.4 **Recall:** if $\mathcal{B} = \{\vec{v}_1, \dots, \vec{v}_n\}$ a basis for $\mathbb{R}^n$, and $\vec{x} \in \mathbb{R}^n$. "Coords of $\vec{x}$ in terms of this basis"

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} \iff \vec{x} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n$$
$$= \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & & | \end{bmatrix} \underbrace{\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}}_{[\vec{x}]_{\mathcal{B}}}$$

FACT $\underbrace{\vec{x}}_{\text{"std coordinates"}} = \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & & | \end{bmatrix} [\vec{x}]_{\mathcal{B}}$, $\quad \underbrace{\begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & & | \end{bmatrix}^{-1}}_{\text{why do we know this matrix is invertible?}} \vec{x} = [\vec{x}]_{\mathcal{B}}$

e.g. let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$ basis for $\mathbb{R}^2$

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ becomes $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ in $\mathcal{B}$-coords. Accordingly,

$$\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

the vector $\vec{x} = 2\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix}$.

Then $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix}2\\1\end{bmatrix}$. Accordingly, $\begin{bmatrix}1 & 2\\1 & -1\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}4\\1\end{bmatrix}$

Also: $\begin{bmatrix}1 & 2\\1 & -1\end{bmatrix}^{-1}\underbrace{\begin{bmatrix}4\\1\end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix}2\\1\end{bmatrix}}_{[\vec{x}]_{\mathcal{B}}}$

Matrix for a linear transformation with respect to a basis

Let $T: \mathbb{R}^n \longrightarrow \mathbb{R}^n$. If A is the matrix of T, then $A = \left[T(\vec{e}_1), T(\vec{e}_2), \ldots, T(\vec{e}_n)\right]$

Let $\mathcal{B} = \{\vec{v}_1, \ldots, \vec{v}_n\}$ be a basis of $\mathbb{R}^n$. Then the matrix of T with respect $\mathcal{B}$ is

$$M = \left[[T(\vec{v}_1)]_{\mathcal{B}}, [T(\vec{v}_2)]_{\mathcal{B}}, \ldots, [T(\vec{v}_n)]_{\mathcal{B}}\right]$$

then $M[\vec{x}]_{\mathcal{B}} = [T(\vec{x})]_{\mathcal{B}}$