Q1: if $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ then T has an associated matrix

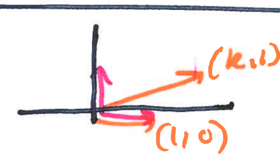
But: $\frac{d}{dx} : \underbrace{C^1(\mathbb{R})}_{\text{differentiable functions on } \mathbb{R}.} \longrightarrow C^1(\mathbb{R})$

- $\frac{d}{dx}(f+g) = \frac{d}{dx}f + \frac{d}{dx}g,$
- $\frac{d}{dx}(c \cdot f) = c \frac{d}{dx} f$

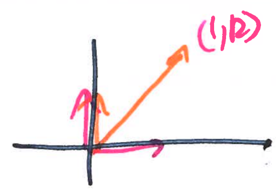
Linear function! Matrix? $\frac{d}{dx} \underbrace{\vec{e}_1}_{???}$

Q2. "Shearing?"

$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$



$\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$



$$\begin{bmatrix} 1 & k_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & k_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & k_1+k_2 \\ 0 & 1 \end{bmatrix}$$

Q3: How to tell if a matrix is augmented?

$$\begin{aligned} 2x_1 + x_2 &= 0 \\ 3x_1 + 2x_2 &= -7 \end{aligned} \sim \left[\begin{array}{cc|c} 2 & 1 & 0 \\ 3 & 2 & -7 \end{array}\right] \qquad \begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & -7 \end{bmatrix}$$

$\hookrightarrow$ $2x_1 + x_2 + 0x_3 = ?$

$3x_1 + 2x_2 - 7x_3 = ?$

$$\rightarrow \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -7 \end{bmatrix}$$

Matrix inverses:

- computing
- practice
- Determinants

Recall an $n \times n$ matrix A is "invertible" $\Leftrightarrow$ $\mathrm{rref}(A) = I_n$

$$= \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$$

$\Leftrightarrow \exists$ an $n \times n$ matrix "A^{-1}" such that $AA^{-1} = A^{-1}A = I_n$

Given a lin sys $A\vec{x} = \vec{b}$. If A is invertible:

$$\underbrace{A^{-1}A}_{I_n}\vec{x} = A^{-1}\vec{b} \Rightarrow I_n\vec{x} = A^{-1}\vec{b}$$

$$\Rightarrow \boxed{\vec{x} = A^{-1}\vec{b}}$$

So, to solve $A\vec{x} = \vec{b}$, find A^{-1}. Then $\vec{x} = A^{-1}\vec{b}$.

To solve $A\vec{x} = \vec{c}$, $\vec{x} = A^{-1}\vec{c}$

We only have to compute A^{-1} once in order to solve many diff. systems!

Computing A^{-1}: (pp. 90-91)

Given $A = \begin{bmatrix} 3 & 4 \\ 0 & 1 \end{bmatrix}$. $A^{-1} = ?$

$$3x_1 + 4x_2 = y_1$$
$$x_2 = y_2$$

Solve for x_1 and x_2 in terms of y_1 and y_2!

$\hookrightarrow A\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \boxed{?}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$$\begin{aligned} 3x_1 + 4x_2 &= y_1 \\ x_2 &= y_2 \end{aligned} \quad \overset{R_1 - 4R_2}{\leadsto} \quad \begin{aligned} 3x_1 &= y_1 - 4y_2 \\ x_2 &= y_2 \end{aligned}$$

$$\overset{\frac{1}{3}R_1}{\leadsto} \quad \begin{aligned} x_1 &= \tfrac{1}{3}y_1 - \tfrac{4}{3}y_2 \\ x_2 &= y_2 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1/3 & -4/3 \\ 0 & 1 \end{bmatrix}}_{A^{-1}} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

In general:

$$[A \,\vdots\, I_n] \xrightarrow{\text{RREF}} [\underbrace{I_n}_{I_n} \,\vdots\, A^{-1}]$$

Problem 1 Find the inverse of the matrix,

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}^{-1} = \ ?$$

Problem 2 Do 76. Then 67, 70, and 72

there exists a vector $\vec{b}$ in $\mathbb{R}^n$ such that the system $A\vec{x} = \vec{b}$ is inconsistent.

52. For

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 2 & 4 \\ 0 & 3 & 6 \\ 1 & 4 & 8 \end{bmatrix}.$$

[12] This will always be the case for a "productive" economy. See Exercise 103.

For two invertible $n \times n$ matrices A and B, determine which of the formulas stated in Exercises 67 through 75 are necessarily true.

67. $(A + B)^2 = A^2 + 2AB + B^2$

68. $(A - B)(A + B) = A^2 - B^2$

69. $A + B$ is invertible, and $(A + B)^{-1} = A^{-1} + B^{-1}$

70. A^2 is invertible, and $(A^2)^{-1} = (A^{-1})^2$

71. $ABB^{-1}A^{-1} = I_n$

72. $ABA^{-1} = B$

73. $(ABA^{-1})^3 = AB^3A^{-1}$

74. $(I_n + A)(I_n + A^{-1}) = 2I_n + A + A^{-1}$

75. $A^{-1}B$ is invertible, and $(A^{-1}B)^{-1} = B^{-1}A$

76. Find all linear transformations T from $\mathbb{R}^2$ to $\mathbb{R}^2$ such that

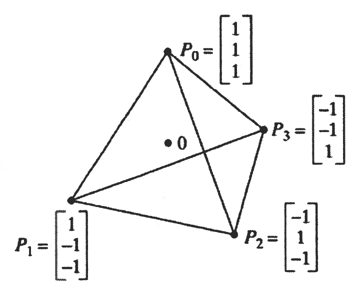
$$T\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \text{and} \quad T\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

Hint: We are looking for the 2×2 matrices A such that

$$A\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \text{and} \quad A\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

These two equations can be combined to form the matrix equation

$$A\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}.$$



Let T from $\mathbb{R}^3$ to $\mathbb{R}^3$ be the rotation about the axis through the points 0 and P_2 that transforms P_1 into P_3. Find the images of the four corners of the tetrahedron under this transformation.

$$P_0 \xrightarrow{T}$$
$$P_1 \to P_3$$
$$P_2 \to$$
$$P_3 \to$$

Let L from $\mathbb{R}^3$ to $\mathbb{R}^3$ be the reflection about the plane

Problem 3

104. The color of light can be represented in a vector

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix},$$

where R = amount of red, G = amount of green, and B = amount of blue. The human eye and the brain transform the incoming signal into the signal

$$\begin{bmatrix} I \\ L \\ S \end{bmatrix},$$

where

$$\begin{aligned} \text{intensity} \quad & I = \frac{R+G+B}{3} \\ \text{long-wave signal} \quad & L = R - G \\ \text{short-wave signal} \quad & S = B - \frac{R+G}{2}. \end{aligned}$$

a. Find the matrix P representing the transformation from

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} \text{ to } \begin{bmatrix} I \\ L \\ S \end{bmatrix}.$$

b. Consider a pair of yellow sunglasses for water sports that cuts out all blue light and passes all red and green light. Find the 3×3 matrix A that represents the transformation incoming light undergoes as it passes through the sunglasses. All the entries of your matrix A will be 0's and 1's.

c. Find the matrix for the composite transformation that light undergoes as it first passes through the sunglasses and then the eye.

d. As you put on the sunglasses, the signal you receive (intensity, long- and short-wave signals) undergoes a transformation. Find the matrix M of this transformation. Feel free to use technology.