"Is $\mathbb{R}^3$ considered a 'flat' shape?"

Recall: intuitively, a subspace of $\mathbb{R}^n$ is a flat, infinite shape containing $\vec{0}$

Yes

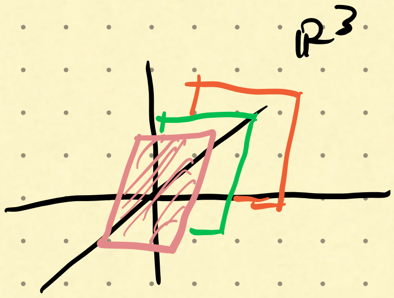
Def A subspace is a set which is the span of some vectors.

Equivalently: $V \subseteq \mathbb{R}^n$ is a subspace $\iff$ $\vec{0} \in V$

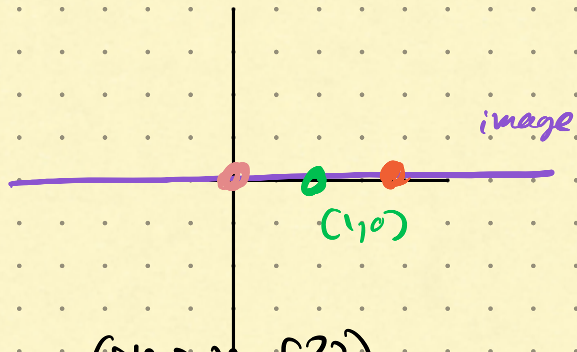
$\vec{v}_1, \vec{v}_2 \in V \Rightarrow \vec{v}_1 + \vec{v}_2 \in V$

$c \in \mathbb{R}, \vec{v}_1 \in V \Rightarrow c\vec{v}_1 \in V$

Kernels: how to visualize them?



$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = A$$



ker: $x - y + 2z = 0$

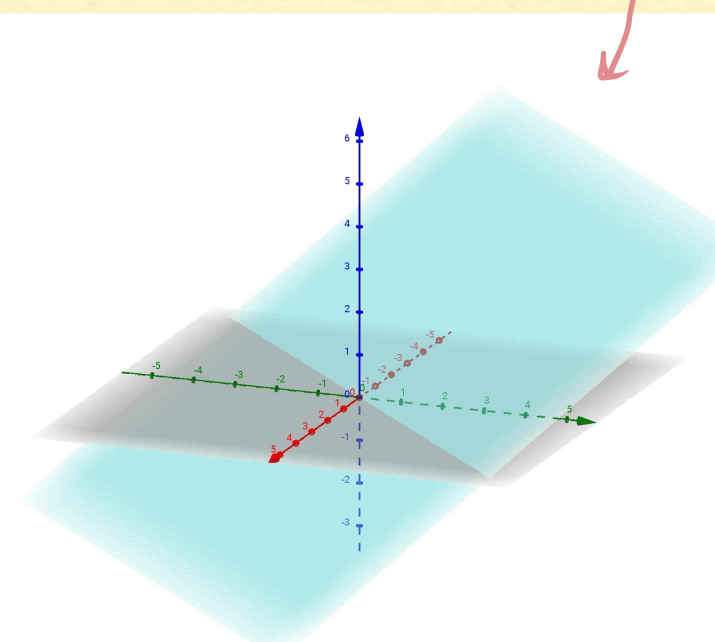


image: $\text{span}\left(\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}-1\\0\end{bmatrix}, \begin{bmatrix}2\\0\end{bmatrix}\right)$

$= \text{span}\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$

$A\begin{bmatrix}x\\y\\z\end{bmatrix} = 0 \iff \begin{bmatrix}x\\y\\z\end{bmatrix}$ in the plane $x - y + 2z = 0$

$A\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} \iff \begin{bmatrix}x\\y\\z\end{bmatrix}$ in the plane $x - y + 2z = 1$

Kernel is the stuff which your transform collapses to a single pt.

How to prepare for midterm?

- Do old exam problems, hw problems
- Check solutions
- Revisit textbook / notes
- Talk to someone!

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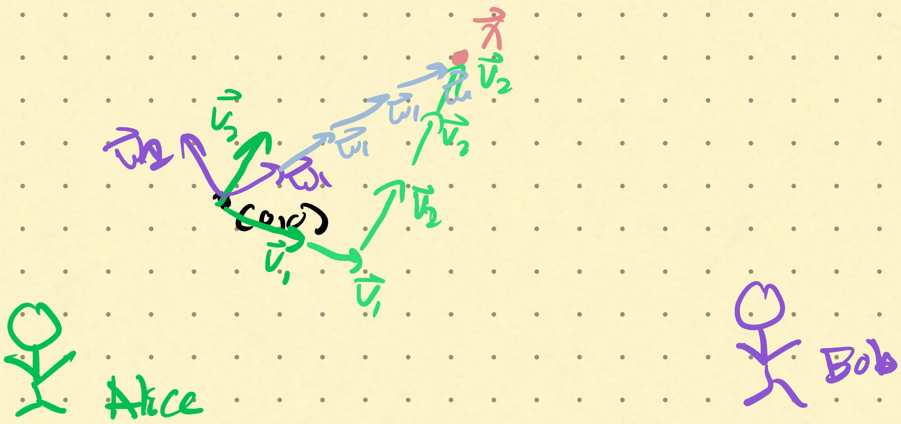
§3.4 Coordinates



Q: What are the coordinates of $\vec{x}$?

A: Depends on how you draw your axes!

Alice: $\vec{x} = 2\vec{v}_1 + 3\vec{v}_2$, so $\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Bob: $\vec{x} = 4\vec{w}_1$, so $\vec{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

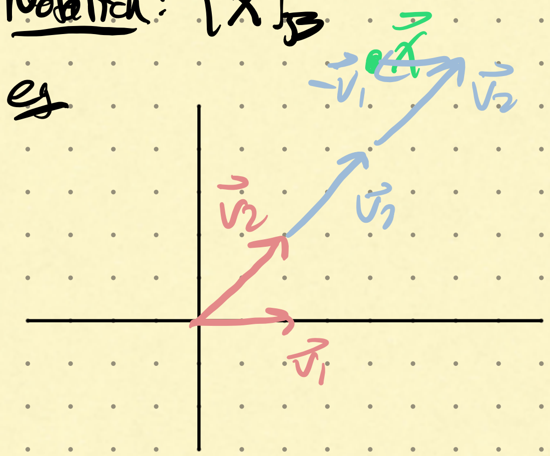
Overarching question: Given a vector $\vec{x}$, how do we write its coordinates using some arbitrary choice of axes? How do we translate between the answers we get using different axes?

axes we're using

Def Let $\mathcal{B} = \{\vec{v}_1, \dots, \vec{v}_n\}$ be a basis of $\mathbb{R}^n$. Let $\vec{x} \in \mathbb{R}^n$. We say the coordinate vector of $\vec{x}$, with respect to $\mathcal{B}$ is $\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$ if $\vec{x} = c_1\vec{v}_1 + \dots + c_n\vec{v}_n$

Notation: $[\vec{x}]_{\mathcal{B}}$

eg



$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\vec{x} = -\vec{v}_1 + 3\vec{v}_2$

So $[\vec{x}]_{\{\vec{v}_1, \vec{v}_2\}} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

eg $\vec{w}_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\vec{w}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$[\vec{x}]_{\{\vec{w}_1, \vec{w}_2\}} = ?$

$\vec{x} = -\frac{1}{2}\vec{w}_1 + 3\vec{w}_2$

$[\vec{x}]_{\{\vec{w}_1, \vec{w}_2\}} = \begin{bmatrix} -1/2 \\ 3 \end{bmatrix}$

exc $\vec{b}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\vec{b}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Find the coords of $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ with respect to $\{\vec{b}_1, \vec{b}_2\}$

$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 3\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{\vec{b}_1} + 2\underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\vec{b}_2} \rightsquigarrow \left[\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right]_{\{\vec{b}_1, \vec{b}_2\}} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

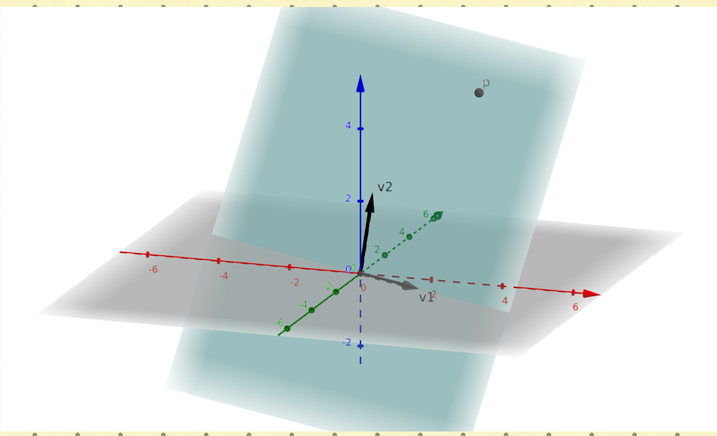
Note: we can do all this ~~for~~ for subspaces too:

If $W \subseteq \mathbb{R}^n$ and $\mathcal{B} = \{\vec{w}_1, \ldots, \vec{w}_d\}$ is a basis of W, and $\vec{x} \in W$, then

$$[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ \vdots \\ c_d \end{bmatrix} \iff \vec{x} = c_1 \vec{w}_1 + \cdots + c_d \vec{w}_d$$

eg. Consider the plane $x + 2y - z = 0$, $\vec{p} = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$ (in plane)

Q Find the coords of $\vec{p}$ w.r.t. the basis $\underbrace{\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}}_{\vec{v}_1}, \underbrace{\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}}_{\vec{v}_2}$



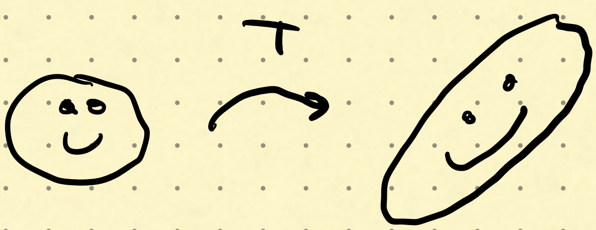
A Want $\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ such that

$$c_1 \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \vec{p}$$

Solve: $\begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$

Answer: $\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 5/2 \end{bmatrix} = [\vec{p}]_{\{\vec{v}_1, \vec{v}_2\}}$

Linear transformations The matrix corresp. to a given lin. transformation also depends on your choice of axes.



stretch x2 in diagonal direction.

Q What is the matrix of T?

$\vec{w}_1 = \begin{bmatrix}1\\1\end{bmatrix}$, $\vec{w}_2 = \begin{bmatrix}-1\\1\end{bmatrix}$

$\vec{v}_1 = \begin{bmatrix}1\\0\end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix}0\\1\end{bmatrix}$

Matrix: $\begin{bmatrix}3/2 & 1/2\\1/2 & 3/2\end{bmatrix}$

(Sep 14 notes)

From Bob's perspective, T is stretching by 2 in the $\vec{w}_1$ direction

Matrix: $\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}$