Office hours:

4 - 5 pm Tuesdays / 1-2 pm Wednesdays

Same link as section.

email me before coming.

weisman@math.utexas.edu

First quiz to next Thursday, 2/3

Google "Quest UT"

Sign in w/ EID.

Dot products (inner product, scalar product)

Cross products (vector product)

Basic properties of vector operations on sheet.

$$(\vec{a}+\vec{b})\times(\vec{c}) = \vec{a}\times\vec{c} + \vec{b}\times\vec{c} \quad ? \quad ✓$$

Computing dot products:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \cdot \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = x_1x_2 + y_1y_2 + z_1z_2$$

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 6 \end{pmatrix} = 4 + (-2)\cdot 6 = -8$$

$$\langle x_1, y_1, z_1 \rangle \cdot \langle x_2, y_2, z_2 \rangle$$

$$a\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} ax_1 \\ ay_1 \\ az_1 \end{pmatrix}$$

$$\left(a\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}\right) \cdot \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

$$\begin{pmatrix} ax_1 \\ ay_1 \\ az_1 \end{pmatrix} \cdot \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = ax_1x_2 + ay_1y_2 + az_1z_2$$

$$= a\left[\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \cdot \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}\right]$$

$$(a\vec{u})\cdot\vec{v} = a(\vec{u}\cdot\vec{v})$$

$$\boxed{\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}}$$

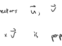
Can check using definition of dot product.

Cross products

vectors $\vec{u}$, $\vec{v}$

$\vec{u}\times\vec{v}$ is perpendicular to both $\vec{u}$ and $\vec{v}$

has length $\sin(\angle \vec{u},\vec{v})$.



Three special vectors $\vec{i}$, $\vec{j}$, $\vec{k}$.

"Standard basis vectors":

$$\vec{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Order matters!

$\vec{i}\times\vec{j} = \vec{k}$	$\vec{j}\times\vec{i} = -\vec{k}$
$\vec{j}\times\vec{k} = \vec{i}$	$\vec{k}\times\vec{j} = -\vec{i}$
$\vec{k}\times\vec{i} = \vec{j}$	$\vec{i}\times\vec{k} = -\vec{j}$

$$\vec{i}\times\vec{i} = \vec{j}\times\vec{j} = \vec{k}\times\vec{k} = 0.$$

i j k (i) (j)

$$(3\vec{i} + 4\vec{j} - 6\vec{k}) \times (\vec{i} - \vec{j} + \vec{k})$$

$$3\vec{i}\times(\vec{i}-\vec{j}+\vec{k}) + 4\vec{j}\times(\vec{i}-\vec{j}+\vec{k}) - 6\vec{k}\times(\vec{i}-\vec{j}+\vec{k})$$

$$-3\vec{k} - 3\vec{j} + -4\vec{k} + 4\vec{i} - 6\vec{j} - 6\vec{i}$$

$$\boxed{-2\vec{i} - 9\vec{j} - 7\vec{k}}$$

Method II: determinants

$$\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right)$$

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$\det\left(\begin{bmatrix} \overset{+}{a} & \overset{-}{b} & \overset{+}{c} \\ d & e & f \\ g & h & i \end{bmatrix}\right) =$$

$$a\cdot\det\begin{pmatrix} e & f \\ h & i \end{pmatrix} - b\det\begin{pmatrix} d & f \\ g & i \end{pmatrix} + c\det\begin{pmatrix} d & e \\ g & h \end{pmatrix} =$$

$$a(ei - fh) - b(di - gf) + c(dh - ge).$$

$$(3\vec{i} + 4\vec{j} - 6\vec{k}) \times (\vec{i} - \vec{j} + \vec{k}) =$$

(1, -1, 1)

$$\det\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & -6 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\vec{i}\cdot\det\begin{pmatrix} 4 & -6 \\ -1 & 1 \end{pmatrix} - \vec{j}\det\begin{pmatrix} 3 & -6 \\ 1 & 1 \end{pmatrix} + \vec{k}\det\begin{pmatrix} 3 & 4 \\ 1 & -1 \end{pmatrix}$$

$$\vec{i}(4-6) - \vec{j}(3+6) + \vec{k}(-3-4)$$

$$-2\vec{i} - 9\vec{j} - 7\vec{k}.$$