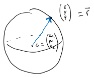
11, 12, 15 on Quest HW.

11. As r varies, the equation $(r-a)\cdot(r-b)=0$ describes a sphere.

Find the center of this sphere.

Why?

Write down equation for sphere w/ center c and radius D.



$$\left\| \begin{pmatrix} x - x_c \\ y - y_c \\ z - z_c \end{pmatrix} \right\|^2 = D^2$$

$$\|\vec{r} - \vec{c}\|^2 = D^2$$

(★) $(\vec{r}-\vec{c})\cdot(\vec{r}-\vec{c}) = D^2$

$(\vec{r}-\vec{a})\cdot(\vec{r}-\vec{b}) = 0$ ← this is the equation for a sphere

Find a vector $\vec{c}$ so this equation is in form (★).

That vector $\vec{c}$ is center of sphere.

→ Should be equivalent to an equation w/ a different form.

$(\vec{r}-\vec{a})\cdot(\vec{r}-\vec{b}) =$

$\vec{r}\cdot\vec{r} - \vec{a}\cdot\vec{r} - \vec{r}\cdot\vec{b} + \vec{a}\cdot\vec{b} = 0$

(★): equivalent to $\vec{r}\cdot\vec{r} - \vec{c}\cdot\vec{r} - \vec{r}\cdot\vec{c} + \vec{c}\cdot\vec{c} = D^2$

Make a substitution to see that these are the same.

(Δ) $\vec{r}\cdot\vec{r} - (\vec{a}+\vec{b})\cdot\vec{r} + \vec{a}\cdot\vec{b} = 0$

(★) $\vec{r}\cdot\vec{r} - 2\vec{c}\cdot\vec{r} + \vec{c}\cdot\vec{c} - D^2 = 0$

both hold for all $\vec{r}$

$\vec{a}+\vec{b} = 2\vec{c}$, $\vec{c} = \frac{\vec{a}+\vec{b}}{2}$

$$\vec{r}\cdot\vec{r} - (\vec{a}+\vec{b})\cdot\vec{r} + \frac{(\vec{a}+\vec{b})}{2}\cdot\frac{(\vec{a}+\vec{b})}{2} - D^2 = 0$$

$$\vec{r}\cdot\vec{r} - (\vec{a}+\vec{b})\cdot\vec{r} + \frac{(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})}{4} - D^2$$

$$\vec{r}\cdot\vec{r} - (\vec{a}+\vec{b})\cdot\vec{r} + \frac{\vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b}}{4} - D^2$$

(Constants)

$$\frac{\vec{a}\cdot\vec{a} + 2\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b}}{4} - D^2 = \vec{a}\cdot\vec{b}$$

(from eq. (Δ))

(Solve for D)

$x^2 + ax + d = 0$

$x^2 + (b+c)x + k = 0$

these are equivalent for all x

their coefficients must be equal.

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12. Find equation for surface consisting of pts $P(x,y,z)$ s.t. distance from P to Q is equal to distance from P to the plane $y=5$.

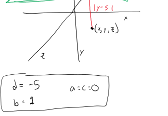
$Q = (x_Q, y_Q, z_Q)$

distance from P to Q?

$$\sqrt{(x-x_Q)^2 + (y-y_Q)^2 + (z-z_Q)^2}$$

distance from P to plane $y=5$?

$|y-5|$



$d = -5$ $a = c = 0$
$b = 1$

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15. Find a vector $\vec{v}$ orthogonal to plane through points P, Q, R

Cross product!

- Find two vectors lying in this plane
- take their cross product.

$\vec{PQ} = Q - P$

$\vec{QR} = R - Q$