427L: Practice problems for 1/27/22

1. Find a unit vector $\vec{v}$ such that the angle between $\vec{v}$ and $\vec{u} = \langle 1, 4 \rangle$ is $\pi/6$. How many such unit vectors are there?

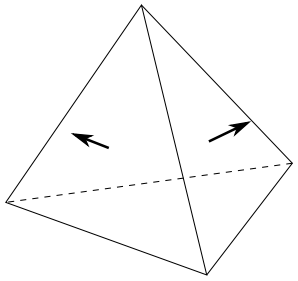
2. Find a unit vector $\vec{v}$ such that the angle between $\vec{v}$ and $\vec{u} = \langle -1, 2, 1 \rangle$ is $\pi/6$. What does the set of such unit vectors look like?

3. Let $P = (1, -2, 0)$ and let $Q = (-3, 6, 8)$, and let A be the set of points X in $\mathbb{R}^3$ such that the angle between $\vec{PQ}$ and $\vec{PX}$ is $\pi/6$.

   Describe what this set looks like, and write down an equation (or equations) describing this set in Cartesian coordinates.

Challenge question

A *regular tetrahedron* is a 4-sided solid, each of whose faces is an equilateral triangle. (That is, it is a triangular pyramid all of whose edges have the same length). You might be familiar with it as the shape of a 4-sided die.



Each face of the tetrahedron has a *normal vector*, perpendicular to that face and pointing outside the tetrahedron. Find the angle between the normal vectors of two faces of the tetrahedron.